CatalyseR

Catalysing Success in JEE

JEE Advanced 2026

Paper 2

Complete Worked Solutions

Mathematics ◇ **Physics** ◇ **Chemistry**

54 fully solved questions — step-by-step reasoning, key insights, common pitfalls and final answers.

Three Sections per Subject

Single Correct · One or More Correct · Numerical Value

A CatalyseR Solutions Booklet • For JEE Advanced Aspirants

This booklet reproduces each question and works through a complete solution. Answers are derived independently; students should cross-check with the official answer key released by the IITs.

Contents

How to use this booklet

Each solution is laid out in the same way, so you always know where to look:

- The shaded box reproduces the **original question** exactly as it appeared in the paper.
- The gold **"How to think about it"** box gives the plan of attack in plain language.
- Numbered **Steps** carry out the working in full.
- Teal **Key point** and red **Common mistake** boxes appear where they genuinely help.
- The green **Final Answer** box states the result.

Difficulty & marks at a glance

The paper has four sections per subject. Marking follows the official JEE Advanced scheme used in Paper 2:

Section	Question type	Questions	Full marks each
Section 1	Single correct option	Q1–Q4	+3
Section 2	One or more correct options	Q5–Q9	+4
Section 3	Numerical value	Q10–Q14	+4
Section 4	Numerical value (paragraph)	Q15–Q18	+2

Across all three subjects (54 questions), the spread of difficulty in this booklet is roughly:

Difficulty	Mathematics	Physics	Chemistry
Easy	2	1	2
Medium	7	9	9
Hard	9	8	7

A complete answer key for all 54 questions is provided on the last page.

Mathematics

Q.1 *Section 1 — Single correct option*

Topic: Vectors and Cross Product • Difficulty: Easy • Marks: +3

Let $\vec{a}, \vec{b}$ be two vectors, and let P, Q and R be the points with position vectors $\vec{a}$, $\vec{b}$ and $\vec{a}+\vec{b}$, respectively, with respect to the origin O. If $|\vec{a}+\vec{b}| = \sqrt{21}$, $|\vec{a}-\vec{b}| = 3$, and $\vec{a}$ and $(\vec{a}-\vec{b})$ are perpendicular to each other, then the area of the triangle OPR is

(A) $\sqrt{3}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{3\sqrt{3}}{2}$ (D) $\frac{3}{2}$

How to think about it: Triangle OPR has vertices at O, $\vec{a}$ and $\vec{a}+\vec{b}$. Its area is $\frac{1}{2}$ the magnitude of the cross product of the two edge vectors from O. Reduce that to $\frac{1}{2}|\vec{a}\times\vec{b}|$ and find $|\vec{a}\times\vec{b}|$ from the three given conditions.

Step 1: Express the area as a cross product.

The edges from O are $\overrightarrow{OP} = \vec{a}$ and $\overrightarrow{OR} = \vec{a}+\vec{b}$. Hence

$$\text{Area} = \tfrac{1}{2}\left|\vec{a}\times(\vec{a}+\vec{b})\right| = \tfrac{1}{2}\left|\vec{a}\times\vec{a}+\vec{a}\times\vec{b}\right| = \tfrac{1}{2}\left|\vec{a}\times\vec{b}\right|,$$

since $\vec{a}\times\vec{a} = \vec{0}$.

Step 2: Convert the given magnitudes into dot products.

From $|\vec{a}+\vec{b}|^2 = 21$ and $|\vec{a}-\vec{b}|^2 = 9$:

$$|\vec{a}|^2 + |\vec{b}|^2 + 2\,\vec{a}\cdot\vec{b} = 21, \qquad |\vec{a}|^2 + |\vec{b}|^2 - 2\,\vec{a}\cdot\vec{b} = 9.$$

Subtracting gives $4\,\vec{a}\cdot\vec{b} = 12$, so $\vec{a}\cdot\vec{b} = 3$. Adding gives $|\vec{a}|^2 + |\vec{b}|^2 = 15$.

Step 3: Use the perpendicularity condition.

Since $\vec{a}\perp(\vec{a}-\vec{b})$,

$$\vec{a}\cdot(\vec{a}-\vec{b}) = 0 \;\Rightarrow\; |\vec{a}|^2 - \vec{a}\cdot\vec{b} = 0 \;\Rightarrow\; |\vec{a}|^2 = \vec{a}\cdot\vec{b} = 3.$$

Then $|\vec{b}|^2 = 15 - 3 = 12$.

Step 4: Compute $|\vec{a}\times\vec{b}|$.

$$|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2 = 3\cdot 12 - 3^2 = 27,$$

so $|\vec{a}\times\vec{b}| = 3\sqrt{3}$ and the area $= \frac{1}{2}(3\sqrt{3}) = \dfrac{3\sqrt{3}}{2}$.

▸ **Key point:** The identity $|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2$ (Lagrange's identity) converts dot-product data straight into a cross-product magnitude.

Final Answer: (C) $\dfrac{3\sqrt{3}}{2}$

Q.2 *Section 1 — Single correct option*

Topic: Parabola and Tangents • Difficulty: Medium • Marks: +3

Let T be the tangent to the parabola $y^2 = 16x$ at the point $(64, 32)$. Let L be the tangent to the same parabola at another point (x_1, y_1) on the parabola. If L and T are perpendicular to each other, then the distance between the point (x_1, y_1) and the focus of the parabola, is

(A) $\frac{15}{4}$ (B) 4 (C) $\frac{17}{4}$ (D) 5

How to think about it: Two tangents to a parabola are perpendicular only when the product of their slopes is -1. Find the slope of T, deduce the slope of L, locate (x_1, y_1), then use the focal-distance property distance to focus $= x_1 + a$.

Step 1: Identify the parabola.

$y^2 = 16x$ means $4a = 16$, so $a = 4$ and the focus is $(4, 0)$.

Step 2: Slope of tangent T.

Differentiating $y^2 = 16x$: $2y\,y' = 16$, so $y' = \frac{8}{y}$. At $(64, 32)$ the slope is $\frac{8}{32} = \frac{1}{4}$.

Step 3: Slope of L and the point of tangency.

For $L \perp T$, slope of $L = -4$. At (x_1, y_1) the slope is $\frac{8}{y_1} = -4$, so $y_1 = -2$. From $y_1^2 = 16x_1$, $x_1 = \frac{(-2)^2}{16} = \frac{1}{4}$. Thus $(x_1, y_1) = \left(\frac{1}{4}, -2\right)$.

Step 4: Distance to the focus.

By the focal-distance property of a parabola, the distance from a point on $y^2 = 4ax$ to the focus equals $x_1 + a$:

$$\text{distance} = x_1 + a = \tfrac{1}{4} + 4 = \frac{17}{4}.$$

△ **Common mistake:** Computing the straight-line distance from $\left(\frac{1}{4}, -2\right)$ to $(4, 0)$ by the distance formula also works, but it is slower and error-prone. The focal-distance property $x_1 + a$ is the quick route.

Final Answer: (C) $\frac{17}{4}$

Q.3 *Section 1 — Single correct option*

Topic: Differential Equations • Difficulty: Medium • Marks: +3

Let $y : (-\infty, \infty) \to (0, \infty)$ be the solution of the differential equation

$$\frac{dy}{dx} = \frac{e^{5x}y^3 + y^3}{e^x + e^x y^4},$$

satisfying $y(0) = \frac{1}{\sqrt{2}}$. Then the value of $y(\log_e 2)$ is

(A) $\sqrt{\frac{5+\sqrt{35}}{2}}$ (B) $\sqrt{\frac{7+\sqrt{53}}{2}}$ (C) $\frac{7+\sqrt{53}}{2}$ (D) $\frac{5+\sqrt{35}}{2}$

How to think about it: The right-hand side factorises so the equation separates: all the y's on one side, all the x's on the other. Integrate, fix the constant with the initial condition, then substitute $x = \log_e 2$.

Step 1: Separate the variables.

Factor numerator and denominator:

$$\frac{dy}{dx} = \frac{y^3\,(e^{5x}+1)}{e^x\,(1+y^4)} \;\Rightarrow\; \frac{1+y^4}{y^3}\,dy = \left(e^{4x}+e^{-x}\right)dx.$$

Step 2: Integrate both sides.

$$\int \left(y^{-3}+y\right)dy = \int \left(e^{4x}+e^{-x}\right)dx \;\Rightarrow\; \frac{y^2}{2} - \frac{1}{2y^2} = \frac{e^{4x}}{4} - e^{-x} + C.$$

Step 3: Determine the constant.

At $x=0$, $y^2 = \frac{1}{2}$ so $\frac{y^2}{2} - \frac{1}{2y^2} = \frac{1}{4} - 1 = -\frac{3}{4}$. The right side at $x = 0$ is $\frac{1}{4} - 1 + C = -\frac{3}{4} + C$. Hence $C = 0$.

Step 4: Evaluate at $x = \log_e 2$.

Here $e^{4x} = 2^4 = 16$ and $e^{-x} = \frac{1}{2}$, so the right side is $\frac{16}{4} - \frac{1}{2} = \frac{7}{2}$. Therefore

$$\frac{y^2}{2} - \frac{1}{2y^2} = \frac{7}{2} \Rightarrow y^2 - \frac{1}{y^2} = 7.$$

Putting $u = y^2$: $u^2 - 7u - 1 = 0$, so $u = \frac{7 \pm \sqrt{53}}{2}$. Since $y \in (0,\infty)$ we need $u = y^2 > 0$, forcing $u = \frac{7+\sqrt{53}}{2}$. Hence $y = \sqrt{\frac{7+\sqrt{53}}{2}}$.

▶ **Key point:** The codomain $(0,\infty)$ is doing real work: it rejects the negative root $\frac{7-\sqrt{53}}{2}$ of the quadratic in $u = y^2$.

Final Answer: (B) $\sqrt{\frac{7+\sqrt{53}}{2}}$

Q.4 *Section 1 — Single correct option*

Topic: Definite Integration • Difficulty: Medium • Marks: +3

The value of the definite integral

$$\int_0^2 \frac{1}{3^x+3}\,dx$$

is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{\log_e 3}{3}$ (D) $\frac{\log_e 3}{2}$

How to think about it: A direct antiderivative is messy. Instead use the king's-rule substitution $x \to (a+b)-x$ over the limits $[0,2]$, add the two forms of the integral, and watch the integrand collapse to a constant.

Step 1: Write the integral and its reflected form.

Let $I = \int_0^2 \frac{dx}{3^x+3}$. Replacing x by $2-x$:

$$I = \int_0^2 \frac{dx}{3^{\,2-x}+3}.$$

Step 2: Simplify the reflected integrand.

Since $3^{\,2-x} = \frac{9}{3^x}$,

$$\frac{1}{3^{\,2-x}+3} = \frac{1}{\frac{9}{3^x}+3} = \frac{3^x}{9+3\cdot 3^x} = \frac{3^x}{3\,(3^x+3)}.$$

Step 3: Add the two expressions for I.

$$2I = \int_0^2 \left[\frac{1}{3^x+3} + \frac{3^x}{3\,(3^x+3)}\right] dx = \int_0^2 \frac{3+3^x}{3\,(3^x+3)}\, dx = \int_0^2 \frac{1}{3}\, dx = \frac{2}{3}.$$

Step 4: Solve for I.

$2I = \frac{2}{3} \Rightarrow I = \frac{1}{3}$.

▶ **Key point:** Whenever the limits are symmetric about their midpoint, the substitution $x \to (a+b)-x$ followed by adding the original integral is worth trying — it often turns an awkward integrand into a constant.

Final Answer: (B) $\frac{1}{3}$

Q.5 *Section 2 — One or more correct options*

Topic: Polynomials and Higher-Order Derivatives • Difficulty: Hard • Marks: +4

Let $\mathbb{R}$ denote the set of all real numbers. Consider the polynomial function $f:\mathbb{R}\to\mathbb{R}$ defined by

$$f(x) = \frac{d^{10}}{dx^{10}}\left((x^2-1)^{10}\right), \qquad \text{for all } x \in \mathbb{R}.$$

Here $\frac{d^{10}}{dx^{10}}\left((x^2-1)^{10}\right)$ is the 10^{th} order derivative of the function $(x^2-1)^{10}$. Then which of the following statements is(are) TRUE?

(A) The coefficient of x^8 in the polynomial $f(x)$ is $(-10)\left(\frac{18!}{8!}\right)$

(B) The value of $f(1)+f(-1)$ is equal to $10!\,2^{11}$

(C) The degree of the polynomial $f(x)$ is 10

(D) The constant term of the polynomial $f(x)$ is $-\left(\frac{10!}{5!}\right)$

How to think about it: Expand $(x^2-1)^{10}$ by the binomial theorem; it contains only even powers x^{2k}. The 10^{th} derivative kills every term with $2k < 10$ and shifts the rest down by 10 powers. Read off each requested coefficient term by term.

Step 1: Binomial expansion.

$$(x^2-1)^{10} = \sum_{k=0}^{10} \binom{10}{k} (-1)^{10-k} x^{2k}.$$

Differentiating x^{2k} ten times gives $\dfrac{(2k)!}{(2k-10)!}\, x^{2k-10}$ when $2k \ge 10$, and 0 otherwise.

Step 2: Degree — statement (C).

The highest power in $(x^2-1)^{10}$ is x^{20}; after ten differentiations it becomes x^{10}. So $\deg f = 10$. **(C) is TRUE.**

Step 3: Coefficient of x^8 — statement (A).

We need $2k-10=8$, i.e. $k=9$. The $k=9$ term of $(x^2-1)^{10}$ is $\binom{10}{9}(-1)^1x^{18} = -10\,x^{18}$. Its 10^{th} derivative is $-10\cdot\dfrac{18!}{8!}\,x^8$, so the coefficient of x^8 is $(-10)\dfrac{18!}{8!}$. **(A) is TRUE.**

Step 4: $f(1)+f(-1)$ — statement (B).

Write $(x^2-1)^{10} = (x-1)^{10}(x+1)^{10}$ and apply Leibniz's rule. At $x=1$ every derivative of $(x-1)^{10}$ vanishes except the 10^{th} (which equals $10!$), so only one Leibniz term survives:

$$f(1) = \binom{10}{10}\,[10!]\,[(x+1)^{10}]_{x=1} = 10!\cdot 2^{10}.$$

Similarly $f(-1) = 10!\cdot 2^{10}$, hence $f(1)+f(-1) = 2\cdot 10!\cdot 2^{10} = 10!\,2^{11}$. **(B) is TRUE.**

Step 5: Constant term — statement (D).

The constant term needs $2k-10=0$, i.e. $k=5$. The $k=5$ term is $\binom{10}{5}(-1)^5x^{10} = -\binom{10}{5}x^{10}$, whose 10^{th} derivative is $-\binom{10}{5}\cdot 10!$. Now $\binom{10}{5} = \dfrac{10!}{5!\,5!}$, so the constant term is $-\dfrac{(10!)^2}{5!\,5!}$, *not* $-\dfrac{10!}{5!}$. **(D) is FALSE.**

▶ **Key point:** $f(x) = \dfrac{d^{10}}{dx^{10}}(x^2-1)^{10}$ is, up to the constant $2^{10}\,10!$, the Legendre polynomial $P_{10}(x)$ (Rodrigues' formula). That is why $f(\pm 1)$ are so clean.

Final Answer: (A), (B) and (C)

Q.6 *Section 2 — One or more correct options*

Topic: Quadratic Equations and Arithmetic Progression • Difficulty: Medium • Marks: +4

Let a, b, c be positive integers in arithmetic progression such that the equation

$$ax^2+bx+c=0$$

has only integer solutions. Then which of the following statements is(are) TRUE?

(A) $c-b$ is an integer multiple of a
(B) Both the roots of the equation $ax^2+bx+c=0$ are odd integers
(C) If $c=15$, then $ab=8$

(D) If $b = 8$, then $x = 3$ is a root of the equation $ax^2 + bx + c = 0$

How to think about it: With $a, b, c > 0$ both roots are negative. Write the roots as $-p, -q$ (p, q positive integers), turn the AP condition $2b = a + c$ into a relation between p and q, and factor it. There is essentially only one pair (p, q).

Step 1: Set up the roots.

Since $a, b, c > 0$, the sum of roots $-b/a < 0$ and the product $c/a > 0$, so both roots are negative integers; write them as $-p, -q$ with $p, q \in \mathbb{Z}^+$. Then $p + q = b/a$ and $pq = c/a$, i.e. $b = a(p + q)$, $c = a\,pq$.

Step 2: Apply the AP condition.

a, b, c in AP means $2b = a + c$:

$$2a(p + q) = a + a\,pq \;\Rightarrow\; 2(p + q) = 1 + pq \;\Rightarrow\; pq - 2p - 2q + 4 = 3,$$

that is $(p - 2)(q - 2) = 3$.

Step 3: Solve for p, q.

With p, q positive integers, the only factor pair of 3 giving positive p, q is $\{1, 3\}$, so $\{p-2, q-2\} = \{1, 3\}$ and $\{p, q\} = \{3, 5\}$. Hence the roots are -3 and -5, and

$$b/a = p + q = 8, \qquad c/a = pq = 15 \;\Rightarrow\; b = 8a,\; c = 15a.$$

So $(a, b, c) = (a, 8a, 15a)$ for any positive integer a — indeed an AP with common difference $7a$.

Step 4: Test each statement.

