PCM 25 July, 2021 (SHIFT-2)
Physics

1. A heat engine has an efficiency of $\frac{1}{6}$ When the temperature of sink is reduced by $62°C$, its efficiency get doubled. The temperature of the source is :

(A) $99°C$ (B) $124°C$ (C) $62°C$ (D) $37°C$

Sol. (A)

$$\eta = \left(1 - \frac{T_2}{T_1}\right)$$

$$\frac{T_2}{T_1} = 1 - \eta = 1 - \frac{1}{6} \qquad .....(1)$$

$$\frac{T_2 - 62}{T_1} = 1 - \frac{1}{3} \qquad .....(2)$$

Equation $\frac{(1)}{(2)}$:

$$\Rightarrow \qquad \frac{T_2}{T_2 - 62} = \frac{5}{6} \times \frac{3}{2} = \frac{5}{4}$$

$$\Rightarrow \qquad T_2 = 5 \times 62$$

From eq. (1)

$$T_1 = \frac{T_2}{1-\eta} = \frac{5 \times 62}{1 - \frac{1}{6}} = 5 \times 62 \times \frac{6}{5} = 372K \; ; \; T_1 = 372 - 273 = 99°C$$

2. Consider a planet in some solar system which has a mass double the mass of earth and density equal to the average density of earth. If the weight of an object on earth is W, the weight of the same object on that planet will be:

(A) 2 W (B) $\sqrt{2}W$ (C) W (D) $2^{\frac{1}{3}}W$

Sol. (D)

$2M_E = M_P$

$$2\rho \times \frac{4}{3}R_E^3 = \rho \times \frac{4}{3}\pi R_P^3 \text{ (same density)}$$

$R_P = 2^{1/3} R_E$

$$g_P = \frac{GM_P}{R_P^2} \text{ (acceleration due to gravity)}$$

$$g_P = \frac{G2M_E}{(2^{1/3}R_E)^2} = \frac{G2M_E}{2^{2/3}R_E^2}$$

$g_P = 2^{1/3} g_e$

Weight on planet = $2^{1/3}$ weight on earth

$W_P = 2^{1/3} W$

3. An electron moving with speed v and a photon moving with speed c, have same D-Broglie wavelength. The ratio of kinetic energy of electron to that of photon is:

(A) $\frac{v}{3c}$ (B) $\frac{3c}{v}$ (C) $\frac{v}{2c}$ (D) $\frac{2c}{v}$

Sol. (C)

De-Broglie wavelength is given by $\lambda = \frac{h}{p}$

$KE_{pn} = mc^2 = pc$ ...(1)

$KE_e = \frac{1}{2}mv^2 = \frac{pv}{2}$ ...(2)

$$\frac{KE_e}{KE_{pn}} = \frac{pv/2}{pc} = \frac{v}{2c}$$

4. Two vectors $\vec{X}$ and $\vec{Y}$ have equal magnitude. The magnitude of $\left(\vec{X} - \vec{Y}\right)$ is n times the magnitude of $\left(\vec{X} + \vec{Y}\right)$. The angle between $\vec{X}$ and $\vec{Y}$ is

(A) $\cos^{-1}\left(\frac{n^2+1}{-n^2-1}\right)$ (B) $\cos^{-1}\left(\frac{n^2-1}{-n^2-1}\right)$

(C) $\cos^{-1}\left(\frac{-n^2-1}{n^2-1}\right)$ (D) $\cos^{-1}\left(\frac{n^2+1}{n^2-1}\right)$

Sol. (B)

$$\left|\vec{X}-\vec{Y}\right| = n\left|\vec{X}+\vec{Y}\right|$$

$$\left|\vec{X}\right|^2+\left|\vec{Y}\right|^2-2\left|\vec{X}\right|\left|\vec{Y}\right|\cos\theta = n^2\left[\left|\vec{X}\right|^2+\left|\vec{Y}\right|^2+2\left|\vec{X}\right|\left|\vec{Y}\right|\cos\theta\right]$$

As $\left|\vec{X}\right| = \left|\vec{Y}\right|$

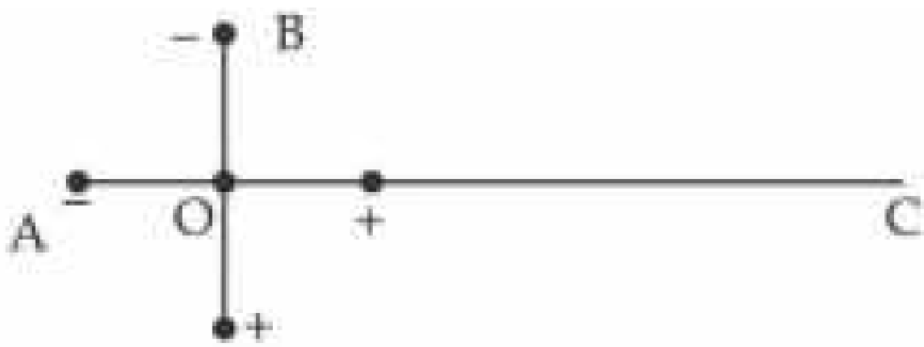
$$2\left|\vec{X}\right|^2-2\left|\vec{X}\right|^2\cos\theta = 2n^2\left|\vec{X}\right|^2+2n^2\left|\vec{X}\right|^2\cos\theta$$

$$1-\cos\theta = n^2+n^2\cos\theta$$

$$\cos\theta = \frac{1-n^2}{1+n^2}$$

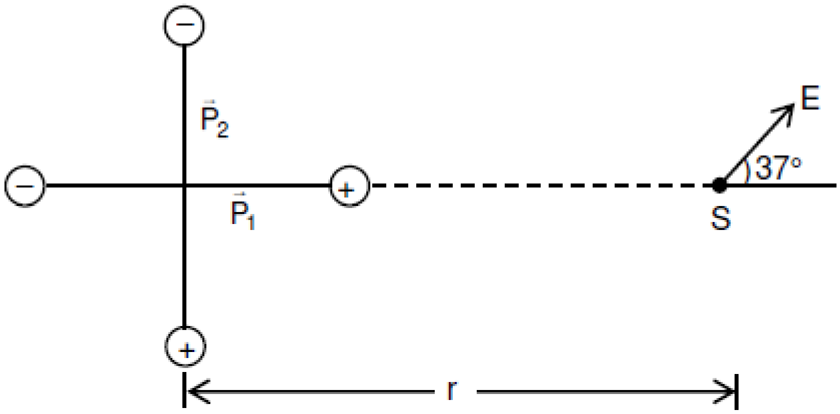
$$\theta = \cos^{-1}\frac{1-n^2}{1+n^2}$$

5. Two ideal electric dipoles A and B, having their dipole moment p_1 and p_2 respectively are placed on a plane with their centres at O as shown in the figure. At point C on the axis of dipole A, the resultant electric field is making an angle of $37°$ with the axis. The ratio of the dipole moment of A and B, $\frac{P_1}{P_2}$ is: (take $\sin 37° = \frac{3}{5}$)



(A) $\frac{3}{2}$ (B) $\frac{2}{3}$ (C) $\frac{3}{8}$ (D) $\frac{4}{3}$

Sol. (B)



Electric field due to $\vec{P}_1$ at axis point S

$$E_{axis} = \frac{2KP_1}{r^3}$$

$$\Rightarrow \qquad E\cos 37^\circ = \frac{2KP_1}{r^3} \qquad \ldots(1)$$

Electric field due to $\vec{P}_2$ at perpendicular bisector at point S.

$$E_{\perp} = \frac{KP_2}{r^3}$$

$$\Rightarrow \qquad E\sin 37^\circ = \frac{KP_2}{r^3} \quad \ldots.(2)$$

$$\therefore \qquad \frac{\frac{2KP_1}{r^3}}{\frac{KP_2}{r^3}} = \frac{E\cos 37^\circ}{E\sin 37^\circ} \qquad \Rightarrow \qquad \frac{2P_1}{P_2} = \frac{4}{3} \Rightarrow \frac{P_1}{P_2} = \frac{2}{3}$$

6. A prism of refractive index μ and angle of prism A is placed in the position of minimum angle of deviation. If minimum angle of deviation is also A, then in terms of refractive index value of A is:

(A) $\sin^{-1}\left(\sqrt{\frac{\mu-1}{2}}\right)$ (B) $\sin^{-1}\left(\frac{\mu}{2}\right)$

(C) $\cos^{-1}\left(\frac{\mu}{2}\right)$ (D) $2\cos^{-1}\left(\frac{\mu}{2}\right)$

Sol. (D)