(A) $c - b = 15a - 8a = 7a$, an integer multiple of a. **TRUE.**
(B) The roots are -3 and -5, both odd. **TRUE.**
(C) If $c = 15$ then $15a = 15 \Rightarrow a = 1$, $b = 8$, so $ab = 8$. **TRUE.**
(D) If $b = 8$ then $a = 1$, $c = 15$ and $x^2 + 8x + 15 = (x + 3)(x + 5) = 0$, whose roots are $-3, -5$. Thus $x = 3$ is *not* a root ($x = -3$ is). **FALSE.**

△ **Common mistake:** In (D) the trap is sign: the root is $x = -3$, not $x = 3$. Positive coefficients a, b, c always force *negative* roots.

Final Answer: (A), (B) and (C)

Q.7 *Section 2 — One or more correct options*

Topic: Three-Dimensional Geometry • Difficulty: Hard • Marks: +4

Let L be the straight line joining the points $P(1, 2, -1)$ and $Q(2, 3, 1)$. Let S be the foot of the perpendicular drawn from the point $R(4, -1, 5)$ to the line L. Another line passing through R intersects L at a point T such that the point S divides the line segment PT internally in the ratio $|PS| : |ST| = 1 : 2$, where $|PS|$ and $|ST|$ are the lengths of the line segments PS and ST, respectively. Then which of the following statements is(are) TRUE?

(A) The orthocentre of the triangle PRT is $\left(\frac{23}{5}, -4, \frac{31}{5}\right)$
(B) The orthocentre of the triangle PRT is $(4, 3, 5)$
(C) The area of the triangle PRT is $6\sqrt{5}$

(D) The area of the triangle PRT is $18\sqrt{5}$

How to think about it: Find S as the foot of perpendicular on L. Since S divides PT in $1:2$, $T = P + 3(S - P)$. With P, R, T known, the area comes from a cross product and the orthocentre from two "altitude" dot-product conditions plus the plane of the triangle.

Step 1: Foot of perpendicular S.

Direction of L is $\vec{d} = Q - P = (1,1,2)$. A general point of L is $P + t\vec{d} = (1+t,\, 2+t,\, -1+2t)$. Requiring $(S - R)\cdot\vec{d} = 0$:

$$(t-3) + (t+3) + 2(2t-6) = 0 \Rightarrow 6t - 12 = 0 \Rightarrow t = 2,$$

so $S = (3,4,3)$.

Step 2: Locate T.

S divides PT with $|PS| : |ST| = 1 : 2$, so S is one-third of the way from P to T and $T = P + 3(S-P)$. Here $S - P = (2,2,4)$, hence $T = (1,2,-1) + 3(2,2,4) = (7,8,11)$.

Step 3: Area of $\triangle PRT$.

With $\overrightarrow{PR} = (3,-3,6)$ and $\overrightarrow{PT} = (6,6,12)$,

$$\overrightarrow{PR}\times\overrightarrow{PT} = (-72,\, 0,\, 36), \qquad |\overrightarrow{PR}\times\overrightarrow{PT}| = \sqrt{72^2 + 36^2} = 36\sqrt{5}.$$

Area $= \frac{1}{2}(36\sqrt{5}) = 18\sqrt{5}$. So **(D) is TRUE, (C) is FALSE.**

Step 4: Orthocentre H.

H lies in the plane of $\triangle PRT$, whose normal is $\overrightarrow{PR}\times\overrightarrow{PT} \parallel (-2,0,1)$:

$$-2(x-1) + (z+1) = 0 \;\Rightarrow\; z = 2x - 3.$$

Altitude from $R \perp PT$: $(H - R)\cdot(1,1,2) = 0 \Rightarrow x + y + 2z = 13$.
Altitude from $P \perp RT$: $(H - P)\cdot(1,3,2) = 0 \Rightarrow x + 3y + 2z = 5$.
Subtracting gives $2y + 8 = 0$, so $y = -4$. Then $x + 2z = 17$ with $z = 2x - 3$ yields $5x = 23$, $x = \frac{23}{5}$, $z = \frac{31}{5}$. Hence $H = \left(\frac{23}{5}, -4, \frac{31}{5}\right)$. **(A) is TRUE, (B) is FALSE.**

▶ **Key point:** An orthocentre in 3D is pinned down by two perpendicularity (altitude) conditions *plus* the requirement that it lie in the triangle's own plane — three equations for three unknowns.

Final Answer: (A) and (D)

Q.8 *Section 2 — One or more correct options*

Topic: Differential Equations and Calculus • Difficulty: Medium • Marks: +4

Let $y = f(x)$ be the real valued function defined on the interval $(0,\infty)$, satisfying $y(1) = 0$ and the differential equation

$$x\frac{dy}{dx} = y - x^3.$$

Then which of the following statements is(are) TRUE?

(A) The function f has a local minimum at $x = \dfrac{1}{\sqrt{3}}$

(B) The function f has a local maximum at $x = \dfrac{1}{\sqrt{3}}$

(C) The function f is increasing in the interval $(1,2)$

(D) If $g(x) = 4x^3 - 5x^2 + \frac{3}{2}x$ for $x > 0$, then the number of elements in the set $\{x \in (0, \infty) : f(x) = g(x)\}$ is 2

How to think about it: The equation is linear once written as $y' - y/x = -x^2$. Solve it with an integrating factor, fix the constant from $y(1) = 0$, then analyse the explicit f for monotonicity and intersections with g.

Step 1: Solve the linear ODE.

Rewrite $\frac{dy}{dx} - \frac{y}{x} = -x^2$. The integrating factor is $e^{-\int dx/x} = 1/x$, so $\frac{d}{dx}\left(\frac{y}{x}\right) = -x$, giving $\frac{y}{x} = -\frac{x^2}{2} + C$. With $y(1) = 0$, $C = \frac{1}{2}$. Hence

$$f(x) = \frac{x - x^3}{2}.$$

Step 2: Critical points — statements (A), (B).

$f'(x) = \frac{1 - 3x^2}{2} = 0$ at $x = \frac{1}{\sqrt{3}}$ (taking $x > 0$), and $f''(x) = -3x < 0$ there. So f has a *local maximum* at $x = \frac{1}{\sqrt{3}}$. **(B) is TRUE, (A) is FALSE.**

Step 3: Monotonicity on $(1, 2)$ — statement (C).

For $x > \frac{1}{\sqrt{3}} \approx 0.577$, $f'(x) = \frac{1 - 3x^2}{2} < 0$. On $(1, 2)$ therefore f is *decreasing*. **(C) is FALSE.**

Step 4: Intersections with g — statement (D).

Set $f(x) = g(x)$:

$$\frac{x - x^3}{2} = 4x^3 - 5x^2 + \frac{3}{2}x \;\Rightarrow\; 9x^3 - 10x^2 + 2x = 0 \;\Rightarrow\; x\,(9x^2 - 10x + 2) = 0.$$

Discard $x = 0$ (not in $(0, \infty)$). The quadratic gives $x = \frac{5 \pm \sqrt{7}}{9}$; both roots are positive (≈ 0.262 and ≈ 0.850). So there are exactly 2 solutions in $(0, \infty)$. **(D) is TRUE.**

Final Answer: (B) and (D)

Q.9 *Section 2 — One or more correct options*

Topic: Matrices and Complex Numbers • Difficulty: Hard • Marks: +4

Let $\mathbb{R}$ denote the set of all real numbers and let $i = \sqrt{-1}$. Consider the matrices

$$S = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad \text{and} \quad T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

Let a, b, c, d be real numbers such that $ST = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Let $H = \{x + iy : \; x, y \in \mathbb{R} \text{ and } y > 0\}$. Then which of the following statements is(are) TRUE?

(A) $\frac{b + ia}{d + ic} = i$

(B) If $\omega = \frac{-1 + i\sqrt{3}}{2}$, then $\frac{a\omega + b}{c\omega + d} = \omega$

(C) If m is an integer greater than 2 such that $(ST)^2 = (ST)^m$, then m is an integer multiple of 8

(D) If $z \in H$, then $\dfrac{az+b}{cz+d} \in H$

How to think about it: First compute ST to get a, b, c, d. Then (A) is a direct complex division; (B) uses $1 + \omega + \omega^2 = 0$; (C) hinges on the order of the matrix ST; (D) is the upper-half-plane criterion for a Möbius map.

Step 1: Compute ST.

$$ST = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix},$$

so $a = 0,\ b = -1,\ c = 1,\ d = 1$.

Step 2: Statement (A).

$\dfrac{b+ia}{d+ic} = \dfrac{-1+0i}{1+i} = \dfrac{-1(1-i)}{(1+i)(1-i)} = \dfrac{-1+i}{2}$, which is not i. **(A) is FALSE.**

Step 3: Statement (B).

$a\omega + b = -1$ and $c\omega + d = \omega + 1$. Since $1 + \omega + \omega^2 = 0$, $\omega + 1 = -\omega^2$, so

$$\frac{a\omega+b}{c\omega+d} = \frac{-1}{-\omega^2} = \frac{1}{\omega^2} = \omega \quad (\text{using } \omega^3 = 1).$$

(B) is TRUE.

Step 4: Statement (C).

$M = ST$ has $\det M = 1$ and trace 1, so its characteristic equation is $\lambda^2 - \lambda + 1 = 0$, with roots $e^{\pm i\pi/3}$ — primitive 6^{th} roots of unity. Hence $M^6 = I$. The condition $(ST)^2 = (ST)^m$ gives $M^{m-2} = I$, i.e. $6 \mid (m-2)$, so $m = 8, 14, 20, \ldots$. Since $m = 14$ works but is not a multiple of 8, the statement fails. **(C) is FALSE.**

Step 5: Statement (D).

$\dfrac{az+b}{cz+d} = \dfrac{-1}{z+1}$ is a Möbius transformation with real entries and determinant $ad - bc = 1 > 0$. For such a map, $\operatorname{Im}\left(\dfrac{az+b}{cz+d}\right) = \dfrac{(ad-bc)\operatorname{Im} z}{|cz+d|^2} > 0$ whenever $\operatorname{Im} z > 0$. So it maps H into H. **(D) is TRUE.**

▶ **Key point:** A real Möbius map $z \mapsto \dfrac{az+b}{cz+d}$ preserves the upper half-plane exactly when $ad - bc > 0$.

Final Answer: (B) and (D)

Q.10 *Section 3 — Numerical value*

Topic: Functions and Combinatorics • Difficulty: Hard • Marks: +4

Let $\mathbb{N}$ denote the set of all positive integers. Consider the sets

$$A = \{1, 2, 3, 4, 5\} \quad \text{and} \quad B = \{1, 2, 3, 4, 5, 6, 7\}.$$

Let S be the set of all functions $f : A \to B$ such that $f(2) \neq 2$ and $f(4) \neq 4$. Consider the set

$$T = \{ f \in S : \text{there exists a function } g : B \to \mathbb{N} \text{ such that } g(f(x)) = 2^x \text{ for all } x \in A \}.$$

Then the number of elements in the set T is ________.

How to think about it: The hidden condition is injectivity: a well-defined g with $g(f(x)) = 2^x$ can exist only if f never sends two different inputs to the same output. So T is exactly the injective functions in S. Count them by inclusion–exclusion.

Step 1: Reduce T to "injective".

If $f(x_1) = f(x_2)$ then $g(f(x_1)) = g(f(x_2))$, forcing $2^{x_1} = 2^{x_2}$, i.e. $x_1 = x_2$. So f must be injective. Conversely, any injective f admits such a g (define g on the image by $g(f(x)) = 2^x$ and arbitrarily elsewhere). Hence $T = \{\text{injective } f : A \to B \text{ with } f(2) \neq 2,\ f(4) \neq 4\}$.

Step 2: Count injective functions $A \to B$.

With $|A| = 5$, $|B| = 7$, the total number of injective maps is $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 = 2520$.

Step 3: Inclusion–exclusion on the forbidden values.

Let X be the injective maps with $f(2) = 2$ and Y those with $f(4) = 4$.

$$|X| = 6 \cdot 5 \cdot 4 \cdot 3 = 360, \quad |Y| = 360, \quad |X \cap Y| = 5 \cdot 4 \cdot 3 = 60.$$

Step 4: Combine.

$$|T| = 2520 - |X| - |Y| + |X \cap Y| = 2520 - 360 - 360 + 60 = 1860.$$

► **Key point:** "A consistent g exists" is just a disguise for "f is one-to-one," because the targets 2^x are all distinct.

Final Answer: 1860

Q.11 *Section 3 — Numerical value*

Topic: Probability and Expectation • Difficulty: Medium • Marks: +4

A bookshelf contains 6 distinct books of Mathematics and 5 distinct books of Physics. From these 11 books, 6 books are chosen at random. Let X be the absolute value of the difference between the number of Mathematics books chosen and the number of Physics books chosen. If α is the mean of the random variable X, then the value of 77α is ________.

How to think about it: If m Mathematics books are chosen then $6 - m$ Physics books are, so $X = |2m - 6|$. Tabulate the probability of each m using $\binom{6}{m}\binom{5}{6-m}$, form $E[X]$, then multiply by 77.

Step 1: Set up the distribution.

Total ways to pick 6 of 11 books: $\binom{11}{6} = 462$. If m Mathematics books are chosen ($1 \leq m \leq 6$, since at most 5 Physics books exist), then $X = |m - (6 - m)| = |2m - 6|$.

Step 2: Count favourable selections for each m.

m	$X = \|2m-6\|$	$\binom{6}{m}\binom{5}{6-m}$
1	4	$6 \cdot 1 = 6$
2	2	$15 \cdot 5 = 75$
3	0	$20 \cdot 10 = 200$
4	2	$15 \cdot 10 = 150$
5	4	$6 \cdot 5 = 30$
6	6	$1 \cdot 1 = 1$

(The counts sum to $6 + 75 + 200 + 150 + 30 + 1 = 462$, as a check.)

Step 3: Expected value.

$$E[X] = \frac{1}{462}\left[4(6+30) + 2(75+150) + 0(200) + 6(1)\right] = \frac{144 + 450 + 6}{462} = \frac{600}{462} = \frac{100}{77}.$$

Step 4: Required value.

$\alpha = E[X] = \frac{100}{77}$, so $77\alpha = 100$.

Final Answer: 100

Q.12 *Section 3 — Numerical value*

Topic: Statistics — Mean and Variance • Difficulty: Easy • Marks: +4

Consider a data consisting of 10 observations $x_1, x_2, \ldots, x_{10}$, whose mean is 5 and variance is 7. If the mean and the variance of the first 8 observations $x_1, x_2, \ldots, x_8$ are 4 and 3.5, respectively, and $x_9 < x_{10}$, then the value of $3x_9 + 2x_{10}$ is ________.

How to think about it: Means give the sums; variances give the sums of squares. Subtract the "first 8" totals from the "all 10" totals to get $x_9 + x_{10}$ and $x_9^2 + x_{10}^2$, then solve the resulting quadratic.

Step 1: Use the means.

$\sum_1^{10} x_i = 10 \cdot 5 = 50$ and $\sum_1^{8} x_i = 8 \cdot 4 = 32$, so $x_9 + x_{10} = 18$.

Step 2: Use the variances.

Variance $= \overline{x^2} - (\bar{x})^2$ gives $\sum_1^{10} x_i^2 = 10(7 + 25) = 320$ and $\sum_1^{8} x_i^2 = 8(3.5 + 16) = 156$. Hence $x_9^2 + x_{10}^2 = 320 - 156 = 164$.

Step 3: Find x_9 and x_{10}.

From $(x_9 + x_{10})^2 = 324$, $2x_9x_{10} = 324 - 164 = 160$, so $x_9x_{10} = 80$. They are the roots of $t^2 - 18t + 80 = 0$, namely $t = 8$ and $t = 10$. With $x_9 < x_{10}$, $x_9 = 8$ and $x_{10} = 10$.

Step 4: Required value.

$3x_9 + 2x_{10} = 3(8) + 2(10) = 24 + 20 = 44$.

Final Answer: 44

Q.13 *Section 3 — Numerical value*

Topic: Conic Sections — Ellipse, Hyperbola, Parabola • Difficulty: Hard • Marks: +4

Consider the ellipse E given by $\frac{x^2}{18}+\frac{y^2}{12}=1$. Let H be the hyperbola whose eccentricity is the reciprocal of the eccentricity of E and whose foci are the same as that of E. Let P and Q be the points of intersection of H and the parabola $\sqrt{5}\,y=x^2$ in the first quadrant. Let d be the distance between P and Q. If a and b are the integers such that $d^2=a+b\sqrt{5}$, then the value of $a-b$ is _________.

How to think about it: Get the ellipse's eccentricity and foci, build the hyperbola from the reciprocal eccentricity and the same foci, intersect it with the parabola to find P and Q, then compute d^2 and read off a, b.

Step 1: Ellipse data.

For E: $a^2=18$, $b^2=12$, so $c^2=18-12=6$, $c=\sqrt{6}$. Eccentricity $e_E=\frac{\sqrt{6}}{\sqrt{18}}=\frac{1}{\sqrt{3}}$; foci $(\pm\sqrt{6},0)$.

Step 2: Build the hyperbola H.

$e_H=\frac{1}{e_E}=\sqrt{3}$ with the same foci, so $c_H=\sqrt{6}$ and $a_H=\frac{c_H}{e_H}=\sqrt{2}$, $a_H^2=2$, $b_H^2=c_H^2-a_H^2=4$. Thus $H:\frac{x^2}{2}-\frac{y^2}{4}=1$.

Step 3: Intersect H with the parabola $x^2=\sqrt{5}\,y$.

Substituting $x^2=\sqrt{5}\,y$ into H:

$$\frac{\sqrt{5}\,y}{2}-\frac{y^2}{4}=1 \;\Rightarrow\; y^2-2\sqrt{5}\,y+4=0 \;\Rightarrow\; y=\sqrt{5}\pm 1.$$

Both values are positive; with $x>0$ (first quadrant), $x^2=\sqrt{5}\,y=5\pm\sqrt{5}$.

Step 4: Compute d^2.

Take P with $y_P=\sqrt{5}+1$ and Q with $y_Q=\sqrt{5}-1$, so $y_P-y_Q=2$.

$$(x_P-x_Q)^2=x_P^2+x_Q^2-2x_Px_Q=10-2\sqrt{(5+\sqrt{5})(5-\sqrt{5})}=10-2\sqrt{20}=10-4\sqrt{5}.$$

Hence $d^2=(x_P-x_Q)^2+(y_P-y_Q)^2=(10-4\sqrt{5})+4=14-4\sqrt{5}$.

Step 5: Read off a, b.

$d^2=14-4\sqrt{5}=a+b\sqrt{5}$ gives $a=14$, $b=-4$, so $a-b=14-(-4)=18$.

Final Answer: 18

Q.14 *Section 3 — Numerical value*

Topic: Continuity of Functions • Difficulty: Hard • Marks: +4

For a real number α, let $[\alpha]$ denote the greatest integer less than or equal to α. For a finite set S, let $|S|$ denote the number of elements in the set S. Consider the functions $f:(-3,3)\to(-\infty,\infty)$ and $g:(-3,3)\to(-\infty,\infty)$ defined by

$$f(x)=[x^3]\,\log_e\big(1+\sin^2(\pi(x-[x]))\big)$$

and

$$g(x)=x^3\,\sin^2\big(\pi\log_e(1+x-[x])\big).$$

Let $A = \{x \in (-3,3) : f \text{ is discontinuous at } x\}$ and $B = \{x \in (-3,3) : g \text{ is discontinuous at } x\}$. Then the value of $|A| + 2|B| - |A \cap B|$ is ________.

How to think about it: Both functions mix a fractional part $\{x\} = x - [x]$ with a floor. Identify exactly where each "jumpy" factor is discontinuous, and where the other factor multiplies that jump by zero (which restores continuity).

Step 1: Continuity of f.

The factor $\log_e(1 + \sin^2(\pi\{x\}))$ is continuous everywhere: as $\{x\}$ runs over $[0,1)$, $\sin(\pi\{x\})$ tends to 0 at both ends, so this factor is continuous even across integers (where it equals 0). The factor $[x^3]$ jumps exactly where x^3 is an integer. The product is discontinuous where $[x^3]$ jumps *and* the log-factor is nonzero, i.e. where $x^3 \in \mathbb{Z}$ but $x \notin \mathbb{Z}$.

Step 2: Count $|A|$.

For $x \in (-3,3)$, $x^3 \in (-27, 27)$; the integers strictly inside are $-26, \ldots, 26$ — that is 53 values, each giving one $x = k^{1/3}$. Of these, x is itself an integer only when k is a perfect cube: $k \in \{-8, -1, 0, 1, 8\}$, i.e. 5 values. Hence $|A| = 53 - 5 = 48$.

Step 3: Continuity of g.

Here x^3 is continuous everywhere, while $\sin^2(\pi \log_e(1 + \{x\}))$ jumps at every integer (its left limit is $\sin^2(\pi \log_e 2) \neq 0$, its right limit is 0). So g is discontinuous at an integer n unless $n^3 = 0$. The integers in $(-3,3)$ are $-2, -1, 0, 1, 2$; the jump is killed only at $n = 0$. Hence $|B| = 4$.

Step 4: Combine.

A consists of non-integers, B consists of integers, so $A \cap B = \varnothing$ and $|A \cap B| = 0$. Therefore

$$|A| + 2|B| - |A \cap B| = 48 + 2(4) - 0 = 56.$$

△ **Common mistake:** It is tempting to count $x^3 \in \mathbb{Z}$ as 53 discontinuities of f. But at the 5 perfect-cube points x is an integer, the log-factor is 0, and f is actually *continuous* there.

Final Answer: 56

Q.15 & Q.16 *Section 4 — Question stem*

Question Stem for Question Nos. 15 and 16.

Consider the curve C_1 given by $y = e^{-x}$ for $x \in [0, 10\pi]$, and the curve C_2 given by $y = e^{-x}(\sin x + \cos x)$ for $x \in [0, 10\pi]$. Let n be the total number of points of intersection of the curves C_1 and C_2. Suppose that $\alpha_1, \alpha_2, \ldots, \alpha_n \in [0, 10\pi]$ are the x-coordinates of the points of intersection of the curves C_1 and C_2 such that $\alpha_1 < \alpha_2 < \cdots < \alpha_n$.

Q.15 *Section 4 — Numerical value*

Topic: Curves and Points of Intersection • Difficulty: Medium • Marks: +2

The value of n is ________.

How to think about it: Setting $C_1 = C_2$ cancels the always-positive factor e^{-x}, leaving a simple trigonometric equation. Count its solutions in $[0, 10\pi]$.

Step 1: Equation of intersection.

$e^{-x} = e^{-x}(\sin x + \cos x)$ and $e^{-x} \neq 0$ give $\sin x + \cos x = 1$, i.e. $\sqrt{2}\sin\left(x + \frac{\pi}{4}\right) = 1$, so $\sin\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$.

Step 2: Solve over $[0, 10\pi]$.

This gives $x + \frac{\pi}{4} = \frac{\pi}{4} + 2k\pi$ or $\frac{3\pi}{4} + 2k\pi$, hence

$$x = 2k\pi \qquad \text{or} \qquad x = \frac{\pi}{2} + 2k\pi.$$

In $[0, 10\pi]$: $x = 2k\pi$ gives $0, 2\pi, 4\pi, 6\pi, 8\pi, 10\pi$ — six values; $x = \frac{\pi}{2} + 2k\pi$ gives $\frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}, \frac{17\pi}{2}$ — five values (the next, $\frac{21\pi}{2} > 10\pi$, is excluded).

Step 3: Total.

$n = 6 + 5 = 11$.

Final Answer: $n = 11$

Q.16 *Section 4 — Numerical value*

Topic: Area Enclosed by Curves • Difficulty: Hard • Marks: +2

Let β be the area of the region enclosed between the curves C_1, C_2, and the lines $x = \alpha_1$ and $x = \alpha_4$. Then the value of

$$-\frac{1}{\pi}\log_e\left(\beta - 2\,e^{-\pi/2}\right)$$

is __________.

How to think about it: Between consecutive intersection points the sign of $C_1 - C_2$ is fixed, so split $[\alpha_1, \alpha_4]$ at α_2, α_3. A single neat antiderivative handles all three pieces.

Step 1: First four intersection abscissae.

From Q.15, $\alpha_1 = 0,\ \alpha_2 = \frac{\pi}{2},\ \alpha_3 = 2\pi,\ \alpha_4 = \frac{5\pi}{2}$.

Step 2: An antiderivative of $C_1 - C_2$.

$C_1 - C_2 = e^{-x}(1 - \sin x - \cos x)$. Using $\int e^{-x}\sin x\,dx = -\frac{1}{2}e^{-x}(\sin x + \cos x)$ and $\int e^{-x}\cos x\,dx = \frac{1}{2}e^{-x}(\sin x - \cos x)$,

$$\int e^{-x}(1 - \sin x - \cos x)\,dx = e^{-x}(\cos x - 1) \equiv F(x).$$

Step 3: Evaluate F at the breakpoints.

$F(0) = 0, \quad F(\frac{\pi}{2}) = -e^{-\pi/2}, \quad F(2\pi) = 0, \quad F(\frac{5\pi}{2}) = -e^{-5\pi/2}.$

Step 4: Assemble the area.

On $(0, \frac{\pi}{2})$ and $(2\pi, \frac{5\pi}{2})$, $C_2 > C_1$; on $(\frac{\pi}{2}, 2\pi)$, $C_1 > C_2$. Taking absolute values,

$$\beta = -\left[F(\tfrac{\pi}{2}) - F(0)\right] + \left[F(2\pi) - F(\tfrac{\pi}{2})\right] - \left[F(\tfrac{5\pi}{2}) - F(2\pi)\right] = 2e^{-\pi/2} + e^{-5\pi/2}.$$

Step 5: Final value.

$\beta - 2e^{-\pi/2} = e^{-5\pi/2}$, so $-\dfrac{1}{\pi}\log_e\!\left(e^{-5\pi/2}\right) = -\dfrac{1}{\pi}\left(-\dfrac{5\pi}{2}\right) = \dfrac{5}{2} = 2.5.$

Final Answer: 2.5

Q.17 & Q.18 *Section 4 — Question stem*

Question Stem for Question Nos. 17 and 18.

Consider the ellipses given by

$$x^2 + 4y^2 = 1 \qquad \text{and} \qquad 4x^2 + y^2 = 1.$$

Q.17 *Section 4 — Numerical value*

Topic: Conic Sections — Tangents and Angles • Difficulty: Medium • Marks: +2

Let P be the point in the first quadrant where the given ellipses intersect. If θ is the acute angle between the tangents to the given ellipses at the point P, then the value of $4\tan\theta$ is _________.

How to think about it: Find the first-quadrant intersection, get each tangent slope by implicit differentiation, and use the angle-between-lines formula.

Step 1: Intersection point P.

Subtracting the ellipse equations: $(x^2 + 4y^2) - (4x^2 + y^2) = 0 \Rightarrow 3y^2 = 3x^2 \Rightarrow x = y$ in the first quadrant. Then $x^2 + 4x^2 = 1$ gives $x = \dfrac{1}{\sqrt{5}}$, so $P = \left(\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$.

Step 2: Tangent slopes at P.

For $x^2 + 4y^2 = 1$: $y' = -\dfrac{x}{4y}$, giving slope $m_1 = -\dfrac{1}{4}$ at P.

For $4x^2 + y^2 = 1$: $y' = -\dfrac{4x}{y}$, giving slope $m_2 = -4$ at P.

Step 3: Angle between the tangents.

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{-\frac{1}{4} + 4}{1 + 1}\right| = \frac{15/4}{2} = \frac{15}{8}.$$

Step 4: Required value.

$4\tan\theta = 4 \cdot \dfrac{15}{8} = \dfrac{15}{2} = 7.5.$

Final Answer: 7.5

Q.18 *Section 4 — Numerical value*

Topic: Area of the Common Region of Two Ellipses • Difficulty: Hard • Marks: +2

If α is the area of the common region that lies inside both the given ellipses, then the value of $\cot\alpha$ is _________.

How to think about it: The two ellipses are reflections of each other across $y = x$, so the common region has fourfold symmetry. Compute the area of one symmetric slice; the integral collapses beautifully to an inverse sine.

Step 1: Use the symmetry.

The ellipses meet at $\left(\pm\frac{1}{\sqrt{5}}, \pm\frac{1}{\sqrt{5}}\right)$. In the first quadrant, above the line $y = x$ the binding boundary is $x^2 + 4y^2 = 1$; below it, $4x^2 + y^2 = 1$. By the $x \leftrightarrow y$ symmetry the two halves have equal area, so the first-quadrant common area is twice the part above $y = x$.

Step 2: Area of the slice above $y = x$.

For $0 \le x \le \frac{1}{\sqrt{5}}$, y runs from x up to $\frac{1}{2}\sqrt{1 - x^2}$:

$$\text{Area}_A = \int_0^{1/\sqrt{5}} \left(\frac{\sqrt{1 - x^2}}{2} - x \right) dx = \frac{1}{4}\left[\frac{2}{5} + \arcsin\frac{1}{\sqrt{5}}\right] - \frac{1}{10} = \frac{1}{4}\arcsin\frac{1}{\sqrt{5}}.$$

Step 3: Total common area.

First-quadrant common area $= 2\,\text{Area}_A = \frac{1}{2}\arcsin\frac{1}{\sqrt{5}}$, so

$$\alpha = 4 \cdot \frac{1}{2}\arcsin\frac{1}{\sqrt{5}} = 2\arcsin\frac{1}{\sqrt{5}}.$$

Step 4: Evaluate $\cot\alpha$.

Let $\phi = \arcsin\frac{1}{\sqrt{5}}$, so $\sin\phi = \frac{1}{\sqrt{5}}$, $\cos\phi = \frac{2}{\sqrt{5}}$. Then $\sin\alpha = \sin 2\phi = \frac{4}{5}$, $\cos\alpha = \cos 2\phi = \frac{3}{5}$, hence $\cot\alpha = \frac{3/5}{4/5} = \frac{3}{4} = 0.75$.

▶ **Key point:** The constant terms in Step 2 cancel exactly — the $\frac{1}{4} \cdot \frac{2}{5}$ piece and the $\frac{1}{10}$ piece — which is what makes the area a clean inverse sine.

Final Answer: $\cot\alpha = 0.75$

Physics

Q.1 *Section 1 — Single correct option*

Topic: Current Electricity — Drift Velocity • Difficulty: Medium • Marks: +3

A metal wire of cross-sectional area $0.5\,\text{mm}^2$ and length $100\,\text{m}$ is connected across a battery of e.m.f. $2\,\text{V}$ and internal resistance $1\,\Omega$. The density, atomic mass and electrical conductivity of the metal are $6.35 \times 10^3\,\text{kg m}^{-3}$, $63.5\,\text{gm/mole}$ and $2 \times 10^8\,\text{mho m}^{-1}$, respectively. Assuming one conduction electron per atom of the metal, the drift velocity (in mm s^{-1}) of the electrons in the wire is: [Take Avogadro's number as 6×10^{23} and charge of the electron as $1.6 \times 10^{-19}\,\text{C}$.]

(A) 0.052 (B) 0.104 (C) 0.208 (D) 0.156

How to think about it: Drift velocity is $v_d = I/(nAe)$. So three numbers are needed: the electron density n (from density, atomic mass, Avogadro's number), and the current I (from the e.m.f. and the total resistance, where the wire's resistance comes from its conductivity).

Step 1: Electron number density n.

$$n = \frac{\text{density}}{\text{atomic mass}} \times N_A = \frac{6.35 \times 10^3}{63.5 \times 10^{-3}} \times 6 \times 10^{23} = 10^5 \times 6 \times 10^{23} = 6 \times 10^{28}\ \text{m}^{-3}.$$

Step 2: Resistance of the wire and the current.

$R_{\text{wire}} = \dfrac{L}{\sigma A} = \dfrac{100}{(2 \times 10^8)(0.5 \times 10^{-6})} = 1\,\Omega$. Total resistance $= R_{\text{wire}} + r = 1 + 1 = 2\,\Omega$, so $I = \dfrac{2\,\text{V}}{2\,\Omega} = 1\,\text{A}$.

Step 3: Drift velocity.

$$v_d = \frac{I}{nAe} = \frac{1}{(6 \times 10^{28})(0.5 \times 10^{-6})(1.6 \times 10^{-19})} = \frac{1}{4800}\ \text{m s}^{-1} \approx 0.208\ \text{mm s}^{-1}.$$

▶ **Key point:** The wire's own resistance is not given directly — it must be built from $R = L/(\sigma A)$ before the current can be found.

Final Answer: (C) 0.208

Q.2 *Section 1 — Single correct option*

Topic: Modern Physics — Radioactive Accumulation • Difficulty: Medium • Marks: +3

A nuclear reactor starts producing a radioactive nuclide X from $t = 0$, at a constant rate of α per second. Each decay of X produces energy E_0, which is utilized to heat a liquid of mass m and specific heat s. Assuming no heat loss from the liquid and taking λ as the decay constant of X, the rate of increase in the temperature of the liquid is:

(A) $\dfrac{\alpha E_0}{m\,s}\left(1 - e^{-\lambda t}\right)$ (B) $\dfrac{\alpha E_0}{m\,s}\left(e^{\lambda t} - 1\right)$ (C) $\dfrac{\lambda E_0}{m\,s}\left(1 - e^{-\lambda t}\right)$ (D) $\dfrac{E_0}{m\,s}\left(\alpha - \lambda e^{-\lambda t}\right)$

How to think about it: Production at a constant rate with simultaneous decay is the standard "accumulation" problem: $dN/dt = \alpha - \lambda N$. The heating power is E_0 times the *decay* rate λN.

Step 1: Number of nuclei present.

$\dfrac{dN}{dt} = \alpha - \lambda N$ with $N(0) = 0$ gives $N(t) = \dfrac{\alpha}{\lambda}\left(1 - e^{-\lambda t}\right)$.

Step 2: Rate of energy release.

The decays per second is $\lambda N = \alpha\left(1 - e^{-\lambda t}\right)$, so the power delivered to the liquid is $P = E_0\,\lambda N = \alpha E_0\left(1 - e^{-\lambda t}\right)$.

Step 3: Rate of temperature rise.

From $P = m\,s\,\dfrac{dT}{dt}$,

$$\frac{dT}{dt} = \frac{P}{m\,s} = \frac{\alpha E_0}{m\,s}\left(1 - e^{-\lambda t}\right).$$

△ **Common mistake:** The heating comes from *decays*, not from production. Using the production rate α as the heating rate (giving a constant $\alpha E_0/ms$) ignores that early on very few nuclei have yet been made to decay.

Final Answer: (A) $\dfrac{\alpha E_0}{m\,s}\left(1 - e^{-\lambda t}\right)$

Q.3 *Section 1 — Single correct option*

Topic: Ray Optics — Thin Prism and Dispersion • Difficulty: Medium • Marks: +3

A beam of polychromatic light passes through a thin prism of prism angle 6°. The refractive index of the material of the prism varies with wavelength (λ) as $n(\lambda) = \alpha\lambda + \dfrac{\beta}{\lambda^2}$, where $\alpha = 3\,\mu\text{m}^{-1}$ and $\beta = 0.096\,\mu\text{m}^2$. If λ_{min} is the wavelength at which the angle of minimum deviation D_m is smallest, then the correct value of D_m at λ_{min} is

(A) 6.4° (B) 4.8° (C) 3.2° (D) 2.4°

How to think about it: For a thin prism the minimum deviation is $D_m = (n-1)A$, so the smallest D_m occurs at the smallest n. Minimise $n(\lambda)$ by calculus, find that wavelength, then evaluate D_m.