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\mu = \frac{\sin A}{\sin A/2}$$

$$\mu = 2\cos\frac{A}{2}$$

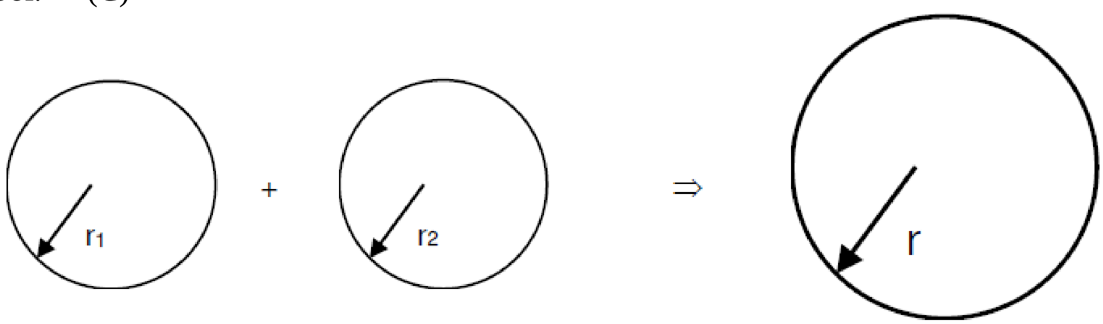
$$A = 2\cos^{-1}\left(\frac{\mu}{2}\right)$$

7. Two spherical soap bubbles of radii r_1 and r_2 in vacuum combine under isothermal conditions. The resulting bubble has a radius equal to:

(A) $\frac{r_1+r_2}{2}$ (B) $\sqrt{r_1 r_2}$

(C) $\sqrt{r_1^2+r_2^2}$ (D) $\frac{r_1 r_2}{r_1+r_2}$

Sol. (C)



By surface energy conservation

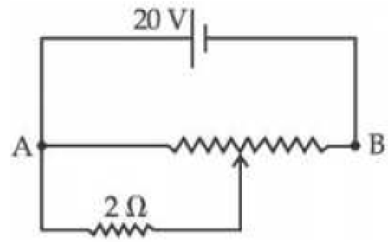
$\sigma A_1 + \sigma A_2 = \sigma A$

$\sigma[2 \times 4\pi r_1^2] + \sigma\,[2 \times 4\pi r_2^2] = \sigma\,[2 \times 4\pi r^2]$

$r_1^2 + r_2^2 = r^2$

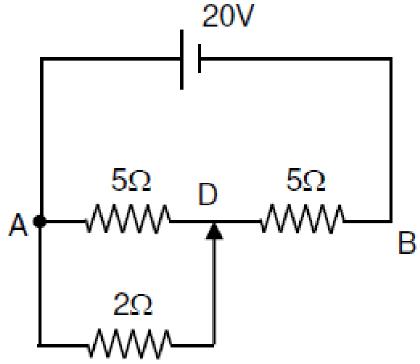
$r = \sqrt{r_1^2 + r_2^2}$

8. The given potentiometer has its wire of resistance 10Ω. When the sliding contact is in the middle of the potentiometer wire, the potential drop across 2Ω resistor is:

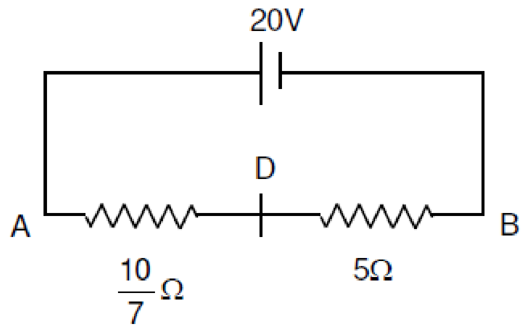


(A) $\frac{40}{9}V$ (B) 5 V (C) $\frac{40}{11}V$ (D) 10 V

Sol. (A)



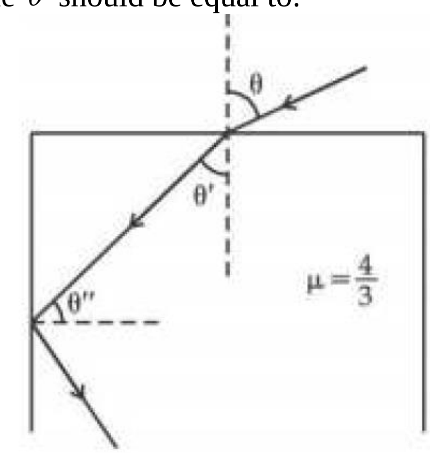
$\Rightarrow$



$$V_{2\Omega} = \frac{20}{\frac{10}{7}+5} \times \frac{10}{7}$$

$$V_{2\Omega} = \frac{40}{9}V$$

9. A ray of light entering from air into a denser medium of refractive index $\frac{4}{3}$, as shown in figure. The light ray suffers total internal reflection at the adjacent surface as shown. The maximum value of angle θ should be equal to:



(A) $\sin^{-1}\frac{\sqrt{5}}{4}$ (B) $\sin^{-1}\frac{\sqrt{7}}{4}$

(C) $\sin^{-1}\frac{\sqrt{7}}{3}$ (D) $\sin^{-1}\frac{\sqrt{5}}{3}$

Sol. (C)

$$1 \times \sin\theta = \frac{4}{3}\sin(90 - C)$$

$$\sin\theta = \frac{4}{3}\cos C$$

$$\text{but } \sin C = \frac{1}{\mu} = \frac{5}{4}$$

$$\cos C = \frac{\sqrt{7}}{4}$$

$$\sin\theta = \frac{4}{3} \times \frac{\sqrt{7}}{4} = \frac{\sqrt{7}}{3}$$

For T.I.R. $\sin\theta \le \frac{\sqrt{7}}{3}$; $\theta = \sin^{-1}\frac{\sqrt{7}}{3}$

10. If q_f is the free charge on the capacitor plates and q_b is the bound charge on the dielectric slab of dielectric constant k placed between the capacitor plates, then bound charge q_b can be expressed as:

(A) $q_b = q_f\left(1+\frac{1}{k}\right)$ (B) $q_b = q_f\left(1-\frac{1}{\sqrt{k}}\right)$

(C) $q_b = q_f\left(1+\frac{1}{\sqrt{k}}\right)$ (D) $q_b = q_f\left(1-\frac{1}{k}\right)$

Sol. (D)

$$\sigma_b = \sigma_f\left(1-\frac{1}{K}\right)$$

$$q_b = q_f\left(1-\frac{1}{k}\right)$$

11. The relation between time t and distance x for a moving body is given as $t = mx^2 + nx$, where m and n are constants. The retardation of the motion is: (Where v stands for velocity)

(A) $2mnv^3$ (B) $2nv^3$

(C) $2mv^3$ (D) $2n^2v^3$

Sol. (C)

$t = mx^2 + nx$

Differentiating w.r.t. t

$1 = 2mxv + nv$

$1 = v(2mx + n)$

Again differentiating w.r.t. t

$$\frac{dv}{dt} \times (2mx + n) + 2mv^2 = 0 \;;\; a = -2mv^3$$

12. The instantaneous velocity of a particle moving in a straight line is given as $v = \alpha t + \beta t^2$, where α and β are constants. The distance travelled by the particle between 1 s and 2 s is:

(A) $\frac{\alpha}{2} + \frac{\beta}{3}$ (B) $\frac{3}{2}\alpha + \frac{7}{3}\beta$

(C) $\frac{3}{2}\alpha + \frac{7}{2}\beta$ (D) $3\alpha + 7\beta$

Sol. (B)

$dx = vdt$

$$x = \int_1^2 \alpha t dt + \int_1^2 \beta t^2 dt$$

$$x = \left[\frac{\alpha t^2}{2}\right]_1^2 + \left[\frac{\beta t^3}{3}\right]_1^2 \;;\; x = \frac{3}{2}\alpha + \frac{7}{3}\beta$$

13. The force is given in terms of time t and displacement x by the equation

$F = A\cos Bx + C\sin Dt$

The dimensional formula of $\frac{AD}{B}$ is:

(A) $\left[ML^2T^{-3}\right]$ (B) $\left[M^2L^2T^{-3}\right]$

(C) $\left[M^0LT^{-1}\right]$ (D) $\left[M^1L^1T^{-2}\right]$

Sol. (A)

Dimension of $A = MLT^{-2}$, $B = L^{-1}$, $D = T^{-1}$

$$\text{Dimension} = \frac{AD}{B} = \frac{MLT^{-2}T^{-1}}{L^{-1}} = ML^2T^{-3}$$

14. Two ions having same mass have charge in the ratio 1 : 2. They are projected normally in a uniform magnetic field with their speeds in the ration 2 : 3. The ratio of the radii of their circular trajectories is:

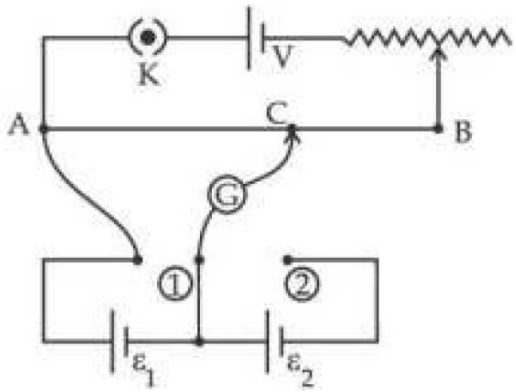
(A) 1 : 4 (B) 2 : 3 (C) 4 : 3 (D) 3 : 1

Sol. (C)

Given $\frac{Q_1}{Q_2}=\frac{1}{2}$ & $\frac{V_1}{V_2}=\frac{2}{3}$

$R=\frac{mv}{qB}$; $\frac{R_1}{R_2}=\frac{V_1}{V_2}\times\frac{Q_2}{Q_1}=\frac{2}{3}\times\frac{2}{1}=\frac{4}{3}$

15. In the given potentiometer circuit arrangement, the balancing length AC is measured to be 250 cm. When the galvanometer connection is shifted from point (1) to point (2) in the given diagram, the balancing length becomes 400 cm. The ration of the emf of two cells, $\frac{\varepsilon_1}{\varepsilon_2}$ is:



(A) $\frac{4}{3}$ (B) $\frac{5}{3}$ (C) $\frac{8}{5}$ (D) $\frac{3}{2}$

Sol. (B)

$$\frac{\varepsilon_1}{\varepsilon_1+\varepsilon_2}=\frac{\lambda\times 250}{\lambda\times 400}$$

$$8\varepsilon_1 = 5\varepsilon_1 + 5\varepsilon_2$$

$$3\varepsilon_1 = 5\varepsilon_2$$

$$\frac{\varepsilon_1}{\varepsilon_2}=\frac{5}{3}$$

16. A force $\vec{F}=\left(40\hat{i}+10\hat{j}\right)N$ acts on a body of mass 5 kg. If the body starts from rest, its position vector $\vec{r}$ at time $t=10\,s$, will be

(A) $\left(100\hat{i}+100\hat{j}\right)m$ (B) $\left(400\hat{i}+400\hat{j}\right)m$

(C) $\left(100\hat{i}+400\hat{j}\right)m$ (D) $\left(400\hat{i}+100\hat{j}\right)m$

Sol. (D)

$\vec{a} = 8\hat{i} + 2\hat{j}$

$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$\vec{s} = \frac{1}{2}(8\hat{i} + 2\hat{j}) \times 100$

$\vec{s} = 400\hat{i} + 100\hat{j}$

17. A balloon was moving upwards with a uniform velocity of 10 m/s. An object of finite mass is dropped from the balloon when it was at a height of 75 m from the ground level. The height of the balloon from the ground when object strikes the ground was around. (Takes the value of g as $10\,m/s^2$)

(A) 125 m (B) 200 m (C) 300 m (D) 250 m

Sol. (A)

For stone

$75 = -10t + \frac{1}{2}gt^2$

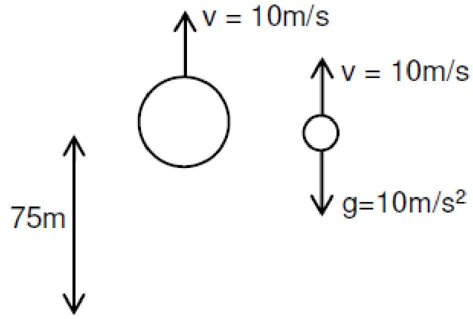
$75 = -10t + 5t^2$

$t^2 - 2t - 15 = 0$

t = 5 sec.

Height of balloon

H = vt + 75



H = 10 × 5 + 75 = 125 m.

18. When radiation of wavelength λ is incident on a metallic surface, the stopping potential of ejected photoelectrons is 4.8 V. If the same surface is illuminated by radiation of double the previous wavelength, then the stopping potential becomes 1.6 V. The threshold wavelength of the metal is:

(A) 8λ (B) 4λ (C) 2λ (D) 6λ

Sol. (B)

$KE = h\nu - W$

$eV = \frac{hc}{\lambda} - W$

For first case

$$4.8 = \frac{hc}{\lambda_0} - W \quad \ldots(i)$$

For second case

$$1.6 = \frac{hc}{2\lambda_0} - W \quad \ldots(ii)$$

From equation (i) and (ii)

$$W = \frac{hc}{4\lambda_0}$$

For λ_{th}

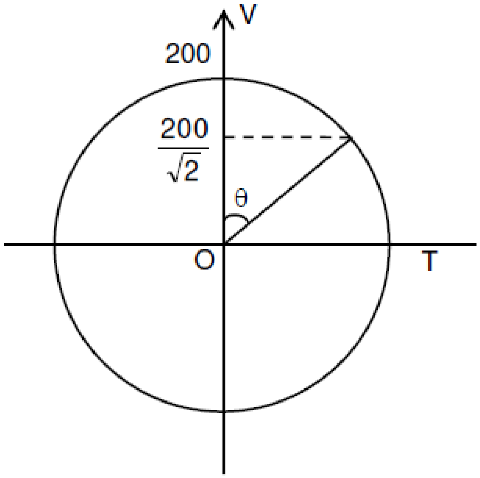
$$W = \frac{hc}{\lambda_{th}}$$

$\Rightarrow \quad \frac{hc}{4\lambda_0} = \frac{hc}{\lambda_{th}} \quad \Rightarrow \quad \lambda_{th} = 4\lambda_0$

19. A 10Ω resistance is connected across 220 V – 50 Hz AC supply. The time taken by the current to change from its maximum value to the rms value is:

(A) 3.0 ms (B) 1.5 ms (C) 4.5 ms (D) 2.5 ms

Sol. (D)



$\omega = 2\pi f$

$= 100\ \pi$ rad/s.

$V = V_0 \sin(\omega t + \frac{\pi}{2})$; $\cos\omega t = \frac{1}{\sqrt{2}}$

$\omega t = \frac{\pi}{4}$; thus $t = \frac{\pi/4}{100\pi} = \frac{1}{400}$ s $= \frac{1000}{400}$ ms $= 2.5$ ms

20. In a simple harmonic oscillation, what fraction of total mechanical energy is in the form of kinetic energy, when the particle is midway between mean and extreme position.

(A) $\frac{3}{4}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$

Sol. (A)

$V_{A/2} = \omega\sqrt{A^2 - x^2}$

$= \omega\sqrt{A^2 - \left(\frac{A}{2}\right)^2} = \omega\left(\frac{\sqrt{3}}{2}A\right) = \frac{\sqrt{3}}{2}V_{max}$

$KE = \frac{1}{2}m\left(\frac{\sqrt{3}}{2}V_{max}\right)^2$

$TE = \frac{1}{2}m(V_{max})^2$

$\frac{KE}{TE} = \frac{3}{4}$ Ans.

Integer

21. A system consists of two types of gas molecules A and B having same number density $2\times10^{25}/m^3$. The diameter of A and B are $10Å$ and $5Å$ respectively. They suffer collision at room temperature. The ratio of average distance covered by the molecule A to that of B between two successive collision is ______$\times10^{-2}$.

Sol. (25)

$$\text{Mean free path} = \frac{1}{\sqrt{2}d^2n}$$

$$\frac{x_1}{x_2} = \left(\frac{d_2}{d_1}\right)^2$$

$$\left(\frac{1}{2}\right)^2 = 0.25 = 25\times10^{-2}$$

22. A light beam of wavelength 500 nm is incident on a metal having work function of 1.25 eV, placed in a magnetic field of intensity B. The electrons emitted perpendicular to the magnetic field B, with maximum kinetic energy are bent into circular are of radius 30 cm. The value of B is _______$\times10^{-7}T$.

Given $hc = 20\times10^{-26}\,J-m$, mass of electron $=9\times10^{-31}kg$.

Sol. (125)

$$\frac{hc}{\lambda} = \phi + KE_{max}$$

$$\frac{1240}{500} = 1.25 + KE_{max}$$

$$KE_{max} = 1.23\text{ eV}$$

$$\text{Now } R = \frac{mv}{qB} = \frac{\sqrt{2mKE}}{qB}$$

$$B = \frac{\sqrt{2mKE}}{qR}$$

$$B = \frac{\sqrt{2\times9.1\times10^{-31}\times1.23\times1.6\times10^{-19}}}{1.6\times10^{-19}\times30\times10^{-2}}$$

$$B = 0.125\times10^{-4}$$

$$B = 125\times10^{-7}$$

23. A solid disc of radius 20 cm and mass 10 kg is rotating with an angular velocity of 600 rpm, about an axis normal to its circular plane and passing through its centre of mass. The retarding torque required to bring the disc at rest in 10 s is ________ $\pi \times 10^{-1}\, Nm$.