Step 1: Minimise $n(\lambda)$.

$\dfrac{dn}{d\lambda} = \alpha - \dfrac{2\beta}{\lambda^3} = 0 \Rightarrow \lambda^3 = \dfrac{2\beta}{\alpha} = \dfrac{2(0.096)}{3} = 0.064$, so $\lambda_{\text{min}} = 0.4\,\mu\text{m}$.

Step 2: Refractive index at λ_{min}.

$$n = \alpha\lambda_{\text{min}} + \frac{\beta}{\lambda_{\text{min}}^2} = 3(0.4) + \frac{0.096}{0.16} = 1.2 + 0.6 = 1.8.$$

Step 3: Minimum deviation.

For a thin prism, $D_m = (n-1)A = (1.8-1)\times 6° = 4.8°$.

Final Answer: (B) 4.8°

Q.4 *Section 1 — Single correct option*

Topic: Gravitation — Radial Oscillations of an Orbit • Difficulty: Hard • Marks: +3

A particle of mass m, and angular momentum ℓ is moving in a circular orbit of radius r_0 under the influence of an attractive force $\vec{F}(r) = -\frac{k}{r^2}\hat{r}$. Keeping its angular momentum unchanged, the particle is displaced radially by a small distance $\delta r \ll r_0$, due to which its radial distance varies periodically. The corresponding time period is:

(A) $\frac{2\pi\ell^3}{mk^2}$ (B) $2\pi\sqrt{\frac{m}{k}}$ (C) $\frac{2\pi\ell^3}{3mk^2}$ (D) $\frac{2\pi\ell^3}{5mk^2}$

How to think about it: Reduce the orbital motion to one dimension using the effective potential $V_{\text{eff}}(r) = U(r) + \frac{\ell^2}{2mr^2}$. Small radial displacements oscillate with $\omega^2 = V''_{\text{eff}}(r_0)/m$.

Step 1: Effective potential.

With $\vec{F} = -\frac{k}{r^2}\hat{r}$, $U(r) = -\frac{k}{r}$, so $V_{\text{eff}}(r) = -\frac{k}{r} + \frac{\ell^2}{2mr^2}$.

Step 2: Circular-orbit condition.

$V'_{\text{eff}}(r_0) = 0$ gives $\frac{k}{r_0^2} = \frac{\ell^2}{mr_0^3}$, hence $r_0 = \frac{\ell^2}{mk}$ and $\frac{\ell^2}{m} = kr_0$.

Step 3: Restoring constant.

$V''_{\text{eff}}(r) = -\frac{2k}{r^3} + \frac{3\ell^2}{mr^4}$. At r_0, using $\frac{\ell^2}{m} = kr_0$,

$$V''_{\text{eff}}(r_0) = -\frac{2k}{r_0^3} + \frac{3k}{r_0^3} = \frac{k}{r_0^3}.$$

Step 4: Period of radial oscillation.

$\omega^2 = \frac{V''_{\text{eff}}(r_0)}{m} = \frac{k}{mr_0^3}$. Substituting $r_0 = \frac{\ell^2}{mk}$ gives $\omega^2 = \frac{m^2k^4}{\ell^6}$, so

$$T = \frac{2\pi}{\omega} = \frac{2\pi\ell^3}{mk^2}.$$

▶ **Key point:** The radial period equals the orbital period here only because the force is inverse-square; the effective-potential method shows this cleanly.

Final Answer: (A) $\frac{2\pi\ell^3}{mk^2}$

Q.5 *Section 2 — One or more correct options*

Topic: Ray Optics — Prisms and Minimum Deviation • Difficulty: Hard • Marks: +4

Consider two isosceles prisms 1 and 2 with prism angles A_1 and A_2 and refractive indices n_1 and n_2, respectively, as shown in the figure. The faces a_1b_1 and a_2b_2 are parallel to each other and perpendicular to the mirror M. If a ray of light is incident on the face a_1c_1 and emerges from the face a_2c_2, then the correct statement(s) is/are:

(A) If both the prisms are at minimum deviation condition, then $\frac{n_2}{n_1} = \sin\left(\frac{A_1}{2}\right) / \sin\left(\frac{A_2}{2}\right)$.

(B) If prism 2 is at minimum deviation condition, then $\sin i_1 = n_2 \sin\left(\frac{A_2}{2}\right)$ is always true.

(C) If both the prisms 1 and 2 are thin and are at minimum deviation condition with angles of deviation δ_{m1} and δ_{m2}, respectively, then $\theta = \frac{\delta_{m1}}{2(n_1-1)} + \frac{\delta_{m2}}{2(n_2-1)}$.

(D) If prism 1 is at minimum deviation condition, then $\sin i_2 = n_1 \sin\left(\frac{A_1}{2}\right)$ is always true.

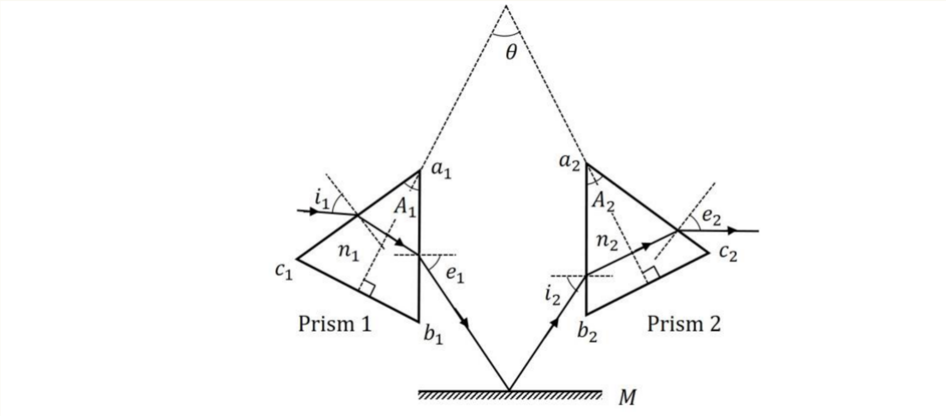

Figure from the original question paper.

How to think about it: The decisive geometric fact: the ray leaves face a_1b_1, reflects off the horizontal mirror, and enters face a_2b_2. Because $a_1b_1 \parallel a_2b_2$ (both vertical) and the mirror is horizontal, the emergence angle at a_1b_1 equals the incidence angle at a_2b_2: $e_1 = i_2$.

Step 1: Establish $e_1 = i_2$.

A ray leaving a_1b_1 at angle e_1 to its (horizontal) normal travels down, reflects off the horizontal mirror — which simply flips the vertical component — and meets the parallel face a_2b_2 at the same angle. Hence $i_2 = e_1$.

Step 2: Statement (D).

If prism 1 is at minimum deviation, the internal ray makes $A_1/2$ with the normal at the exit face a_1b_1, so Snell's law there gives $\sin e_1 = n_1 \sin(A_1/2)$. With $e_1 = i_2$ this is exactly $\sin i_2 = n_1 \sin(A_1/2)$. **(D) is TRUE.**

Step 3: Statement (A).

If *both* prisms are at minimum deviation, then additionally $\sin i_2 = n_2 \sin(A_2/2)$ at the entry face a_2b_2. Combining with Step 2, $n_1 \sin(A_1/2) = n_2 \sin(A_2/2)$, i.e. $\frac{n_2}{n_1} = \frac{\sin(A_1/2)}{\sin(A_2/2)}$. **(A) is TRUE.**

Step 4: Statement (B).

If only prism 2 is at minimum deviation, the entry-face relation is $\sin i_2 = n_2 \sin(A_2/2)$, and since $i_2 = e_1$ this gives $\sin e_1 = n_2 \sin(A_2/2)$ — it involves e_1, *not* the incidence angle i_1 on prism 1. So (B) replaces e_1 by i_1 incorrectly. **(B) is FALSE.**

Step 5: Statement (C).

For thin prisms, $\delta_m = (n-1)A$, so $A_1 = \frac{\delta_{m1}}{n_1 - 1}$ and $A_2 = \frac{\delta_{m2}}{n_2 - 1}$. From the geometry the angle θ between the outer faces equals $A_1 + A_2$, hence $\theta = \frac{\delta_{m1}}{n_1-1} + \frac{\delta_{m2}}{n_2-1}$. Statement (C) carries an extra factor of $\frac{1}{2}$ in each denominator. **(C) is FALSE.**

Final Answer: (A) and (D)

Q.6 *Section 2 — One or more correct options*

Topic: Charged Particle in Electric and Magnetic Fields • Difficulty: Hard • Marks: +4

In a vacuum chamber, a particle of charge $1\,\mu$C and mass 1 mg is projected with a velocity $(\hat{\imath} + 2\hat{\jmath})\,\text{ms}^{-1}$ from the XZ plane at time $t = 0$ in an electric field of $1\hat{\imath}\,\text{Vm}^{-1}$. At $t = 0.2$ s, the electric field is switched off and a magnetic field of $6\hat{\jmath}$ T is switched on. The acceleration due to gravity is $-10\hat{\jmath}\,\text{ms}^{-2}$. Correct option(s) is/are:

(A) The vertical distance of the particle from the XZ plane at $t = 0.3$ s is 15 cm.
(B) The vertical distance of the particle from the XZ plane at $t = 0.4$ s is 10 cm.
(C) The radius of the trajectory of the particle for $t > 0.2$ s is 20 cm.
(D) The particle will be in the XZ plane at $t = 0.35$ s.

How to think about it: Treat the motion in two phases. Phase 1 (0 to 0.2 s): constant acceleration from the electric force and gravity. Phase 2 ($t > 0.2$ s): gravity along $\hat{\jmath}$ plus a magnetic field along $\hat{\jmath}$. Since the magnetic force has no $\hat{\jmath}$-component, the y-motion is pure free-fall while the x–z motion is circular.

Step 1: Phase 1 accelerations ($0 \le t \le 0.2$ s).

$a_E = \dfrac{qE}{m} = \dfrac{(10^{-6})(1)}{10^{-6}} = 1\,\text{ms}^{-2}$ along x; gravity gives $-10\,\text{ms}^{-2}$ along y. With $\vec{v}_0 = (1, 2, 0)$ and start on the XZ plane ($y = 0$):

$$v_x = 1 + t, \quad v_y = 2 - 10t, \qquad y = 2t - 5t^2.$$

At $t = 0.2$ s: $\vec{v} = (1.2,\ 0,\ 0)$ and $y = 0.4 - 0.2 = 0.2$ m.

Step 2: Phase 2 — the y-motion.

For $t > 0.2$ s the magnetic force ($\vec{v} \times \hat{\jmath}$) has no y-component, so y obeys free-fall from $y = 0.2$, $v_y = 0$. Writing $\tau = t - 0.2$: $y = 0.2 - 5\tau^2$.

Step 3: Test (A), (B), (D).

$t = 0.3$ s ($\tau = 0.1$): $y = 0.2 - 0.05 = 0.15$ m $= 15$ cm. **(A) TRUE.**
$y = 0$ when $5\tau^2 = 0.2 \Rightarrow \tau = 0.2$, i.e. $t = 0.4$ s — so at $t = 0.4$ s the particle is *on* the XZ plane ($y = 0$), not at 10 cm. **(B) FALSE.** And it returns to the plane at $t = 0.4$ s, not 0.35 s. **(D) FALSE.**

Step 4: Radius of the circular part — (C).

The speed in the plane perpendicular to $\vec{B}$ is $v_\perp = 1.2\,\text{ms}^{-1}$ (the x-component at $t = 0.2$ s). The radius is

$$r = \frac{mv_\perp}{qB} = \frac{(10^{-6})(1.2)}{(10^{-6})(6)} = 0.2\text{ m} = 20\text{ cm}.$$

(C) TRUE.

▶ **Key point:** With $\vec{B} \parallel \hat{\jmath}$, the magnetic force never has a y-component — so gravity alone governs the vertical motion, and the path is a helix whose "radius" is set only by the in-plane speed.

Final Answer: (A) and (C)

Q.7 *Section 2 — One or more correct options*

Topic: Electrostatics — Locus of Equal Potential • Difficulty: Medium • Marks: +4

Two charges $Q_1 = q$ and $Q_2 = mq$ are placed at the points $P_1(a, b)$ and $P_2(ma, mb)$, respectively, in the XY plane, where $a, b \neq 0$ and $m \neq 0, 1$. If V_1 is the potential at a point in the XY plane due to charge Q_1 and V_2 is the potential at that point due to charge Q_2. Correct statement(s) for the points at which $|V_1| = |V_2|$ is/are:

(A) For $m = -1$, locus of these points is $ax + by = 0$.

(B) For $m = 2$, the locus of these points is a circle of radius $\frac{2}{3}\sqrt{a^2 + b^2}$ centered at $\left(\frac{2}{3}a, \frac{2}{3}b\right)$.

(C) For $m = -2$, the locus of these points is a circle of radius $2\sqrt{a^2 + b^2}$ centered at $(2a, 2b)$.

(D) For $m = -3$, locus of these points is $3bx + 3ay = 0$.

How to think about it: $|V_1| = |V_2|$ means $\frac{|q|}{r_1} = \frac{|mq|}{r_2}$, i.e. $r_2 = |m|\,r_1$, so $r_2^2 = m^2 r_1^2$. Expand this — it gives a circle in general (an Apollonius circle), and a straight line when the $x^2 + y^2$ term drops out.

Step 1: The condition.

$|V_1| = |V_2| \Rightarrow \frac{|q|}{r_1} = \frac{|m||q|}{r_2} \Rightarrow r_2^2 = m^2 r_1^2$, where $r_1^2 = (x-a)^2 + (y-b)^2$ and $r_2^2 = (x-ma)^2 + (y-mb)^2$.

Step 2: Expand and simplify.

Subtracting $m^2 r_1^2$ from r_2^2 and dividing by the common factor $(m - 1)$ gives the locus

$$(m+1)(x^2 + y^2) - 2max - 2mby = 0.$$

Step 3: Examine each case.

(A) $m = -1$**:** the $x^2 + y^2$ term vanishes; $2ax + 2by = 0 \Rightarrow ax + by = 0$. **TRUE.**

(B) $m = 2$**:** $3(x^2 + y^2) - 4ax - 4by = 0$, a circle with centre $\left(\frac{2}{3}a, \frac{2}{3}b\right)$ and radius $\sqrt{(\frac{2}{3}a)^2 + (\frac{2}{3}b)^2} = \frac{2}{3}\sqrt{a^2 + b^2}$. **TRUE.**

(C) $m = -2$**:** $-(x^2 + y^2) + 4ax + 4by = 0 \Rightarrow x^2 + y^2 - 4ax - 4by = 0$, a circle with centre $(2a, 2b)$ and radius $2\sqrt{a^2 + b^2}$. **TRUE.**

(D) $m = -3$**:** $-2(x^2 + y^2) + 6ax + 6by = 0$ — the $x^2 + y^2$ term survives, so the locus is a *circle*, not the line $3bx + 3ay = 0$. **FALSE.**

Final Answer: (A), (B) and (C)

Q.8 *Section 2 — One or more correct options*

Topic: Electric Dipole — Rotational Dynamics • Difficulty: Hard • Marks: +4

Consider an electric dipole comprising two charges $+q$ and $-q$ each with mass m, separated by a fixed distance d and initially at rest with its dipole moment pointing along $\hat{i}$. A uniform electric field $E\hat{j}$ is turned on at time $t = 0$ and it is turned off at $t = t_f$, when the dipole moment makes an angle θ_f with $\hat{i}$. Neglecting any sources of energy loss, correct option(s) is/are:

(A) The center of mass of the dipole is deflected towards $\hat{j}$ in the presence of the field.

(B) If the magnitude of the final angular velocity $\omega_f = \sqrt{\frac{2qE}{md}}$, then $\theta_f = \frac{\pi}{6}$.

(C) If $\theta_f = \pi/3$, then the change in kinetic energy of the dipole is given by $2\sqrt{3}\,qEd$.

(D) For $\theta_f = \pi/4$, the dipole rotates around its center of mass with a constant angular velocity after $t > t_f$.

How to think about it: A uniform field exerts zero *net* force on a dipole, so the centre of mass never moves — the dipole only rotates about it. The work done by the torque equals the gain in rotational kinetic energy.

Step 1: Centre of mass — (A).

Net force $= qE + (-q)E = 0$. Starting from rest, the centre of mass stays put. **(A) is FALSE.**

Step 2: Moment of inertia and the work–energy relation.

$I = 2m(d/2)^2 = \dfrac{md^2}{2}$. With $p = qd$ and the field along $\hat{j}$, the torque magnitude is $pE\cos\theta$ (measuring θ from $\hat{i}$). The work done turning from 0 to θ_f is

$$W = \int_0^{\theta_f} qdE\cos\theta\,d\theta = qdE\sin\theta_f = \tfrac{1}{2}I\omega_f^2.$$

Hence $\omega_f^2 = \dfrac{4qE\sin\theta_f}{md}$.

Step 3: Statement (B).

Setting $\omega_f^2 = \dfrac{2qE}{md}$ gives $\dfrac{4qE\sin\theta_f}{md} = \dfrac{2qE}{md} \Rightarrow \sin\theta_f = \tfrac{1}{2} \Rightarrow \theta_f = \dfrac{\pi}{6}$. **(B) is TRUE.**

Step 4: Statement (C).

Change in kinetic energy $= W = qdE\sin\theta_f$. For $\theta_f = \pi/3$, $W = qdE\sin 60^\circ = \dfrac{\sqrt{3}}{2}qEd$, not $2\sqrt{3}\,qEd$. **(C) is FALSE.**

Step 5: Statement (D).

After t_f the field is off, so the torque is zero; angular momentum is conserved and the rigid dipole rotates about its (stationary) centre of mass at the constant angular velocity ω_f — true for any θ_f, including $\pi/4$. **(D) is TRUE.**

Final Answer: (B) and (D)

Q.9 *Section 2 — One or more correct options*

Topic: Thermodynamics — Processes of an Ideal Gas • Difficulty: Medium • Marks: +4

Ten moles of an ideal monoatomic gas, initially in state $\boldsymbol{a}$ at atmospheric pressure and temperature $T_a = 27°\text{C}$, is enclosed in a metal cylinder of volume V_0 fitted with a frictionless piston. The gas is suddenly compressed to state $\boldsymbol{b}$ with volume $V_0/3$. Now, keeping the piston stationary, the cylinder is submerged in a water bath of temperature 11°C until the gas reaches the temperature of the water bath, which is denoted as state $\boldsymbol{c}$. Finally, while still in the water bath, the piston is brought slowly to its initial position, which is denoted as state $\boldsymbol{f}$. If R is universal gas constant, then the correct option(s) is/are: [Given: $9^{1/3} = 2.08$]

(A) The schematic P-V diagram of the processes described above is as shown in the figure below.
(B) The change in internal energy in going from state $\boldsymbol{a}$ to state $\boldsymbol{b}$ is $4860R$.
(C) The net change in the internal energy in the whole process is $-240R$.

(D) The pressure and temperature of the state ***b*** are 2.08 times the atmospheric pressure and 624 K, respectively.

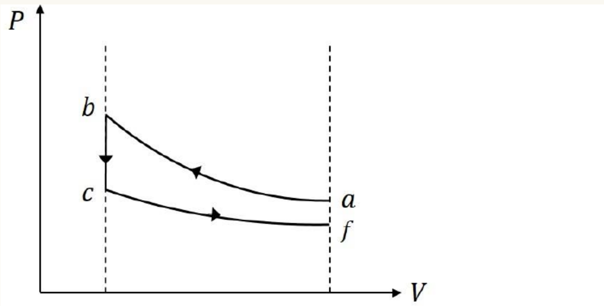

Figure from the original question paper.

How to think about it: Three processes: $a \to b$ a sudden (adiabatic) compression, $b \to c$ cooling at constant volume, $c \to f$ a slow (isothermal) expansion in the bath. Internal energy depends only on temperature, so track T at each state.

Step 1: State b from the adiabatic compression.

Monoatomic gas: $\gamma = 5/3$. Adiabatically, $T_b = T_a(V_a/V_b)^{\gamma-1} = 300 \cdot 3^{2/3} = 300 \cdot 9^{1/3} = 300(2.08) = 624\,\text{K}$. And $P_b = P_a(V_a/V_b)^{\gamma} = P_a \cdot 3^{5/3} = P_a \cdot 3(2.08) = 6.24\,P_a$. So $P_b = 6.24\times$atmospheric, *not* $2.08\times$. **(D) is FALSE** (the temperature 624 K is right, the pressure is not).

Step 2: Statement (B).

$\Delta U_{a\to b} = nC_V\Delta T = 10 \cdot \frac{3}{2}R\,(624-300) = 15R(324) = 4860R$. **(B) is TRUE.**

Step 3: Statement (C).

The final temperature is the bath temperature, $T_f = 284\,\text{K}$. Internal energy is a state function, so $\Delta U_{\text{net}} = nC_V(T_f - T_a) = 15R(284-300) = -240R$. **(C) is TRUE.**

Step 4: Statement (A).

$a \to b$: adiabat rising steeply to small volume. $b \to c$: vertical (constant volume) drop as the gas cools. $c \to f$: isotherm at 284 K expanding back to V_0. Since at V_0 the temperature (284 K) is below T_a (300 K), f lies just below a — exactly the schematic shown. **(A) is TRUE.**

Final Answer: (A), (B) and (C)

Q.10 *Section 3 — Numerical value*

Topic: Measurements — Screw Gauge and Zero Error • Difficulty: Easy • Marks: +4

Two thin wires, Wire-1 of diameter 0.650 mm and Wire-2 of unknown diameter d are given. To obtain the value of d, the diameters of the two wires are measured with a screw gauge. The screw gauge has a pitch of 0.5 mm and there are 100 divisions on the circular scale (CS). The smallest division on the linear scale (LS) is 0.5 mm. The table shows the readings of LS and CS for the measurements.

	Readings	
	LS (mm)	CS
Wire-1	0.5	42
Wire-2	1.5	95

The value of d (in μm) is ________.

How to think about it: First get the least count. The Wire-1 reading should equal its known diameter — any discrepancy is the screw-gauge *zero error*. Subtract that error from the Wire-2 reading.

Step 1: Least count.

$$\text{LC} = \frac{\text{pitch}}{\text{CS divisions}} = \frac{0.5}{100} = 0.005\,\text{mm}.$$

Step 2: Reading for Wire-1 and the zero error.

Measured reading $= 0.5 + 42(0.005) = 0.5 + 0.21 = 0.71\,\text{mm}$. The true diameter is $0.650\,\text{mm}$, so the zero error $= 0.71 - 0.650 = +0.060\,\text{mm}$.

Step 3: Corrected diameter of Wire-2.

Measured reading $= 1.5 + 95(0.005) = 1.5 + 0.475 = 1.975\,\text{mm}$. Correcting,

$$d = 1.975 - 0.060 = 1.915\,\text{mm} = 1915\,\mu\text{m}.$$

▶ **Key point:** A known reference wire is the trick that reveals the instrument's zero error — there is no separate "zero reading" given here.

Final Answer: $d = 1915\ \mu$m

Q.11 *Section 3 — Numerical value*

Topic: Wave Optics — Error Analysis • Difficulty: Medium • Marks: +4

In a single slit diffraction experiment, a slit of width (0.016 ± 0.002) mm is used to measure the wavelength of a monochromatic light source. In the diffraction pattern, the angular distance between the central maximum and first minimum is measured to be $(2^\circ \pm 40')$. The value of the fractional error in the measurement of wavelength is: [Given: $\sin(2^\circ) = 0.035$]

How to think about it: The first minimum satisfies $a\sin\theta = \lambda$, so $\lambda = a\sin\theta$. The fractional error adds the fractional error in a to the fractional error in $\sin\theta$.

Step 1: Relation and error formula.

From $\lambda = a\sin\theta$,

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta a}{a} + \frac{\Delta(\sin\theta)}{\sin\theta}, \qquad \Delta(\sin\theta) = \cos\theta\,\Delta\theta.$$

Step 2: Fractional error in the slit width.

$$\frac{\Delta a}{a} = \frac{0.002}{0.016} = 0.125.$$

Step 3: Fractional error from the angle.

$\Delta\theta = 40' = \frac{40}{60} \times \frac{\pi}{180} \approx 0.01164\,\text{rad}$. With $\cos 2^\circ \approx 1$ and $\sin 2^\circ = 0.035$,

$$\frac{\Delta(\sin\theta)}{\sin\theta} \approx \frac{\Delta\theta}{\sin 2^\circ} = \frac{0.01164}{0.035} \approx 0.333.$$

Step 4: Total fractional error.

$$\frac{\Delta\lambda}{\lambda} = 0.125 + 0.333 \approx 0.46.$$

(Equivalently $\frac{1}{8} + \frac{1}{3} = \frac{11}{24} \approx 0.458$.)

Final Answer: $\frac{\Delta\lambda}{\lambda} \approx 0.46$

Q.12 *Section 3 — Numerical value*

Topic: Ray Optics — Polarisation by Reflection • Difficulty: Medium • Marks: +4

As shown in the figure, a ray AB of unpolarized light enters from water of refractive index $n_w = 4/3$ into a medium of refractive index $n_p = 4/\sqrt{3}$ after passing through a glass plate of refractive index $n_g = 1.5$ and a layer of water. At a particular incident angle i the reflected ray CD is polarized in the direction as shown in the figure. The value of i (in degrees) is:

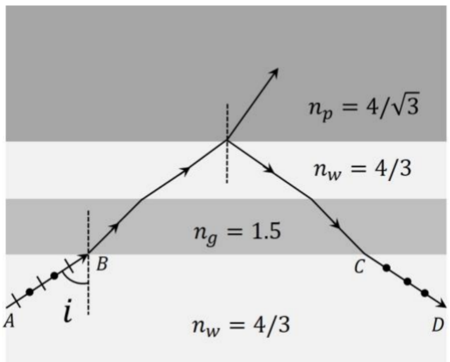

Figure from the original question paper.

How to think about it: Reflected light is completely polarized only when the angle of incidence at the reflecting interface equals Brewster's angle for that interface. Since the first medium and the upper layer are both water (same n), the ray's angle is unchanged across the parallel slabs.

Step 1: Which interface polarizes the reflection?

The ray angle inside the upper water layer equals the entry angle i, because all the slabs are parallel and the bottom medium and the layer above the glass are both water — the glass merely shifts the ray. The clean Brewster condition appears at the upper water/n_p interface.

Step 2: Brewster angle at the water/n_p interface.

$$\tan\theta_B = \frac{n_p}{n_w} = \frac{4/\sqrt{3}}{4/3} = \frac{4}{\sqrt{3}} \cdot \frac{3}{4} = \sqrt{3} \Rightarrow \theta_B = 60^\circ.$$

Step 3: Relate to the incidence angle i.

Since the ray's angle in the upper water layer equals i (same refractive index as the bottom water), $i = \theta_B = 60^\circ$.

▶ **Key point:** Across parallel layers, the angle in any layer depends only on that layer's index; identical first and intermediate media make i equal to the angle at the deciding interface.

Final Answer: $i = 60°$

Q.13 *Section 3 — Numerical value*

Topic: Current Electricity — Half-Deflection Method • Difficulty: Medium • Marks: +4

As shown in the figure, the resistance of a galvanometer G can be found by the half-deflection method. Here the resistance R_2 is adjusted such that when the key K is closed the deflection in the galvanometer becomes half of the value as compared to when K is open. Half-deflection is obtained at $R_2 = 4\,\Omega$ and thus the galvanometer resistance is found to be $6\,\Omega$. In this half-deflection condition the current (in mA) through the resistor R_1 is:

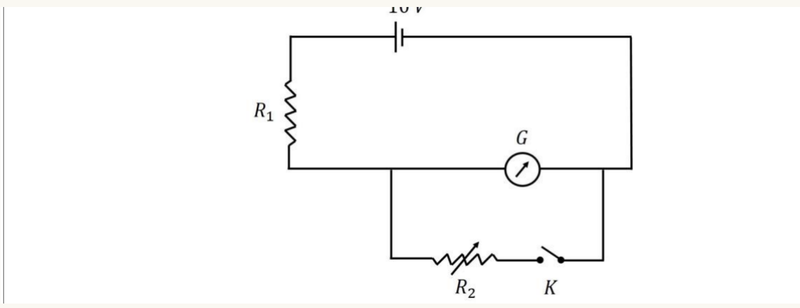

Figure from the original question paper.

How to think about it: The half-deflection method gives $G = \dfrac{R_1 R_2}{R_1 - R_2}$. Use it to find R_1, then compute the battery current through R_1 with R_2 in parallel with G.

Step 1: Find R_1.

With $G = \dfrac{R_1 R_2}{R_1 - R_2}$, $6 = \dfrac{4R_1}{R_1 - 4}$, so $6(R_1 - 4) = 4R_1 \Rightarrow 2R_1 = 24 \Rightarrow R_1 = 12\,\Omega$.

Step 2: Equivalent resistance with K closed.

R_2 is then in parallel with G: $R_2 \parallel G = \dfrac{4 \times 6}{4 + 6} = 2.4\,\Omega$. Total resistance $= R_1 + 2.4 = 14.4\,\Omega$.

Step 3: Current through R_1.

R_1 carries the full battery current:

$$I = \frac{E}{R_{\text{total}}} = \frac{10}{14.4} \approx 0.69444\,\text{A} = 694.44\,\text{mA}.$$

Final Answer: ≈ 694.44 **mA**

Q.14 *Section 3 — Numerical value*

Topic: Units and Dimensions • Difficulty: Medium • Marks: +4

In a new system of units, the units of mass, length, time and current are 5 kg, 5 m, 5 s and 5 A, respectively. If μ_0 and ϵ_0 are the permeability and permittivity of free space, respectively, then in this new system of units, the magnitude of one SI unit of $\sqrt{\mu_0/\epsilon_0}$, is:

How to think about it: $\sqrt{\mu_0/\epsilon_0}$ is the impedance of free space — it has the dimensions of resistance. Express $1\,\Omega$ in the new units by replacing each base unit.

Step 1: Dimensions of $\sqrt{\mu_0/\epsilon_0}$.

$\sqrt{\mu_0/\epsilon_0}$ has the dimensions of resistance, $[R] = \mathrm{M\,L^2\,T^{-3}\,I^{-2}}$.

Step 2: Re-express $1\,\Omega$ in the new units.

Each new base unit is $5\times$ the SI unit, so $1\,\mathrm{kg} = \frac{1}{5}$(new mass unit), and similarly for length, time, current. Thus

$$1\,\Omega = \left(\tfrac{1}{5}\right)^1\left(\tfrac{1}{5}\right)^2\left(\tfrac{1}{5}\right)^{-3}\left(\tfrac{1}{5}\right)^{-2} \times (\text{new unit}) = \left(\tfrac{1}{5}\right)^{1+2-3-2}(\text{new unit}).$$

Step 3: Evaluate.

The exponent is $1+2-3-2=-2$, so $1\,\Omega = \left(\frac{1}{5}\right)^{-2} = 25$ new units.

Final Answer: 25

Q.15 & Q.16 *Section 4 — Question stem*

Question Stem for Question Nos. 15 and 16.

A container of height $2\,\mathrm{m}$, length $2\,\mathrm{m}$ and breadth $1\,\mathrm{m}$ is made of insulating vertical walls and two large area horizontal metal plates (M_1 and M_2) which extend far beyond the vertical walls in all directions. The container is partitioned into two equal chambers with a thin insulating vertical wall. The partition wall contains a small hole of cross-sectional area $\sqrt{10}\,\mathrm{cm}^2$ near its bottom edge. Initially the hole is closed and the left chamber of the container is completely filled with a liquid of dielectric constant $\epsilon_r = 15$ and the right chamber is empty ($\epsilon_r = 1$). At time $t = 0$, the hole is opened and the liquid flows from the left chamber to the right chamber. In both the chambers, the space above the liquid has $\epsilon_r = 1$ and is maintained at atmospheric pressure. The schematic of the container at a time $t > 0$ is shown in the figure. [Given: acceleration due to gravity is $10\,\mathrm{ms}^{-2}$.]

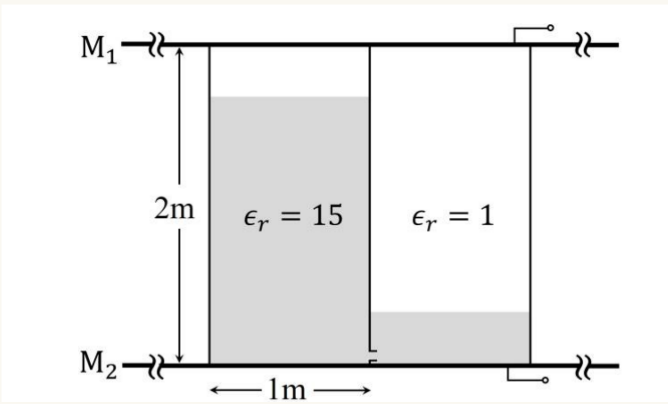

Figure from the original question paper.

Q.15 *Section 4 — Numerical value*

Topic: Fluid Mechanics — Torricelli's Law • Difficulty: Hard • Marks: +2

The height (in m) of the liquid in left chamber at $t = 500\,\mathrm{s}$ is _________.

How to think about it: Each chamber has base area $1\,m^2$ and the total liquid volume is fixed. The flow speed through the bottom hole obeys Torricelli's law, driven by the height difference. This gives a separable differential equation for the left height.

Step 1: Set up the heights.

Each chamber has base $1 \times 1 = 1\,m^2$; total liquid volume $= 2\,m^3$. With $h_L + h_R = 2$, the driving head is $\Delta h = h_L - h_R = 2(h_L - 1)$.

Step 2: Torricelli flow equation.

Let $x = h_L - 1$ (so $x : 1 \to 0$). With hole area $a = \sqrt{10} \times 10^{-4}\,m^2$,

$$\frac{dx}{dt} = -a\sqrt{2g\,\Delta h} = -a\sqrt{4gx} = -2a\sqrt{g}\,\sqrt{x}.$$

Step 3: Integrate.

$\dfrac{dx}{\sqrt{x}} = -2a\sqrt{g}\,dt \Rightarrow 2\sqrt{x} = 2 - 2a\sqrt{g}\,t$, hence $\sqrt{x} = 1 - a\sqrt{g}\,t$. Here $a\sqrt{g} = \sqrt{10} \times 10^{-4} \times \sqrt{10} = 10^{-3}\,s^{-1}$, so $x = (1 - 10^{-3}t)^2$.

Step 4: Evaluate at $t = 500\,s$.

$x = (1 - 0.5)^2 = 0.25$, so $h_L = 1 + x = 1.25\,m$.