Sol. (4)

$$\omega = \frac{600 \times 2\pi}{60} = 20\pi \text{ rad/s}$$

$$\omega_f = \omega_i + \alpha_t$$

$$0 = 20\,\pi - \alpha(10)$$

$$\alpha = 2\pi \text{ rad/s}^2$$

$$\tau = I \times \alpha = \frac{mR^2}{2} \times 2\pi = \frac{10 \times 0.04}{2} \times 2\pi = 4 \times 10^{-1}\,\pi$$

$$x = 4$$

24. In a semiconductor, the number density of intrinsic charge carries at $27°C$ is $1.5 \times 10^{16} / m^3$. If the semiconductor is doped with impurity atom, the hole density increases to $4.5 \times 10^{22} / m^3$. The electron density in the doped semiconductor is _______ $10^9 / m^3$.

Sol. (5)

For a doped semi-conductor in thermal equilibrium

$$n_e n_h = n_i^2$$

$$\Rightarrow \quad n_e = \frac{n_i^2}{n_h} = \frac{(1.5 \times 10^{16})^2}{4.5 \times 10^{22}} = 5 \times 10^9 \text{ m}^{-3}$$

25. Two circuits are shown in the figure (a) and (b). AT a frequency of ______ rad/s the average power dissipated in one cycle will be same in both the circuits.

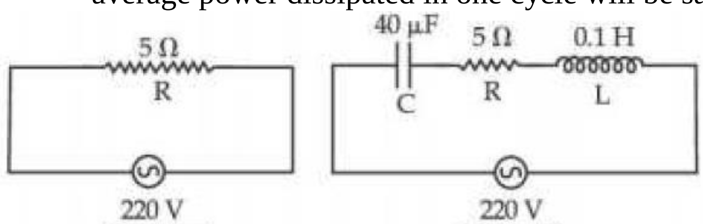


figure (a) figure (b)

Sol. (500)

$P_1 = P_2$

$$\left(\frac{V^2}{R}\right)_1 = \left(\frac{V^2}{Z}\right)_2 \qquad \Rightarrow \quad R = Z$$

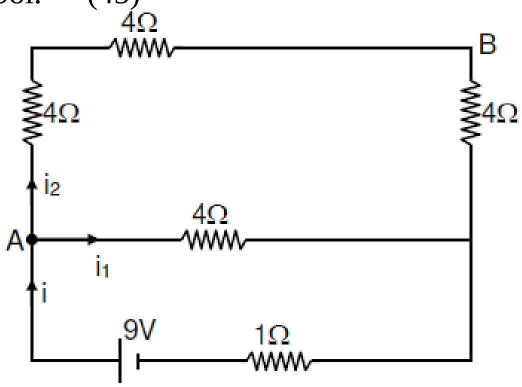
$$R = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2} \qquad \Rightarrow \quad 5 = \sqrt{\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2}$$

$$25 = \left[\omega(0.1) - \frac{1}{\omega(40\times10^{-6})}\right]^2 + 25 \ ; \ \omega^2(0.1) = \frac{1}{40\times10^{-6}}$$

$$\omega^2 = \frac{1}{4}\times10^6 \ ; \quad \omega = 500$$

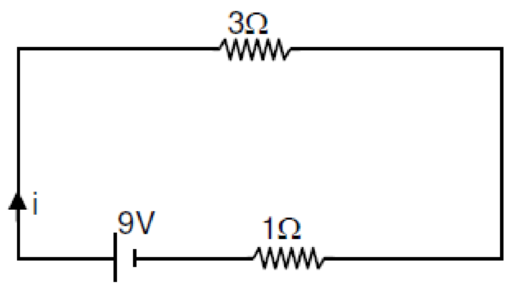
26. A 16Ω wire is bend to from a square loop. A 9 V supply having internal resistance of 1Ω is connected across one of its sides. The potential drop across the diagonals of the square loop is _____$\times10^{-1}V$.

Sol. (45)



$V_{AB} = ??$

$$R_{eq} = \frac{12 \times 4}{12 + 4} = 3$$



$$i = \frac{9}{3+1} = \frac{9}{4}A$$

$$i_2 = \frac{r_2}{r_2 + r_1} i = \frac{1}{4} \times \frac{9}{4} = \frac{9}{16}; \quad V_{AB} = \frac{9}{2} = 4.5V$$

27. From the given data, the amount of energy required to break the nucleus of aluminium $^{27}_{13}Al$ is ________ $x \times 10^{-3} J$.

Mass of neutron = 1.00866 u
Mass of proton = 1.00726 u
Mass of Aluminium nucleus = 27.18846 u
(Assume 1 u corresponds to x J of energy)
(Round off to the nearest integer)

Sol. (27)

Binding Energy = ΔmC^2

$$\Delta m = [13 \times 1.00726 + 14 \times 1.00866 - 27.18846]$$

$$= 0.02716 = 27.16 \times 10^{-3} \text{ u} \approx 27 \text{ u}$$

Binding Energy = 27×10^{-3}J

28. A message signal of frequency 20 kHz and peak voltage of 20 volt is used to modulate a carrier wave of frequency 1 MHz and peak voltage of 20 volt. The modulation index will be______

Sol. (A)

$$\mu = \frac{A_m}{A_c} = \frac{20}{20} = 1$$

29. The nuclear activity of a radioactive element becomes $\left(\frac{1}{8}\right)^{th}$ of its initial value in 30 years. The half-life of radioactive element is _____ years.

Sol. (10)

$A = A_0 e^{-\lambda t}$

For half life

$A/2 = Ae^{-\lambda t_{1/2}}$

$\frac{1}{2} = e^{-\lambda t_{1/2}}$ .....(1)

Given $1/8 = e^{-\lambda 30}$ .....(2)

Solving (1) and (2)

$e^{-3\lambda t_{1/2}} = e^{-\lambda 30}$

$T_{1/2} = 10$ Yrs.

30. A force of $F = (5y + 20)\hat{j}\,N$ acts on a particle. The work done by this force when the particle is moved from y = 10 m to y = 10 m is_______.

Sol. (450)

$$W = \int F.dy$$

$$w = \int_0^{10} (5y + 20)dy = \left[\frac{5y^2}{2} + 20y\right]_0^{10} \Rightarrow \frac{5 \times 100}{2} + 200 = 450 \text{ J}$$

Chemistry

31. Match List I with List II

List-I	List-II
Example of Colloids	Classification
(a) Cheese	(i) dispersion of liquid in liquid
(b) Pumice stone	(ii) dispersion of liquid in gas
(c) Hair cream	(iii) dispersion of gas in solid
(d) Cloud	(iv) dispersion of liquid in solid

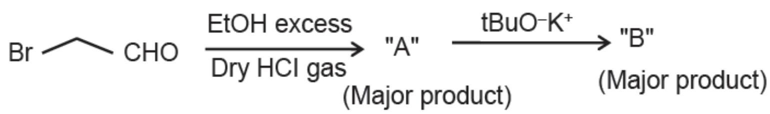
Choose the most appropriate answer from the option given below:

Option

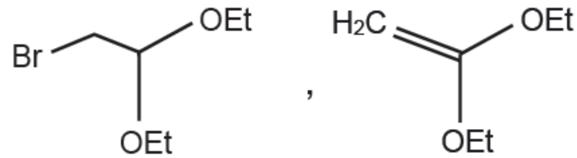
(A) (a) – (iv), (b) – (iii), (c) – (i), (d) – (iii)
(B) (a) – (iv), (b) – (i), (c) – (iii), (d) – (ii)
(C) (a) – (iv), (b) – (iii), (c) – (ii), (d) – (i)
(D) (a) – (iii), (b) – (iv), (c) – (i), (d) – (ii)

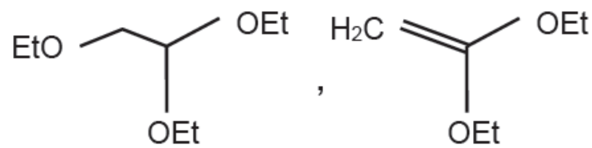
Sol. (A)

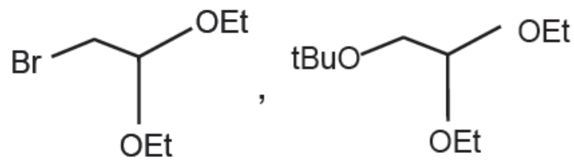
32. Consider the below reaction sequence, Product "A" and Product "B" formed respectively are :

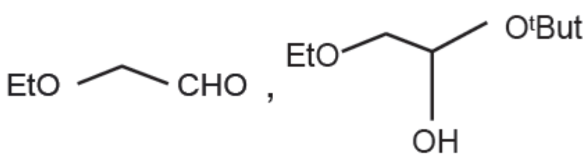


[where Et ⇒ $-C_2H_5$ t-Bu ⇒ $(CH_3)_3C-$]

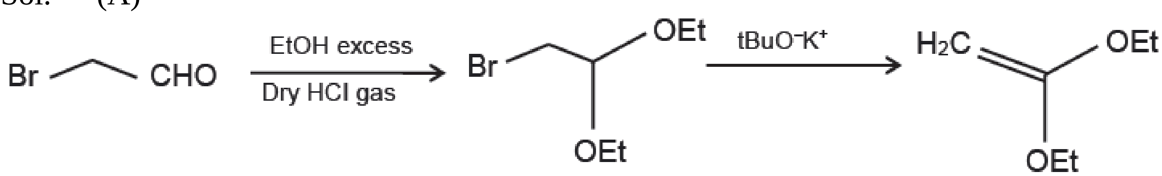
(A) 

(B) 

(C) 

(D) 

Sol. (A)



33. Given below are two statements :

Statement I : Chlorofluoro carbons breakdown by radiation in the visible energy region and release chlorine gas in the atmosphere which then reacts with stratospheric ozone.

Statement II : Atmospheric ozone reacts with nitric oxide to given nitrogen and oxygen gases, which add to the atmosphere.

For the above statements choose the correct answer from the options given below :

Options

(A) Both statement I and II are false

(B) Both statement I and II are correct

(C) Statement I is incorrect but statement II is true

(D) Statement I is incorrect but statement II is False

Sol. (A)

$$CFCs + UV \longrightarrow Cl^{\bullet}$$

$$O_3 + NO \longrightarrow NO_2 + O_2$$

34. Which one of the following metal complexes is most stable ?

(A) $\left[Co(en)_2(NH_3)_2\right]Cl_2$

(B) $\left[Co\left(NH_3\right)_6\right]Cl_2$

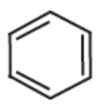
(C) $\left[Co\left(en\right)_3\right]Cl_2$

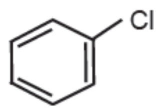
(D) $\left[Co\left(en\right)_2,\left(NH_3\right)_4\right]Cl_2$

Sol. (C)

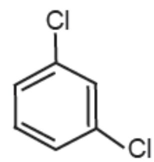
Chelation due to bidentateligand. Greater the chelation greater is the stability.

35. The correct decreasing order of densities of the following compounds:

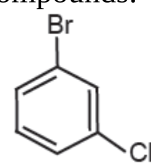

(A)



(B)



(C)



(D)

(A) (D) > (C) > (B) > (A) (B) (C) > (B) > (A) > (D)

(C) (A) > (B) > (C) > (D) (D) (C) > (D) > (A) > (B)

Ans. (A)

Density is directly proportional to molecular mass.

36. Which one of the following metals forms interstitial hydride easily?

(A) Cr (B) Fe (C) Co (D) Mn

Sol. (A)

These are formed by many d-block and f-block elements. However, the metals of group 7, 8 and 9 do not form hydride. Even from group 6, only chromium forms CrH.

37. The spin only magnetic moments (in BM) for free Ti^{3+}, V^{2+} and Sc^{3+} ions respectively are (At. No. Sc : 21; Ti : 22; V : 23)

(A) 1.73, 0, 3.87 (B) 1.73, 3.87, 0 (C) 0, 3.87, 1.73 (D) 3.87, 1.73, 0

Sol. (B)

Ti^{3+} {Unpaired electron = 1}

Sc^{3+} {Unpaired electron = 0}

V^{+2} {Unpaired electron = 3}

$\mu = \sqrt{n\,(n+2)}$ B.M.

38. Identify the process in which change in the oxidation state is five

(A) $MnO_4^- \rightarrow Mn^{2+}$ (B) $CrO_4^{2-} \rightarrow Cr^{3+}$

(C) $C_2O_4^{2-} \rightarrow 2CO_2$ (D) $Cr_2O_7^{2-} \rightarrow 2Cr^{3+}$

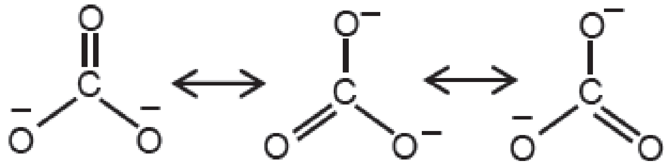
Sol. (A)

$$\underbrace{\overset{+7}{MnO_4^-} \longrightarrow Mn^{2+}}_{+5}$$

39. Identify the species having one π–bond and maximum number of canonical forms from the following

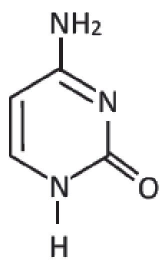
(A) CO_3^{2-} (B) SO_3 (C) O_2 (D) SO_2

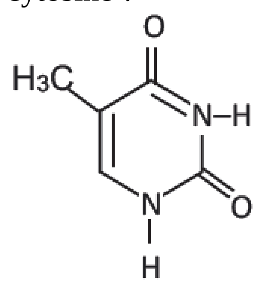
Sol. (A)

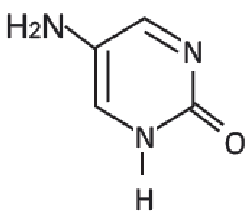


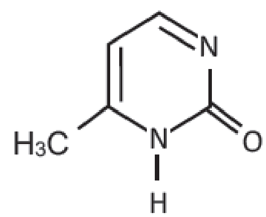
3 canonical structures

40. Which one of the following is correct structure for cytosine ?

(A) 

(B) 

(C) 

(D) 

Sol. (A)

41. The ionic radii of F^- and O^{2-} respectively are $1.33\mathring{A}$ and $1.4\mathring{A}$ while the covalent radius of N is $0.74\mathring{A}$. The correct statement for ionic radius of N^{3-} from the following is:

(A) It is bigger than F^- and N, but smaller than of O^{2-}

(B) It is smaller than O^{2-} and F^- but bigger than of N

(C) It is bigger than O^{2-} and F^-

(D) It is smaller than F^- and N

Sol. (C)

$F^- = 1.33$ Å $N = 0.74$ Å

$O^{2-} = 1.40$ Å $N^{3-} = 1.46$ Å

Ionic radius of N^{3-} is greater than F^- and O^{2-}

size of Anion $\propto$ Magnitude of –ve change

42. Match List I with List II : (Both having metallurgical terms)

List-I | **List –II**

List-I	List –II
(a) Concentration of Ag ore	(i) Reverberatory furnace
(b) Blast furnace	(ii) Pig iron
(c) Blister copper	(iii) Leaching with dilute NaCN solution
(d) Froth floatation method	(iv) sulfide ores

Choose the correct answer from the options given below

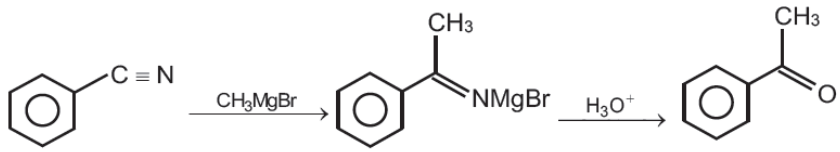
(A) (a) – (iii), (b) – (iv), (c) – (i), (d) – (ii)
(B) (a) – (iv), (b) – (iii), (c) – (ii), (d) – (i)
(C) (a) – (iii), (b) – (ii), (c) – (i), (d) – (iv)
(D) (a) – (iv), (b) – (i), (c) – (iii), (d) – (ii)

Sol. (C)

43. A reaction of benzonitrile with one equivalent CH_3MgBr followed by hydrolysis produces a yellow liquid "P". The compound "P" will give positive__________

(A) Tollen's test
(B) Iodoform test
(C) Ninhydrin's test
(D) Schiff's test

Sol. (B)



(P)

44. In the following the correct bond order sequence is :

(A) $O_2 > O_2^- > O_2^{2-} > O_2^+$
(B) $O_2^{2-} > O_2 > O_2^- > O_2$
(C) $O_2^+ > O_2 > O_2^- > O_2^{2-}$
(D) $O_2^+ > O_2^- > O_2^{2-} > O_2$

Sol. (C)

According to Molecular orbital theory

$$B.O. = \frac{1}{2}[N_b - N_a]$$

$O_2 = 2.0$

$O_2^+ = 2.5$

$O_2^- = 1.5$

$O_2^{2-} = 1.0$

45. Match List I with list II :

List-I	List-II
Elements	Properties
(a) Li	(i) Poor water solubility of I^- salt
(b) Na	(ii) most abundant elements in cell fluid
(c) K	(iii) bicarbonate salt used in fire extinguisher
(d) Cs	(iv) Carbonate salt decomposes easily on heating

Choose the correct from the options given below

(A) (a) - (i), (b) - (ii), (c) - (iii), - (d) - (iv)

(B) (a) - (iv), (b) - (ii), (c) - (iii), - (d) - (i)

(C) (a) - (i), (b) - (ii), (c) - (ii), - (d) - (iv)

(D) (a) - (iv), (b) - (iii), (c) - (ii), - (d) - (i)

Sol. (D)

(i) $Li_2CO_3 \xrightarrow{\Delta} Li_2O + CO_2$

(ii) $NaHCO_3$ is used in dry fire extinguishers.