Final Answer: $h_L = 1.25$ **m**

Q.16 *Section 4 — Numerical value* CatalyseR

Topic: Capacitance — Series Dielectric Layers • Difficulty: Hard • Marks: +2

The difference in the capacitance (in F) between the metal plates at $t = 0$ and that at $t = 500\,s$ is $(8 - n)\epsilon_0$, where ϵ_0 is the permittivity of free space. The value of n is _________.

How to think about it: Within each chamber the liquid layer and the air layer above it sit in *series* between the plates; the two chambers sit in *parallel*. A chamber with liquid height h has $C = \dfrac{\epsilon_0 A}{h/\epsilon_r + (2 - h)}$.

Step 1: Capacitance at $t = 0$.

Left chamber: full of liquid, $h = 2$: $C_L = \dfrac{\epsilon_0}{2/15} = 7.5\,\epsilon_0$. Right chamber: all air, $h = 0$: $C_R = \dfrac{\epsilon_0}{2} = 0.5\,\epsilon_0$. Total $C(0) = 7.5\epsilon_0 + 0.5\epsilon_0 = 8\epsilon_0$.

Step 2: Heights at $t = 500\,s$.

From Q.15, $h_L = 1.25\,m$ and $h_R = 2 - 1.25 = 0.75\,m$.

Step 3: Capacitance at $t = 500\,s$.

Left: $C_L = \dfrac{\epsilon_0}{1.25/15 + 0.75} = \dfrac{\epsilon_0}{5/6} = 1.2\,\epsilon_0$. Right: $C_R = \dfrac{\epsilon_0}{0.75/15 + 1.25} = \dfrac{\epsilon_0}{13/10} = \dfrac{10}{13}\epsilon_0$. Total $C(500) = \dfrac{6}{5}\epsilon_0 + \dfrac{10}{13}\epsilon_0 = \dfrac{128}{65}\epsilon_0$.

Step 4: The difference.

$$C(0) - C(500) = \left(8 - \frac{128}{65}\right)\epsilon_0, \text{ so } n = \frac{128}{65} \approx 1.97.$$

Final Answer: $n \approx 1.97$

Q.17 & Q.18 *Section 4 — Question stem*

Question Stem for Question Nos. 17 and 18.

A uniform circular disk of radius 0.2 m and mass 1 kg is pivoted at its top point C such that it can rotate freely around C in the XY plane, as shown in the figure. Initially, when the disk is at rest, a particle of mass 20 g, travelling along negative x direction in the XY plane with speed $100\,\text{ms}^{-1}$, hits the circumference of the disk at a point P. After collision the particle moves along negative y direction at a speed of $90\,\text{ms}^{-1}$. [Given: the acceleration due to gravity $(g) = -10\hat{j}\,\text{ms}^{-2}$]

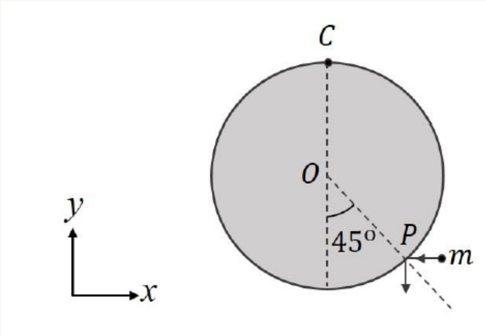

Figure from the original question paper.

Q.17 *Section 4 — Numerical value*

Topic: Rotational Dynamics — Angular Momentum and Energy • Difficulty: Hard • Marks: +2

After the collision the disk starts to rotate around point C in the XY plane. The maximum change in the height (in m) of its center O is _________.

How to think about it: During the collision, angular momentum about the pivot C is conserved (the pivot force has no torque about C). Afterwards the disk swings up like a physical pendulum; the rotational kinetic energy converts into the rise of its centre of mass.

Step 1: Geometry of the impact point.

Take C at the origin; the centre O is at $(0, -R)$ with $R = 0.2$ m. P is on the rim with OP at 45° to the downward vertical, so $P = \big(R\sin 45°,\, -R - R\cos 45°\big) = (0.1414,\, -0.3414)$.

Step 2: Conserve angular momentum about C.

Moment of inertia of the disk about the rim point C: $I_C = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2 = 0.06\,\text{kg m}^2$. With $L = m(x\,v_y - y\,v_x)$,

$$L_{\text{before}} = 0.02\big[0 - (-0.3414)(-100)\big] = -0.6828,$$

$$L_{\text{particle, after}} = 0.02\big[(0.1414)(-90) - 0\big] = -0.2546.$$

Conservation gives $I_C\,\omega = L_{\text{before}} - L_{\text{particle,after}} = -0.4283$, so $|\omega| = \dfrac{0.4283}{0.06} \approx 7.138\,\text{rad s}^{-1}$.

Step 3: Energy conservation as the disk swings up.

The rotational kinetic energy lifts the centre of mass:

$$\tfrac{1}{2}I_C\omega^2 = Mg\,\Delta h_{\max} \;\Rightarrow\; \Delta h_{\max} = \frac{\tfrac{1}{2}(0.06)(7.138)^2}{(1)(10)} = \frac{1.5286}{10} \approx 0.153\,\text{m}.$$

▶ **Key point:** Through the collision, angular momentum about the pivot is the conserved quantity — linear momentum is not, because the pivot delivers an external impulse.

Final Answer: ≈ 0.15 **m**

Q.18 *Section 4 — Numerical value*

Topic: Collisions — Energy Loss • Difficulty: Medium • Marks: +2

Amount of energy loss (in J) in the collision is ________.

How to think about it: Energy loss = (kinetic energy before) − (kinetic energy after). After the collision both the particle and the rotating disk carry kinetic energy.

Step 1: Kinetic energy before the collision.

Only the particle moves: $KE_{\text{before}} = \frac{1}{2}mv^2 = \frac{1}{2}(0.02)(100)^2 = 100\,\text{J}$.

Step 2: Kinetic energy after the collision.

Particle: $\frac{1}{2}(0.02)(90)^2 = 81\,\text{J}$. Disk: $\frac{1}{2}I_C\omega^2 = \frac{1}{2}(0.06)(7.138)^2 \approx 1.53\,\text{J}$ (from Q.17). Total $KE_{\text{after}} \approx 82.53\,\text{J}$.

Step 3: Energy lost.

$$\Delta E = KE_{\text{before}} - KE_{\text{after}} = 100 - 82.53 \approx 17.47\,\text{J}.$$

Final Answer: ≈ 17.47 **J**

Chemistry

Q.1 *Section 1 — Single correct option*

Topic: Electrochemistry — Molar Conductivity • Difficulty: Hard • Marks: +3

At 300 K, the molar conductivities of the aqueous solutions of three salts at two different concentrations are given below:

Salt	Concentration (M)	Molar conductivity ($S\,cm^2\,mol^{-1}$)
$NaNO_3$	0.01	111
	0.04	101
NaCl	0.01	117
	0.04	107
$AgNO_3$	0.01	125
	0.04	116

The conductivity of a saturated aqueous solution of AgCl is $1.40 \times 10^{-6}\,S\,cm^{-1}$ at 300 K. If the solubility of AgCl in water at 300 K is $X\,mol\,L^{-1}$, then $\log_{10}(X^{-1})$ is

(Assume that AgCl dissolved in water ionizes completely and that the molar conductivity of saturated AgCl solution is equal to its limiting molar conductivity.)

(A) 3 (B) 4 (C) 5 (D) 6

How to think about it: Extrapolate each salt's molar conductivity to infinite dilution (plot against $\sqrt{c}$), then combine by the law of independent migration of ions to get $\Lambda°$ of AgCl. Finally use $\Lambda_m = \kappa \times 1000/c$ to find the solubility.

Step 1: Limiting molar conductivities.

For a strong electrolyte $\Lambda_m = \Lambda° - b\sqrt{c}$. Using the two data points ($\sqrt{c} = 0.1$ and 0.2) for each salt:

$$\Lambda°(NaNO_3) = 121, \quad \Lambda°(NaCl) = 127, \quad \Lambda°(AgNO_3) = 134.$$

(For example, $NaNO_3$: $111 - 101 = b(0.1) \Rightarrow b = 100$, so $\Lambda° = 111 + 100(0.1) = 121$.)

Step 2: $\Lambda°$ of AgCl by Kohlrausch's law.

$AgCl \rightarrow Ag^+ + Cl^-$, so

$$\Lambda°(AgCl) = \Lambda°(AgNO_3) + \Lambda°(NaCl) - \Lambda°(NaNO_3) = 134 + 127 - 121 = 140\ S\,cm^2\,mol^{-1}.$$

Step 3: Solubility from the conductivity.

For the saturated solution, $\Lambda_m = \Lambda° = 140$. Using $\Lambda_m = \dfrac{\kappa \times 1000}{c}$,

$$c = \frac{\kappa \times 1000}{\Lambda_m} = \frac{1.40 \times 10^{-6} \times 1000}{140} = 10^{-5}\ mol\,L^{-1}.$$

Step 4: Required logarithm.

$X = 10^{-5}$, so $X^{-1} = 10^5$ and $\log_{10}(X^{-1}) = 5$.

▶ **Key point:** Each given molar conductivity is at a finite concentration — they must be extrapolated to infinite dilution before Kohlrausch's law of independent ionic migration can be applied.

Final Answer: (C) 5

Q.2 *Section 1 — Single correct option*

Topic: Chemical Bonding — Bond Angles • Difficulty: Easy • Marks: +3

The correct order of ONO bond angle in the given species is

(A) $NO_2^+ < NO_2 < NO_3^- < NO_2^-$
(B) $NO_2^- < NO_3^- < NO_2 < NO_2^+$
(C) $NO_3^- < NO_2^- < NO_2 < NO_2^+$
(D) $NO_2^- < NO_3^- < NO_2^+ < NO_2$

How to think about it: Bond angle is set by the electron environment on nitrogen. More non-bonding electron density (a lone pair) squeezes the angle; less non-bonding density opens it up.

Step 1: Identify the species on nitrogen.

NO_2^+ — no lone pair on N, *sp* hybridised, *linear*, 180°.
NO_2 — one *odd* (single) electron on N, bent, ≈ 134°.
NO_3^- — trigonal planar, 120°.
NO_2^- — one full *lone pair* on N, bent, ≈ 115°.

Step 2: Order the angles.

A full lone pair compresses the angle most, an odd electron less so:

$$NO_2^-(115°) < NO_3^-(120°) < NO_2(134°) < NO_2^+(180°).$$

△ **Common mistake:** The single odd electron in NO_2 repels less than a full lone pair, which is why NO_2 has a *wider* angle than NO_2^- — and even wider than the 120° of NO_3^-.

Final Answer: (B) $NO_2^- < NO_3^- < NO_2 < NO_2^+$

Q.3 *Section 1 — Single correct option*

Topic: Organic Chemistry — Ozonolysis and Aldol • Difficulty: Medium • Marks: +3

Natural rubber on complete ozonolysis (O_3/Zn-H_2O) gives compound **X** as the major product. **X** gives positive iodoform and Tollen's tests. **X** on heating with aqueous NaOH gives **Y** as the major product. **Y** is

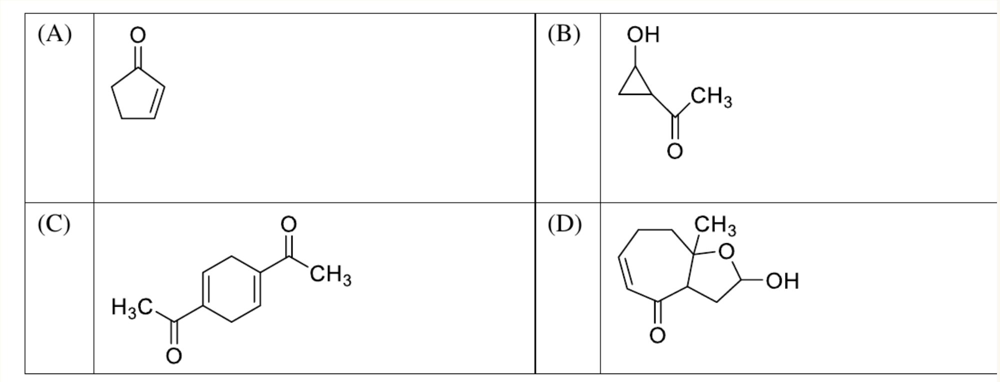

Figure from the original question paper.

How to think about it: Natural rubber is *cis*-1,4-polyisoprene. Ozonolysis cleaves each C=C, giving a single small keto-aldehyde. That keto-aldehyde then undergoes an *intramolecular* aldol condensation with NaOH.

Step 1: Identify X.

The isoprene repeat unit is $-CH_2 - C(CH_3){=}CH - CH_2-$. Cleaving the double bond turns $C(CH_3)$ into a ketone and CH into an aldehyde, giving $\mathbf{X} = CH_3{-}CO{-}CH_2{-}CH_2{-}CHO$ (4-oxopentanal). It has a methyl ketone (positive iodoform) and an aldehyde (positive Tollen's), consistent with the question.

Step 2: Intramolecular aldol condensation.

X has five carbons. With NaOH, the α-carbon (the methyl group next to the ketone) attacks the aldehyde carbon, closing a *five-membered ring*. Loss of water then conjugates the new C=C with the remaining C=O.

Step 3: Identify Y.

The product is the cyclic α, β-unsaturated ketone *cyclopent-2-en-1-one* — structure (A).

▶ **Key point:** A 1,4-keto-aldehyde with five carbons is ideally set up for an intramolecular aldol that closes a strain-free five-membered ring — hence a cyclopentenone.

Final Answer: (A) cyclopent-2-en-1-one

Q.4 *Section 1 — Single correct option*

Topic: Biomolecules — Carbohydrates (Sucralose) • Difficulty: Hard • Marks: +3

A known artificial sweetener **X** is composed of 4-chloro-4-deoxy-α-D-galactose and 1,6-dichloro-1,6-dideoxy-β-D-fructose joined by a glycosidic linkage. Structure of D-galactose is given below. The correct structure of **X** is

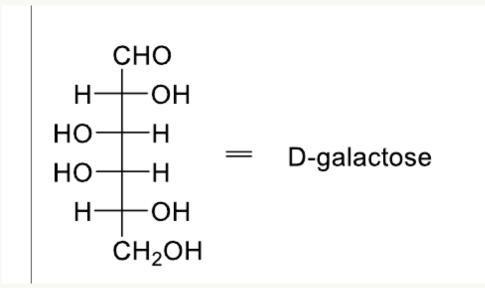

Figure from the original question paper.

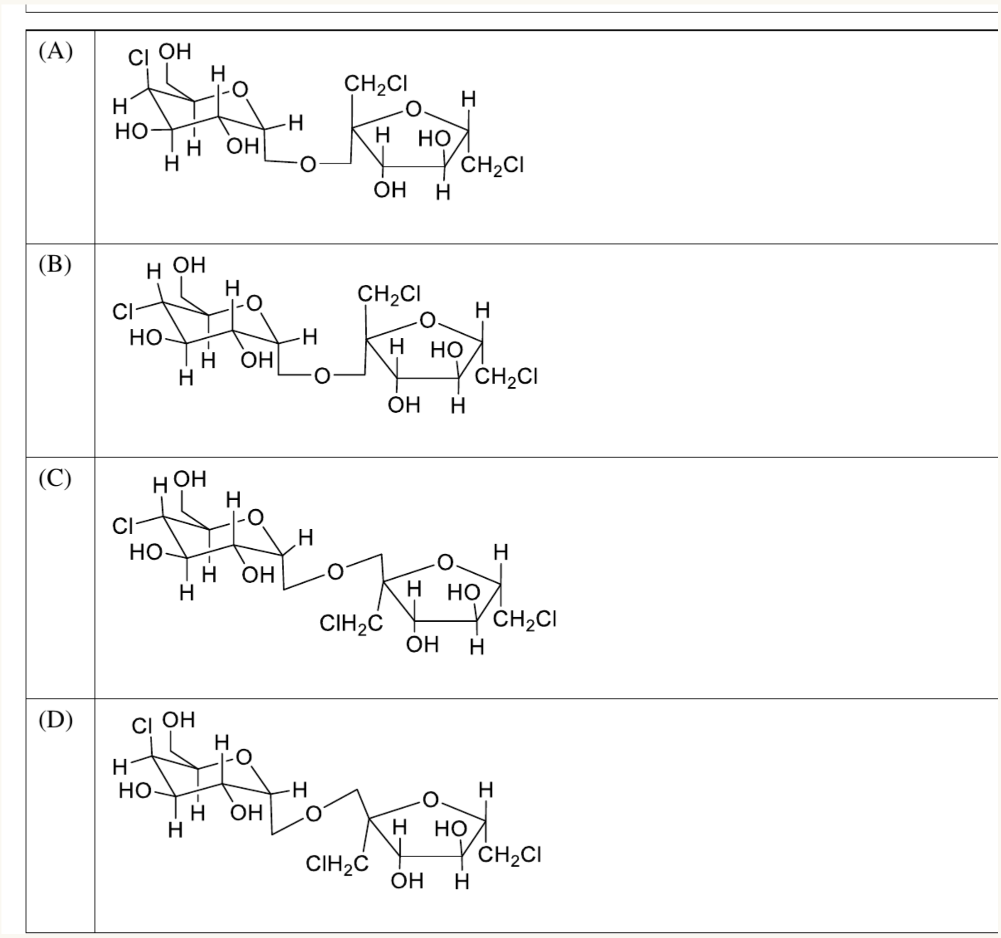

Figure from the original question paper.