(iii) Potassium has vital role in biological systems.

(iv) CsI is least soluble due to smaller hydration enthalpy of its two ions.

46. A biodegradable polyamide can be made from :

(A) Glycine and aminocaproic acid

(B) Glycine and isoprene

(C) Styrene and caproic acid

(D) Hexamethylene diamine and adipic acid

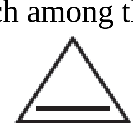
Sol. (A)

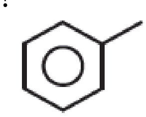
Nylon 2-Nylon 6 (Polyamide copolymer) is biodegradable polymer.

Its monomer units are : Glycine + Aminocaproic acid

$$[\underset{\text{Glycine}}{H_2N-CH_2-COOH} + \underset{\text{Aminocaproic acid}}{NH_2(CH_2)_5-COOH}]$$

47. Which among the following is the strongest acid ?

(A) 

(B) 

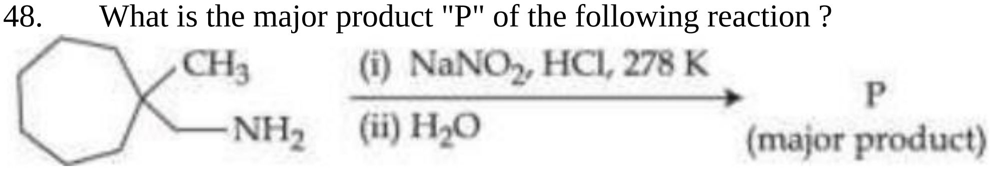
(C) $CH_3CH_2CH_2CH_3$

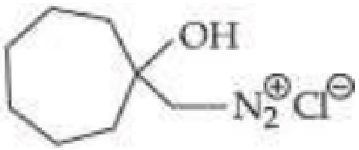
(D) 

Sol. (D)

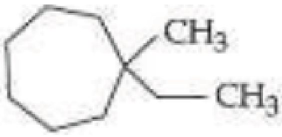
Acidic strength $\propto$ stability of conjugate base.

48. What is the major product "P" of the following reaction ?



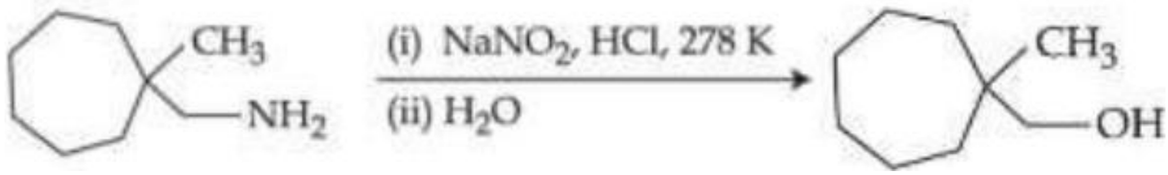
(A) 

(B) 

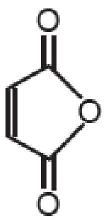
(C) 

(D) 

Sol. (B)



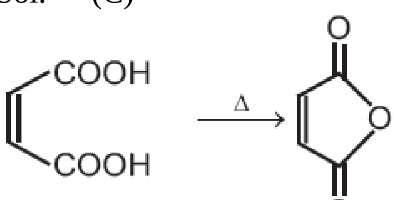
49. Maleic anhydride can be prepared by:



Maleic anhydride

(A) Treating trans-but-2-enedioic acid with alcohol and acid
(B) Heating trans-but-2-enedioic acid
(C) Heating cis-but-2-enedioic acid
(D) Treating cis-but-2-enedioic acid with alcohol and acid

Sol. (C)

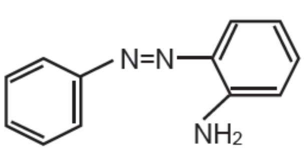


Maleic acid

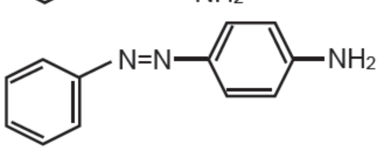
50. Consider the below reaction, the Product "P" is:

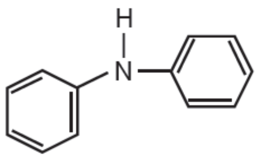
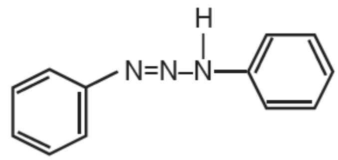
$$C_6H_5NO_2 \xrightarrow{Sn\ +\ HCl} \text{“A”} \xrightarrow[H^+]{C_6H_5N_2^+Cl^-} P$$

(Yellow coloured compound)

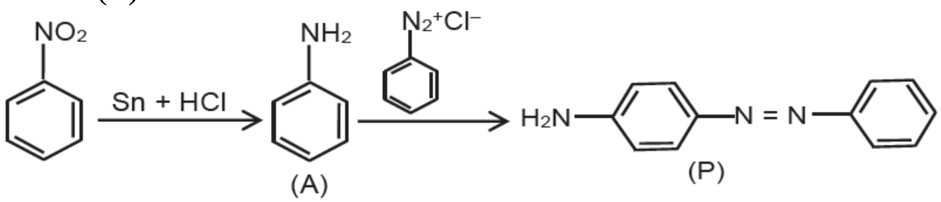
(A) 

(B)



(C)  (D) 

Ans. (B)



Integer

51. When 3.00 g of a substance 'X' is dissolved in 100g of CCl_4, it raises the boiling point by 0.60 K. The molar mass of the substance is ____________ $g\,mol^{-1}$. (Nearest integer)

Sol. (250)

$\Delta T_b = K_b\, m$

$= 5 \times \left(\frac{\text{Wt.} \times 1000}{\text{M.M} \times \text{Mass of solvent (g)}}\right)$

$= 5 \times \frac{3 \times 1000}{\text{M.M} \times 100}$

$0.6 = \frac{150}{\text{M.M}}$

$\text{M.M} = \frac{150}{0.6} = 250$ g/mol.

52. An accelerated electron, has a speed of $5 \times 10^6\, ms^{-1}$ with an uncertainty of 0.02%. The uncertainty in finding its location while in motion is $x \times 10^{-9}\, m$. The value of x is_______. (Nearest integer)

[Use mass of electron $= 9.1 \times 10^{-31}\, Kg. h = 6.63 \times 10^{-34}\, Js, \pi = 3.14$]

Sol. (58)

$\Delta x.\Delta p = \frac{h}{4\pi}$

$\Delta x.m\Delta V = \frac{h}{4\pi}$.

$\Delta v = 5 \times 10^6 \times \frac{0.02}{100}$

$\Delta v \to 1000$ m/s

$\Delta x = \frac{h}{4\pi \times m.\ \Delta V} = \frac{6.63\times10^{-34}}{4\times3.14\times9.1\times10^{-31}\times1000\ m/s} = 5.8 \times 10^{-8}$ s $= 58 \times 10^{-9}$ s

53. The number of significant figures in 0.00340 is __________.

Sol. (3)

Number of significant figures in 0.00340 are 3.

54. For a chemical reaction $A \to B$, it was found that concentration of B is increased by $0.2\,mol\,L^{-1}$ in 30 min. The average rate of the reaction is ___________ $\times10^{-1} mol\,L^{-1}h^{-1}$. (in nearest integer)

Sol. (4)

	$A \longrightarrow B$	
t = 0	a	0
t = 30 min	(a–x)	$x = 0.2\frac{mol}{lit}$

$\text{Avg. rate} = \frac{d[B]}{dt} = \frac{0.2\,mole\ L^{-1}}{0.5\ hours}$

$\Rightarrow$ 0.4 mole L^{-1} hr^{-1}.