How to think about it: **X** is sucralose. Two stereochemical checkpoints decide the answer: (i) the fructofuranose must be β — the C1 (CH_2Cl) group points *up*; (ii) the galactopyranose must have its C4 chlorine *up* (axial), the galacto configuration.

Step 1: Use the fructose ring to eliminate options.

In β-D-fructofuranoside (as in sucrose and sucralose), the C1 group on the anomeric carbon points *up* while the glycosidic oxygen points down. Options (A) and (B) draw CH_2Cl pointing up; options (C) and (D) draw it pointing down — so (C) and (D) are eliminated.

Step 2: Use the galactose ring to choose between (A) and (B).

D-galactose differs from D-glucose at C4: the C4 substituent is *axial-up*. In sucralose this position carries Cl, so the C4 chlorine must point *up*. Option (A) shows the chlorine up (galacto); option (B) shows it down (gluco) — so (B) is wrong.

Step 3: Conclusion.

Only structure (A) has both the β-fructofuranose (CH_2Cl up) *and* the α-galactopyranose (Cl at C4 up).

▶ **Key point:** Sucralose is sucrose with three −OH groups replaced by −Cl, and with the glucose unit's C4 inverted to the galactose configuration — the Cl at C4 sits *axial-up*.

Final Answer: (A)

Q.5 *Section 2 — One or more correct options*

Topic: Chemical Kinetics — First-Order Reaction • Difficulty: Medium • Marks: +4

For a first-order reaction $\mathbf{R} \longrightarrow \mathbf{P}$ at a given temperature, k is the rate constant. For this reaction, at the given temperature, the concentrations of $\mathbf{R}$ and $\mathbf{P}$ at a time t are $[\mathbf{R}]$ and $[\mathbf{P}]$, respectively. The correct graphical representation(s) for this reaction is(are)

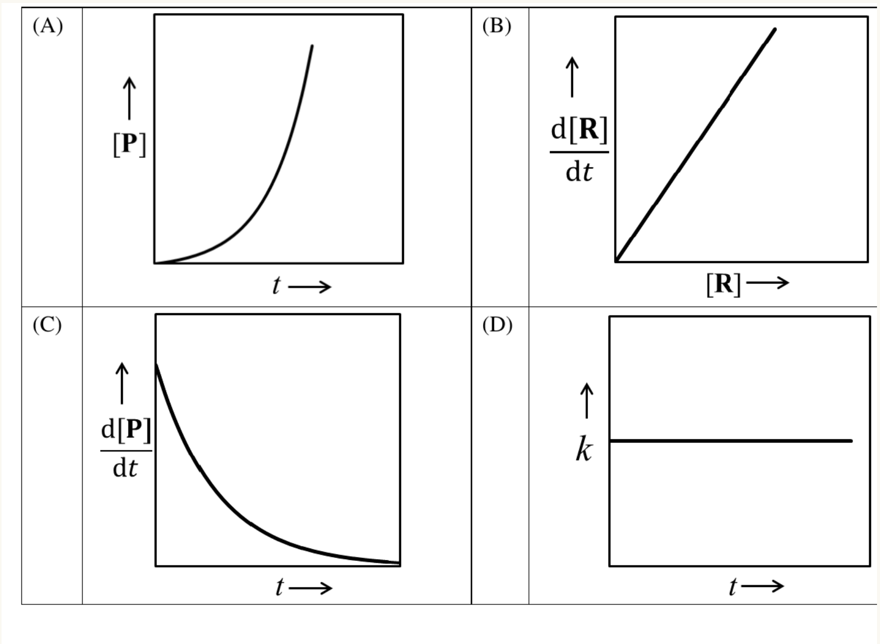

Figure from the original question paper.

How to think about it: Write the first-order results explicitly: $[\mathbf{R}] = [\mathbf{R}]_0 e^{-kt}$, $[\mathbf{P}] = [\mathbf{R}]_0(1 - e^{-kt})$, rate $= k[\mathbf{R}]$. Then check the shape demanded by each graph.

Step 1: Graph (A) — $[\mathbf{P}]$ vs t.

$[\mathbf{P}] = [\mathbf{R}]_0(1 - e^{-kt})$ rises steeply at first and then levels off — it is *concave down*. The graph shown is concave up (accelerating). **(A) is incorrect.**

Step 2: Graph (B) — $\frac{d[\mathbf{R}]}{dt}$ vs $[\mathbf{R}]$.

$\frac{d[\mathbf{R}]}{dt} = -k[\mathbf{R}]$ — a straight line through the origin with *negative* slope. The graph shown has a positive slope. **(B) is incorrect.**

Step 3: Graph (C) — $\frac{d[\mathbf{P}]}{dt}$ vs t.

$\frac{d[\mathbf{P}]}{dt} = k[\mathbf{R}]_0 e^{-kt}$ — starts high and decays exponentially towards zero. This matches the graph shown. **(C) is correct.**

Step 4: Graph (D) — k vs t.

The rate constant depends only on temperature, not on time, so k vs t is a horizontal line, exactly as drawn. **(D) is correct.**

Final Answer: (C) and (D)

Q.6 *Section 2 — One or more correct options*

Topic: p-Block — Noble Gas Compounds • Difficulty: Medium • Marks: +4

Correct statement(s) about the compounds **P**, **Q** and **R** is(are):

$Xe(g) + F_2(g)$ (1:5 ratio) $\xrightarrow{873\,K,\ 7\,bar}$ **P**

P $+ O_2F_2 \xrightarrow{143\,K}$ **Q** $+ O_2$

Q $+ H_2O \xrightarrow{\text{complete hydrolysis}}$ **R** $+ HF$

(A) **P** has two lone pairs of electrons on the central atom.
(B) **Q** has a perfect octahedral geometry.
(C) **Q** can act as a fluorinating agent.
(D) The molecular structure of **R** is trigonal pyramidal.

How to think about it: Identify **P**, **Q**, **R** from the conditions, then apply VSEPR to each.

Step 1: Identify the compounds.

$Xe : F_2 = 1{:}5$ at $873\,K$, $7\,bar$ gives **P** $= XeF_4$. The strong oxidant O_2F_2 further fluorinates it: **Q** $= XeF_6$. Complete hydrolysis of XeF_6 gives **R** $= XeO_3$ (with HF).

Step 2: Statement (A).

XeF_4: Xe has 8 valence electrons; 4 go into Xe−F bonds, leaving 2 lone pairs (square planar). **(A) is TRUE.**

Step 3: Statement (B).

XeF_6 has 6 bond pairs and 1 lone pair — seven electron pairs. The lone pair distorts the shape, so it is a *distorted* octahedron, not a perfect one. **(B) is FALSE.**

Step 4: Statement (C).

XeF_6 is a strong fluorinating agent. **(C) is TRUE.**

Step 5: Statement (D).

XeO_3: Xe has 3 bond pairs and 1 lone pair — a trigonal pyramidal molecule. **(D) is TRUE.**

Final Answer: (A), (C) and (D)

Q.7 *Section 2 — One or more correct options*

Topic: Periodic Properties • Difficulty: Medium • Marks: +4

The correct statement(s) regarding the periodic properties of elements is(are):

(A) Second ionization enthalpy of carbon atom is less than that of boron atom.
(B) Increasing order of ionic radii: $Al^{3+} < Mg^{2+} < Na^+$
(C) Under identical conditions, in solid state, the density of potassium metal is more than density of sodium metal.
(D) The H−H bond is weaker than F−F bond.

How to think about it: Examine each comparison from electronic structure: which electron is being removed, isoelectronic trends, and known bond/density data.

Step 1: Statement (A).

Removing the second electron: from C^+ ($1s^2 2s^2 2p^1$) a $2p$ electron is lost; from B^+ ($1s^2 2s^2$) an electron is pulled out of a *stable filled* $2s$. Breaking the filled $2s$ shell costs more, so $IE_2(C) < IE_2(B)$. **(A) is TRUE.**

Step 2: Statement (B).

Na^+, Mg^{2+}, Al^{3+} are isoelectronic; higher nuclear charge pulls electrons in tighter, so $Al^{3+} < Mg^{2+} < Na^+$. **(B) is TRUE.**

Step 3: Statement (C).

Potassium ($\approx 0.86\,\mathrm{g\,cm^{-3}}$) is actually *less* dense than sodium ($\approx 0.97\,\mathrm{g\,cm^{-3}}$). **(C) is FALSE.**

Step 4: Statement (D).

The H—H bond ($\approx 436\,\mathrm{kJ\,mol^{-1}}$) is far *stronger* than the F—F bond ($\approx 155\,\mathrm{kJ\,mol^{-1}}$); F—F is weak because of lone-pair repulsion between the small fluorine atoms. **(D) is FALSE.**

Final Answer: (A) and (B)

Q.8 *Section 2 — One or more correct options*

Topic: Organic Chemistry — Aromatic Reaction Sequences • Difficulty: Hard • Marks: +4

In the following reaction sequence, **P**, **Q**, **S** and **T** are the major products. The correct statement(s) about **P**, **Q**, **S** and **T** is(are):

(A) **Q** on treatment with ethanol generates an aromatic aldehyde.
(B) **S** gives positive phthalein dye test.
(C) **P** is a dinitro compound.
(D) **T** is a coloured compound.

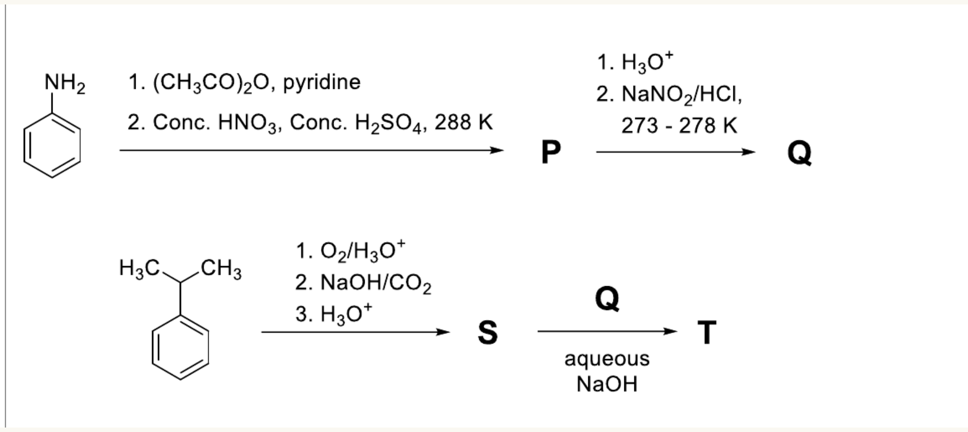

Figure from the original question paper.

How to think about it: Track each sequence: aniline through protection, nitration and diazotisation; and cumene through the cumene process, Kolbe reaction, and azo coupling.

Step 1: Find P and Q.

Aniline $\xrightarrow{(CH_3CO)_2O}$ acetanilide $\xrightarrow{HNO_3/H_2SO_4,\,288\,K}$ **P** = *p*-nitroacetanilide (the bulky $-NHCOCH_3$ directs *para*; only *one* $-NO_2$ enters). Hydrolysis then diazotisation gives **Q** = *p*-nitrobenzenediazonium chloride.

Step 2: Find S and T.

Cumene $\xrightarrow{O_2/H_3O^+}$ phenol (cumene process) $\xrightarrow{NaOH/CO_2,\text{ then }H_3O^+}$ **S** = salicylic acid (Kolbe–Schmitt). Coupling salicylic acid with **Q** in alkali gives **T**, an *azo dye*.

Step 3: Test the statements.

(A) A diazonium salt with ethanol gives ArH + acetaldehyde; acetaldehyde is *aliphatic*, not an aromatic aldehyde. **FALSE.**
(B) **S** (salicylic acid) carries a phenolic $-OH$ and gives a positive phthalein dye test. **TRUE.**
(C) **P** has only one nitro group — *mono*nitro, not dinitro. **FALSE.**
(D) **T** is an azo compound; azo dyes are coloured. **TRUE.**

> △ **Common mistake:** In (A), the trap word is "aromatic." The diazonium/ethanol reaction does yield an aldehyde — but it is acetaldehyde, an *aliphatic* aldehyde.

Final Answer: (B) and (D)

Q.9 *Section 2 — One or more correct options*

Topic: Biomolecules — Sugars • Difficulty: Medium • Marks: +4

The correct statement(s) regarding sugars is(are):

Given: Specific rotations of L-(−)-glucose and L-(+)-fructose are $-52.5°$ and $+92.5°$, respectively.

(A) On treatment with HNO_3, gluconic acid is oxidized to saccharic acid, whereas glucose is not oxidized to saccharic acid.
(B) Fructose gives a positive Fehling's test because it isomerises to glucose and another aldohexose in the presence of Fehling's reagent.
(C) Invert sugar is an equimolar mixture of D-glucose and D-fructose formed after hydrolysis of the corresponding disaccharide.
(D) Specific rotation of invert sugar is $-40°$.

> **How to think about it:** Recall the oxidation chemistry of glucose, the basic-medium isomerisation behind Fehling's test, and how the specific rotation of a mixture is computed.

Step 1: Statement (A).

HNO_3 oxidises *both* terminal groups of glucose ($-CHO$ and $-CH_2OH$) to $-COOH$, giving saccharic acid. So glucose *is* oxidised to saccharic acid by HNO_3. **(A) is FALSE.**

Step 2: Statement (B).

In the basic Fehling's medium, fructose isomerises through an enediol into glucose and mannose — both aldohexoses — which then reduce the reagent. **(B) is TRUE.**

Step 3: Statement (C).

Hydrolysis of sucrose gives an equimolar mixture of D-glucose and D-fructose, called invert sugar. **(C) is TRUE.**

Step 4: Statement (D).

The given data imply D-(+)-glucose = $+52.5°$ and D-(−)-fructose = $-92.5°$. For the equimolar mixture, the specific rotation is the *average*: $\frac{+52.5 + (-92.5)}{2} = -20°$, not $-40°$. **(D) is FALSE.**

△ **Common mistake:** In (D) the trap is to *add* the two rotations ($-40°$). The specific rotation of an equimolar (equal-mass) mixture is their *mean*, $-20°$.

Final Answer: (B) and (C)

Q.10 *Section 3 — Numerical value*

Topic: Atomic Structure — Hydrogen-like Species • Difficulty: Medium • Marks: +4

$\mathbf{X}^{a+}$ and $\mathbf{Y}^{b+}$ are hydrogen-like species. The wavelength of light absorbed during the transition between the states with principal quantum numbers $n = 1$ and $n = 2$ of $\mathbf{X}^{a+}$ is λ. The wavelength of light absorbed during the transition between the states with principal quantum numbers $n = 2$ and $n = 4$ of $\mathbf{Y}^{b+}$ is 9λ. The lowest possible value of $(a + b)$ is ________.

How to think about it: For a hydrogen-like ion of nuclear charge Z, $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$. A hydrogen-like ion $\mathbf{X}^{a+}$ has one electron, so its nuclear charge is $Z_X = a + 1$. Form the ratio of the two wavelength conditions.

Step 1: Wavelength of each transition.

For $\mathbf{X}^{a+}$ ($Z_X = a + 1$), $1 \to 2$: $\frac{1}{\lambda} = RZ_X^2\left(1 - \frac{1}{4}\right) = \frac{3}{4}RZ_X^2$.

For $\mathbf{Y}^{b+}$ ($Z_Y = b + 1$), $2 \to 4$: $\frac{1}{9\lambda} = RZ_Y^2\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{3}{16}RZ_Y^2$.

Step 2: Take the ratio.

Dividing the two equations eliminates R and λ:

$$\frac{1/\lambda}{1/(9\lambda)} = 9 = \frac{\frac{3}{4}Z_X^2}{\frac{3}{16}Z_Y^2} = \frac{4Z_X^2}{Z_Y^2} \Rightarrow \frac{Z_X^2}{Z_Y^2} = \frac{9}{4} \Rightarrow \frac{Z_X}{Z_Y} = \frac{3}{2}.$$

Step 3: Solve for a, b.

$\frac{a+1}{b+1} = \frac{3}{2} \Rightarrow 2a = 3b + 1$. For positive integer charges, the smallest solution is $b = 1,\ a = 2$, giving $Z_Y = 2$ (He^+) and $Z_X = 3$ (Li^{2+}) — both genuine hydrogen-like ions.

Step 4: Required value.

$a + b = 2 + 1 = 3$.

Final Answer: 3

Q.11 *Section 3 — Numerical value*

Topic: Surface Chemistry — Freundlich Adsorption • Difficulty: Medium • Marks: +4

At a given temperature, 0.45 g of acetic acid in 50 mL of water is shaken with 1.0 g of charcoal and the pH of the resulting solution is 3.0. Assume, the adsorption of acetic acid from the aqueous solution by charcoal follows Freundlich isotherm, $\frac{x}{m} = kC^{1/n}$. If the plot of $\log_{10}(x/m)$ against $\log_{10} C$ gives a straight line with slope 1, the value of k in $\mathrm{L\,mol^{-1}}$ is ________.

Given: molar mass of acetic acid = $60\,\mathrm{g\,mol^{-1}}$; $K_a = 1.0 \times 10^{-5}$.

How to think about it: The pH fixes the equilibrium concentration C of acetic acid left in solution (via K_a). From that, find how much was adsorbed, hence x/m. With slope 1, $n = 1$ and $k = (x/m)/C$.

Step 1: Equilibrium concentration C.

$\mathrm{pH} = 3 \Rightarrow [\mathrm{H^+}] = 10^{-3}$. For the weak acid, $K_a = \dfrac{[\mathrm{H^+}]^2}{[\mathrm{HA}]}$, so $[\mathrm{HA}] = \dfrac{(10^{-3})^2}{10^{-5}} = 0.1\,\mathrm{M} = C$.