55. Assuming that $Ba(OH)_2$ is completely ionised in aqueous solution under the given conditions the concentration of H_3O^+ ions in 0.005 M aqueous solution of $Ba(OH)_2$ at 298 K is ________$\times10^{-12} mol\,L^{-1}$. (Nearest integer)

Sol. (1)

$Ba(OH)_2$	$\rightleftharpoons$	Ba^{+2}	+	$2OH^-$
0.005 M		—		—
—		0.005 M		0.010 M

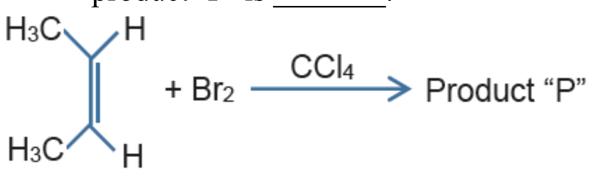
$[OH^-] = 0.01 = 10^{-2}\ M$

Now $[H^+][OH^-] = K_W$

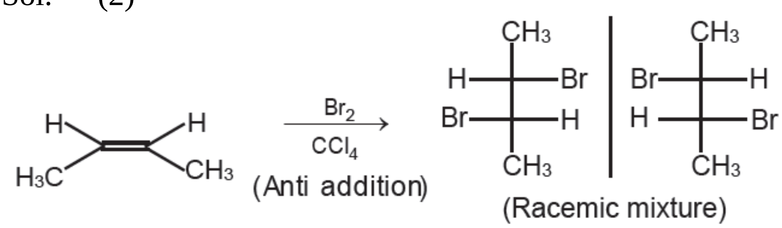
$$[H_3O^+] = \frac{K_w}{[OH^-]}$$

$$= \frac{10^{-14}}{10^{-2}} = 10^{-12}\ M = 1 \times 10^{-12}\ M$$

56. Consider the above chemical reaction. The total number of stereoisomers possible for product 'P' is _______.



Sol. (2)



57. 0.8 g of an organic compound was analysed by Kjeldahl's method for the estimation of nitrogen. If the percentage of nitrogen in the compound was found to be 42%, then ________ mL of $1M\ H_2SO_4$ would have been neutralized by the ammonia evolved during the analysis.

Sol. (12)

$$\%\ N = \frac{1.4\ N.V}{W}$$

$$42 = \frac{1.4 \times (1 \times 2) \times V}{0.8}$$

V = 12 ml

58. An LPG cylinder contains gas at a pressure of 300 kPa at $27°C$. The cylinder can withstand the pressure of $1.2 \times 10^6\ Pa$. The room in which the cylinder is kept catches fire. The minimum temperature at which the bursting of cylinder will take place is __________ $°C$. (Nearest integer)

Sol. (927)

$P_1 = 300$ K Pa $= 3 \times 10^5$ Pa $= 3$ bar

& $T_1 = 300$ K

Maximum that can withstand by cylinder = 12×10^5 Pa = 12 bar

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\Rightarrow \quad \frac{3}{300} = \frac{12}{T_2}$$

$$\Rightarrow \quad T_2 = \frac{300 \times 12}{3} = 1200 \text{ K}$$

$$\Rightarrow \quad T_2 = (1200 - 273) \text{ K} = 927^{\circ}\text{C}$$

59. A system does 200 J of work and at the same time absorbs 150 J of heat. The magnitude of the change in internal energy is __________J. (Nearest integer)

Sol. (50)

q = 150 Joules

W = –200 Joules

$\Delta E = q + w$ (First law of thermodynamics

$\Delta E = 150 + (-200)$

$\Delta E = -50$ Joule

60. Number of electrons present in 4f orbital of Ho^{3+} ion is _______. (Given Atomic No. of Ho = 67)

Sol. (10)

Holmium (Z = 67) : $4f^{11}$, $6s^2$

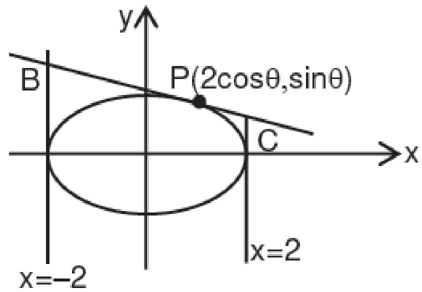
Ho^{+3} : $4f^{10}$

Mathematics

61. If a tangent to the ellipse $x^2 + 4y^2 = 4$ meets the tangents at the extremities of its major axis at B and C, then the circle with BC as diameter passes through the point:

(A) $(-1,1)$ (B) $(\sqrt{3},0)$ (C) (1, 1) (D) $(\sqrt{2},0)$

Sol. (B)



Equation of ellipse is $\frac{x^2}{4}+\frac{y^2}{1}=1$

Equation of tangent to the ellipse at the point P is $\frac{2\cos\theta x}{4}+\frac{y\sin\theta}{1}=1$

Co-ordinates of B $\left(-2, \cot\frac{\theta}{2}\right)$ & C $\left(2, \tan\frac{\theta}{2}\right)$

Equation of circle whose end points of diameter are B and C is

$(x-2)(x+2)+\left(y-\cot\frac{\theta}{2}\right)\left(y-\tan\frac{\theta}{2}\right)=0$

$x^2+y^2-\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)-3=0$

at $y = 0$, $x = \pm\sqrt{3}$

Hence circle is passes through the point $(\pm\sqrt{3}, 0)$

62. The sum of all those terms which are rational numbers in the expansion of $\left(2^{\frac{1}{3}}+3^{\frac{1}{4}}\right)^{12}$ is:

(A) 89 (B) 27 (C) 35 (D) 43

Sol. (D)

General term of $\left(2^{\frac{1}{3}}+3^{\frac{1}{4}}\right)^{12}$ $= T_{r+1} = {}^{12}C_r\,((2)^{1/3})^{12-r}.\,(3^{1/4})^r$

$= {}^{12}C_r\,(2)^{\frac{12-r}{3}}.\,(3)^{\frac{r}{4}}$

For rational numbers, both $\frac{12-r}{3}$ & $\frac{r}{4}$ must be integers simultaneously

Hence possible values of r = 0 or 12

$\therefore$ Sum of rational numbers terms are ${}^{12}C_0\,2^4+{}^{12}C_{12}\,3^3$

$= 16 + 27 = 43$

63. Let X be a random variable such that the probability function of a distribution is given by $P(x=0)=\frac{1}{2}, P(X=j)=\frac{1}{3^j}(j=1,2,3....,\infty)$. Then the mean of the distribution and P(X is positive and even) respectively are:

(A) $\frac{3}{4}$ and $\frac{1}{8}$
(B) $\frac{3}{4}$ and $\frac{1}{9}$
(C) $\frac{3}{8}$ and $\frac{1}{8}$
(D) $\frac{3}{4}$ and $\frac{1}{16}$

Sol. (A)

x_i	0	1	2	3	4	5	....
p_i	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{3^2}$	$\frac{1}{3^3}$	$\frac{1}{3^4}$	$\frac{1}{3^5}$	....

Mean of distribution = $\sum p_i x_i$

$$M = \frac{1}{3}+\frac{2}{3^2}+\frac{3}{3^3}+\frac{4}{3^4}+....... \quad ....(1)$$

$$\frac{M}{3} = \frac{1}{3^2}+\frac{2}{3^3}+\frac{3}{3^4}+....... \quad ....(2)$$

(1) – (2)

$$\frac{2M}{3} = \frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}+.......$$

$$\frac{2M}{3} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2} \Rightarrow M = \frac{3}{4}$$

$$\text{p(x is positive and even)} = \frac{1}{3^2}+\frac{1}{3^4}+\frac{1}{3^6}+.......$$

$$= \frac{1}{8}$$

64. If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$, then the value of r is equal to:

(A) 1 (B) 4 (C) 3 (D) 2

Sol. (D)

$$^nC_r = {}^nC_{r-1} \Rightarrow \frac{n-r+1}{r} = 1 \Rightarrow n + 1 = 2r \quad \ldots\ldots (1)$$

and $^nP_r = {}^nP_{r+1} \Rightarrow \frac{n!}{(n-r)!} = \frac{n!}{(n-r-1)!}$

$\Rightarrow n - r = 1 \quad \ldots\ldots (2)$

Solving (1) & (2) $n + 1 = 2(n - 1) \Rightarrow n = 3$ and $r = 2$

65. If $f(x) = \begin{cases} \int_0^x (5+|1-t|)dt & ; \quad x > 2 \\ 5x+1 & ; \quad x \le 2 \end{cases}$, then

(A) f(x) is everywhere differentiable
(B) f(x) is not differentiable at x = 1
(C) f(x) is continuous but not differentiable at x = 2
(D) f(x) is not continuous at x = 2

Sol. (C)

LHL = $\lim_{x \to 2^-} (5x+1) = 11$

RHL = $\lim_{x \to 2^+} \int_0^x (5+|1-t|)dt = \int_0^1 (5+(1-t))dt + \int_1^2 (5-(1-t))dt = 11$

f(2) = 11

So, f(x) is continuous at x = 2

LHD at x = 2 is $\frac{d}{dx}(5x+1)\Big|_{x=2} = 5$

RHD at x = 2 is $\frac{d}{dx}\int_0^x (5+|1-t|)dt\Big|_{x=2} = 6$

LHD $\neq$ RHD, so function is not differentiable at x = 2.