Step 2: Amount adsorbed.

Initial acetic acid $= 0.45/60 = 0.0075\,\mathrm{mol}$ in $0.05\,\mathrm{L}$. Left in solution $= C \times V = 0.1 \times 0.05 = 0.005\,\mathrm{mol}$. Adsorbed $= 0.0075 - 0.005 = 0.0025\,\mathrm{mol} = 0.0025 \times 60 = 0.15\,\mathrm{g}$.

Step 3: Compute k.

$x/m = 0.15/1.0 = 0.15$. Slope $= 1/n = 1 \Rightarrow n = 1$, so $x/m = kC$ and

$$k = \frac{x/m}{C} = \frac{0.15}{0.1} = 1.5\ \mathrm{L\,mol^{-1}}.$$

Final Answer: $k = 1.5\ \mathrm{L\,mol^{-1}}$

Q.12 *Section 3 — Numerical value*

Topic: Solutions — Colligative Properties • Difficulty: Hard • Marks: +4

In a solvent **S**, a compound **B** is partially dissociated into **C** and **D** as given below:

$$\mathbf{B} \rightleftharpoons 2\mathbf{C} + 2\mathbf{D}$$

B, **C** and **D** are non-volatile in nature. The molar mass of **B** is 10 times the molar mass of **S**. The standard boiling point and the standard enthalpy of vaporization of **S** are $400\,\mathrm{K}$ and $10R\,\mathrm{J\,mol^{-1}}$, respectively ($R$ is the gas constant in $\mathrm{J\,K^{-1}\,mol^{-1}}$). A solution of **B** in **S** with an initial concentration of **B** as 0.25% (mass/mass) has a boiling point of $408\,\mathrm{K}$ at 1 bar pressure. In this solution, the mole percent of **B** that has been dissociated is ________.

How to think about it: Use the thermodynamic form of boiling-point elevation, $\Delta T_b = \dfrac{RT_b^2}{\Delta H_{\mathrm{vap}}} x_{\mathrm{solute}}$, where x_{solute} is the total mole fraction of all dissolved particles. The dissociation $\mathbf{B} \to 2\mathbf{C} + 2\mathbf{D}$ multiplies the particle count.

Step 1: Find the total solute mole fraction.

$\Delta T_b = 408 - 400 = 8\,\mathrm{K}$ and $\dfrac{RT_b^2}{\Delta H_{\mathrm{vap}}} = \dfrac{R(400)^2}{10R} = 16000\,\mathrm{K}$. Hence $x_{\mathrm{solute}} = \dfrac{8}{16000} = 5 \times 10^{-4}$.

Step 2: Particle count with dissociation.

For 1 mol **B** dissociating to fraction α, the particles total $1 + 3\alpha$ (since $\mathbf{B} \to 2\mathbf{C} + 2\mathbf{D}$). In 100 g solution: 0.25 g **B** and 99.75 g **S**. With $M_B = 10M_S$, moles of $\mathbf{B} = \dfrac{0.025}{M_S}$ and moles of $\mathbf{S} = \dfrac{99.75}{M_S}$.

Step 3: Solve for α.

$$x_{\mathrm{solute}} \approx \frac{(1+3\alpha)(0.025)}{99.75} = 5 \times 10^{-4} \;\Rightarrow\; 1 + 3\alpha = 1.995 \;\Rightarrow\; \alpha = 0.3317.$$

Step 4: Mole percent dissociated.

$\alpha \times 100 \approx 33.17\%$.

▶ **Key point:** The stoichiometry $\mathbf{B} \to 2\mathbf{C} + 2\mathbf{D}$ gives the van't Hoff factor $i = 1 + 3\alpha$ — one mole of **B** becomes $1 - \alpha$ of **B** plus $2\alpha + 2\alpha$ of products.

Final Answer: $\approx 33.17\%$

Q.13 *Section 3 — Numerical value*

Topic: Coordination Compounds — Geometry • Difficulty: Hard • Marks: +4

Consider that the coordinating atoms of the ligands in *cis*-$[Co(NH_3)_4Cl_2]Cl$ and *mer*-$[Co(NH_3)_3Cl_3]$ octahedral complexes are at the vertices of an octahedron. The sum of total number of the triangular faces in both the complexes having one N atom and two Cl atoms at their corners is ________.

How to think about it: An octahedron has 8 triangular faces, each formed by three vertices — one from each Cartesian axis. Place the Cl and N donors on the $\pm x, \pm y, \pm z$ vertices and count the faces that read "two Cl, one N."

Step 1: *cis*-$[Co(NH_3)_4Cl_2]^+$.

Put the two (cis) Cl at $+x$ and $+y$; the four N occupy the rest. A face contains both Cl only if it contains $+x$ and $+y$ — there are exactly two such faces, $(+x, +y, +z)$ and $(+x, +y, -z)$, each carrying two Cl and one N. **Count** $= 2$.

Step 2: *mer*-$[Co(NH_3)_3Cl_3]$.

The meridional arrangement places Cl at $+x, -x, +y$ (one Cl−Cl trans pair plus one Cl). Two Cl share a face only if they lie on *different* axes: $+x\&+y$ give faces $(+x, +y, \pm z)$ — two faces; $-x\&+y$ give $(-x, +y, \pm z)$ — two more. ($+x$ and $-x$ never share a face.) Each of these four faces has two Cl and one N. **Count** $= 4$.

Step 3: Total.

$2 + 4 = 6$.

Final Answer: 6

Q.14 *Section 3 — Numerical value*

Topic: Organic Chemistry — Condensation Polymers • Difficulty: Hard • Marks: +4

In the following reaction sequence, major products **X** and **Y** are acyclic monomers. 500 mol of **X** completely reacts with 500 mol of **Y** to give 1 mol of a single biodegradable acyclic copolymer **Z** as the only product. The amount of **Z** formed in grams is ________.

Given: Atomic mass (in amu): H : 1, C : 12, N : 14, O : 16, Br : 80.

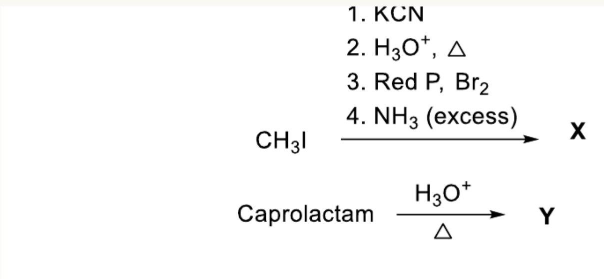

Figure from the original question paper.

How to think about it: Identify the two amino-acid monomers **X** and **Y**, then treat the polymerisation as a condensation: joining 1000 monomer units into one chain forms 999 amide bonds, releasing 999 molecules of water.

Step 1: Identify X.

$CH_3I \xrightarrow{KCN} CH_3CN \xrightarrow{H_3O^+} CH_3COOH \xrightarrow{\text{Red P, Br}_2} BrCH_2COOH \xrightarrow{NH_3} \mathbf{X}$ = glycine, $H_2N{-}CH_2{-}COOH$ ($M = 75$).

Step 2: Identify Y.

Caprolactam on acid hydrolysis ring-opens to **Y** = 6-aminohexanoic acid, $H_2N{-}(CH_2)_5{-}COOH$ ($M = 131$).

Step 3: Mass balance for the copolymer.

Total monomer mass $= 500(75) + 500(131) = 37500 + 65500 = 103000$ g. A linear (acyclic) chain of 1000 units has 999 amide linkages, each releasing one H_2O: water lost $= 999 \times 18 = 17982$ g.

Step 4: Mass of Z.

$$\text{mass of } \mathbf{Z} = 103000 - 17982 = 85018 \text{ g}.$$

▶ **Key point:** For a linear chain of N monomers there are $N-1$ condensation bonds — here $1000-1 = 999$ — and therefore 999 molecules of water are eliminated, not 1000.

Final Answer: 85018 g

Q.15 & Q.16 *Section 4 — Question stem*

Question Stem for Question Nos. 15 and 16.

Two volatile liquids **A** and **B** form an ideal solution. Consider a 5 molal solution of **B** in **A** inside a closed container having a total vapour pressure of 100 mm Hg at 300 K. The vapour pressure of pure **A** at 300 K is 105 mm Hg. Assume that **A** and **B** behave as ideal gases in the vapour phase.

Given: $R = 0.08\,\text{L atm K}^{-1}\,\text{mol}^{-1}$; molar mass of **A** $= 50\,\text{g mol}^{-1}$; molar mass of **B** $= 57\,\text{g mol}^{-1}$; density of liquid **B** at 300 K $= 0.5$ g/mL; 1 atm $= 760$ mm Hg.

Q.15 *Section 4 — Numerical value*

Topic: Solutions — Vapour and Liquid Molar Volumes • Difficulty: Medium • Marks: +2

At 300 K, the ratio of the molar volume of pure **B** in vapour phase to its molar volume in liquid phase is ________.

How to think about it: First fix the vapour pressure of pure **B** from Raoult's law on the 5 molal solution. The vapour molar volume is RT/P_B°; the liquid molar volume is M_B/ρ_B.

Step 1: Mole fractions in the 5 molal solution.

5 mol **B** per 1000 g **A**: moles of **A** $= 1000/50 = 20$, so $x_A = 20/25 = 0.8$, $x_B = 0.2$.

Step 2: Vapour pressure of pure B.

Raoult's law: $100 = x_A P_A^\circ + x_B P_B^\circ = 0.8(105) + 0.2\,P_B^\circ$, so $P_B^\circ = \dfrac{100-84}{0.2} = 80\,\text{mm Hg}$.

Step 3: Molar volumes.

Vapour (ideal gas at P_B°): $V_{\text{vap}} = \dfrac{RT}{P_B^\circ} = \dfrac{0.08\times 300}{80/760} = 228\,\text{L mol}^{-1}$. Liquid: $V_{\text{liq}} = \dfrac{M_B}{\rho_B} = \dfrac{57}{0.5} = 114\,\text{mL/mol} = 0.114\,\text{L mol}^{-1}$.

Step 4: Ratio.

$\dfrac{V_{\text{vap}}}{V_{\text{liq}}} = \dfrac{228}{0.114} = 2000.$

Final Answer: 2000

Q.16 *Section 4 — Numerical value*

Topic: Solutions — Composition of the Vapour Phase • Difficulty: Easy • Marks: +2

The mole fraction of **B** in vapour phase which is in equilibrium with this solution is ________.

How to think about it: In the vapour, the mole fraction of **B** is its partial pressure divided by the total pressure.

Step 1: Partial pressures.

$p_A = x_A P_A^\circ = 0.8(105) = 84\,\text{mm Hg}$; $p_B = x_B P_B^\circ = 0.2(80) = 16\,\text{mm Hg}$.

Step 2: Mole fraction of B in the vapour.

$$y_B = \frac{p_B}{p_A + p_B} = \frac{16}{84+16} = \frac{16}{100} = 0.16.$$

▶ **Key point:** The vapour is always *richer* in the more volatile component; here **B** ($P_B^\circ = 80$) is less volatile than **A** ($P_A^\circ = 105$), so $y_B = 0.16 < x_B = 0.2$.

Final Answer: 0.16

Q.17 & Q.18 *Section 4 — Question stem*

Question Stem for Question Nos. 17 and 18.

Consider the following reaction sequence in which **J**, **K**, **L** and **M** are the major products.

Given: Atomic mass (in amu): H : 1, C : 12, N : 14, O : 16, S : 32, Br : 80, Ba : 137.

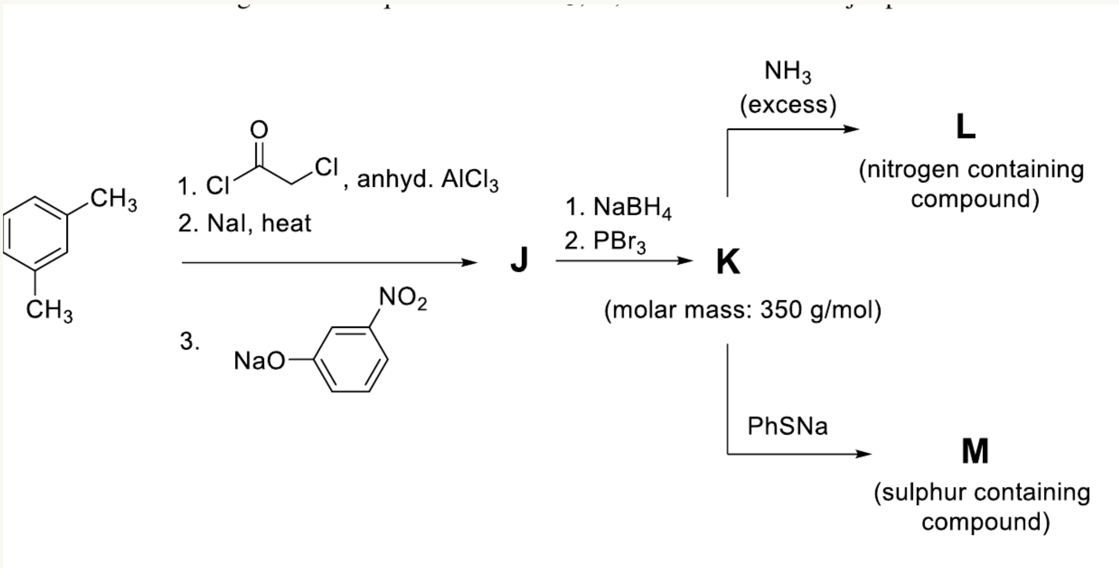

Figure from the original question paper.

Q.17 *Section 4 — Numerical value*

Topic: Organic Chemistry — Nitrogen Estimation (Kjeldahl) • Difficulty: Hard • Marks: +2

The volume of 1 M aqueous H_2SO_4 required to completely neutralize the ammonia evolved from 5.72 g of **L** in Kjeldahl's method of nitrogen estimation is ________ mL.

How to think about it: Work out **L** from the sequence, then recall a crucial limitation of Kjeldahl's method: it estimates amino/amide nitrogen but *not* nitro nitrogen.

Step 1: Trace the sequence to K and L.

Friedel–Crafts acylation of *m*-xylene with chloroacetyl chloride, Finkelstein (NaI), then *p*-nitrophenoxide gives **J** = Ar−CO−CH_2−O−Ar′ (Ar = 2, 4-dimethylphenyl, Ar′ = 4-nitrophenyl). $NaBH_4$ then PBr_3 give **K** = Ar−CHBr−CH_2−O−Ar′ ($C_{16}H_{16}BrNO_3$, $M = 350$, as stated). Excess NH_3 replaces Br by NH_2: **L** = Ar−CH(NH_2)−CH_2−O−Ar′, $C_{16}H_{18}N_2O_3$, $M = 286$.

Step 2: Moles of estimable nitrogen.

L has two N atoms — one amino (−NH_2) and one nitro (−NO_2). Kjeldahl's method estimates *only* the amino nitrogen (nitro nitrogen is not converted to ammonium sulfate). Moles of **L** $= 5.72/286 = 0.02$, so estimable N (and hence NH_3) = 0.02 mol.

Step 3: Acid required.

$2NH_3 + H_2SO_4 \rightarrow (NH_4)_2SO_4$, so H_2SO_4 needed $= 0.02/2 = 0.01$ mol. Volume of 1 M acid $= 0.01/1 = 0.01\,L = 10\,mL$.

△ **Common mistake:** Counting both nitrogens of **L** would double the answer. Kjeldahl's method fails for nitro (and azo, and ring) nitrogen — only the −NH_2 nitrogen is estimated.

Final Answer: 10 **mL**

Q.18 *Section 4 — Numerical value*

Topic: Organic Chemistry — Sulphur Estimation (Carius) • Difficulty: Medium • Marks: +2

In sulphur estimation by Carius method, the amount of $BaSO_4$ formed from 3.79 g of **M** is ________ g.

How to think about it: **M** comes from **K** by substitution with PhSNa. In the Carius method each sulphur atom ends up as one $BaSO_4$.

Step 1: Identify M.

PhSNa replaces the benzylic Br of **K** by −SPh: **M** = Ar−CH(SPh)−CH_2−O−Ar′, $C_{22}H_{21}NO_3S$, with $M = 379\,\mathrm{g\,mol^{-1}}$.

Step 2: Moles of sulphur.

Moles of **M** = 3.79/379 = 0.01. Each molecule has one S, so moles of S = 0.01, giving 0.01 mol $BaSO_4$.

Step 3: Mass of $BaSO_4$.

$M(BaSO_4) = 137 + 32 + 4(16) = 233\,\mathrm{g\,mol^{-1}}$, so

$$\text{mass} = 0.01 \times 233 = 2.33 \text{ g.}$$

Final Answer: 2.33 **g**

Complete Answer Key

A consolidated key for all 54 questions. As always, treat the official answer key released by the IITs as final; if any answer here differs from it, please use the thumbs-down option to flag it.

Q. No.	Mathematics	Physics	Chemistry
1	(C)	(C)	(C)
2	(C)	(A)	(B)
3	(B)	(B)	(A)
4	(B)	(A)	(A)
5	(A), (B), (C)	(A), (D)	(C), (D)
6	(A), (B), (C)	(A), (C)	(A), (C), (D)
7	(A), (D)	(A), (B), (C)	(A), (B)
8	(B), (D)	(B), (D)	(B), (D)
9	(B), (D)	(A), (B), (C)	(B), (C)
10	1860	1915	3
11	100	0.46	1.5
12	44	60	33.17
13	18	694.44	6
14	56	25	85018
15	11	1.25	2000
16	2.5	1.97	0.16
17	7.5	0.15	10
18	0.75	17.47	2.33

— End of Solutions —

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