66. Let $y = y(x)$ be the solution of the differential equation $xdy = (y + x^3 \cos x)dx$ with $y(\pi) = 0$, then $y\left(\frac{\pi}{2}\right)$ is equal to:

(A) $\frac{\pi^2}{4} + \frac{\pi}{2}$
(B) $\frac{\pi^2}{2} - \frac{\pi}{4}$
(C) $\frac{\pi^2}{2} + \frac{\pi}{4}$
(D) $\frac{\pi^2}{4} - \frac{\pi}{2}$

Sol. (A)

$$\frac{xdy - ydx}{x^2} = x \cos x \, dx$$

$$\Rightarrow \int d\left(\frac{y}{x}\right) = \int x \cos x \, dx$$

$$\Rightarrow \frac{y}{x} = x\sin x + \cos x + c$$

$$\Rightarrow 0 = 0 - 1 + c \Rightarrow c = 1$$

$$\Rightarrow y = x^2 \sin x + x\cos x + x$$

$$\Rightarrow f\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4} + 0 + \frac{\pi}{2} = \frac{\pi^2}{4} + \frac{\pi}{2}$$

67. The value of $\cot\dfrac{\pi}{24}$ is:

(A) $\sqrt{2}-\sqrt{3}-2+\sqrt{6}$ (B) $3\sqrt{2}-\sqrt{3}-\sqrt{6}$

(C) $\sqrt{2}+\sqrt{3}+2-\sqrt{6}$ (D) $\sqrt{2}+\sqrt{3}+2+\sqrt{6}$

Sol. (D)

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{2\cos^2\theta}{2\sin\theta\cos\theta}$$

$$= \frac{1+\cos 2\theta}{\sin 2\theta}$$

$$\therefore \cot\frac{\pi}{24} = \frac{1+\cos\frac{\pi}{12}}{\sin\frac{\pi}{12}} \qquad \left(\sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}}\right) \ \& \ \left(\cos\frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}}\right)$$

$$\Rightarrow \frac{1+\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \frac{2\sqrt{2}+\sqrt{3}+1}{\sqrt{3}-1}$$

$$\Rightarrow \frac{\left(2\sqrt{2}+\sqrt{3}+1\right)\left(\sqrt{3}+1\right)}{2}$$

$$\Rightarrow \frac{2\sqrt{6}+2\sqrt{2}+3+\sqrt{3}+\sqrt{3}+1}{2} = \sqrt{6}+\sqrt{2}+\sqrt{3}+2$$

68. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is:

(A) 3 (B) 1 (C) 4 (D) 2

Sol. (B)

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

$R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \begin{vmatrix} \sin x + 2\cos x & \sin x + 2\cos x & \sin x + 2\cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

$$(\sin x + 2\cos x)\begin{vmatrix} 1 & 1 & 1 \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

$C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$$(\sin x + 2\cos x)\begin{vmatrix} 1 & 0 & 0 \\ \cos x & \sin x - \cos x & 0 \\ \cos x & 0 & \sin x - \cos x \end{vmatrix} = 0$$

$(\sin x + 2\cos x)\ (\sin x - \cos x)^2 = 0$

$\sin x = \cos x$ or $\sin x = -2\cos x$

$\tan x = 1$ or $\tan x = -2$

$\because x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

$x = \frac{\pi}{4}$

69. The first of the two samples in a group has 100 items with mean 15 and standard deviation 3. If the whole group has 250 items with mean 15.6 and standard deviation $\sqrt{13.44}$, then the standard deviation of the second sample is:

(A) 6 (B) 5 (C) 4 (D) 8

Sol. (C)

Combined mean = 15.6

$\therefore\ 15.6 = \dfrac{100\times15+150\times\bar{x}_B}{250}$

$\Rightarrow \bar{x}_B = 16$ (mean of sample B)

Combined standard deviation = $\sqrt{13.44}$

$\Rightarrow$ combined variance (σ^2) = 13.44

$$\sigma^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$$

$$13.44 = \ = \frac{\sum x_i^2}{250} - 243.36$$

$\Rightarrow \sum x_i^2 = 64200$ ....(1)

for sample A

$$9 = \frac{\sum x_{iA}^2}{100} - 225$$

$\Rightarrow \sum x_{iA}^2 = 23400$

$\Rightarrow \sum x_{iB}^2 = 64200 - 23400 = 40800$

standard deviation of sample B will be

$$\sqrt{\frac{\sum x_{iB}^2}{n_B} - (\bar{x}_B)^2} = \sqrt{\frac{40800}{150} - 256} = 4 \text{ Ans.}$$

70. If $|\vec{a}| = 2, |\vec{b}| = 5$ and $|\vec{a}\times\vec{b}| = 8$, then $|\vec{a}\cdot\vec{b}|$ is equal to:

(A) 5 (B) 6 (C) 4 (D) 3

Sol. (B)

$$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta|$$

$$8 = 2 \times 5 \times \sin\theta$$

$$\sin\theta = \frac{4}{5} \Rightarrow \cos\theta = \pm\frac{3}{5} \Rightarrow |\cos\theta| = \frac{3}{5}$$

$$|\vec{a}.\vec{b}| = |\vec{a}||\vec{b}||\cos\theta| = 2\times5\times\frac{3}{5} = 6$$

71. The value of the integral $\int_{-1}^{1} \log\left(x+\sqrt{x^2+1}\right)dx$ is :

(A) 1 (B) 2 (C) 0 (D) –1

Sol. (C)

$$I = \int_{-1}^{1} \log\left(x+\sqrt{x^2+1}\right)dx$$

$$f(x) = \log\left(\sqrt{x^2+1}+x\right)$$

$$f(-x) = \log\left(\sqrt{x^2+1}-x\right)$$

$$= -f(x)$$

So f(x) is a odd function.

$\Rightarrow I = 0$

72. If the greatest value of the term independent of 'x' in the expansion of $\left(x\sin\alpha + a\frac{\cos\alpha}{x}\right)^{10}$ is $\frac{10!}{(5!)^2}$, then the value of 'a' is equal to:

(A) 1 (B) 2 (C) –1 (D) –2

Sol. (B)

General term of $\left(x\sin\alpha + a\frac{\cos\alpha}{x}\right)^{10}$ is $T_{r+1} = {}^{10}C_r(x\sin\alpha)^{10-r}\left(\frac{a\cos\alpha}{x}\right)^r$

$T_{r+1} = {}^{10}C_r(\sin\alpha)^{10-r}(\cos\alpha)^r.\ a^r.\ x^{10-2r}$

for term independent of x, $10-2r = 0 \Rightarrow r = 5$

$T_6 = {}^{10}C_5\,(\sin\alpha\cos\alpha)^5\ .\ a^5$

$$T_6 = {}^{10}C_5\,\frac{(\sin 2\alpha)^5}{2^5}.a^5$$

$\Rightarrow T_6$ will be greatest at $\sin 2\alpha = 1$

$$\Rightarrow \frac{10}{(5!)^2} = {}^{10}C_5\left(\frac{a}{2}\right)^5$$

$\Rightarrow a = 2$

Greatest value of term when $\sin 2\alpha = 1$

$2^5 = a^5$

$a = 2$

73. The number of real solutions of equation, $x^2 - |x| - 12 = 0$ is:

(A) 3 (B) 4 (C) 2 (D) 1

Sol. (C)

$|x|^2 - |x| - 12 = 0$

$|x| = 4, -3$ (not possible)

$\Rightarrow |x| = 4 \Rightarrow x = \pm 4$

$\therefore$ Number of real solutions = 2

74. Consider functions $f : A \to B$ and $g : B \to C \left(A, B, C \subseteq R\right)$ such that $\left(gof\right)^{-1}$ exists, then:

(A) f is onto and g is one - one
(B) f is one – one and g is onto
(C) f and g both are onto
(D) f and g both are one – one

Sol. (B)

Let f is not a one-one function then

Let $f(x_1) = f(x_2) = y \in B$ and $g(y) = z$

$\Rightarrow \; gof(x_1) = g(f(x_1)) = g(y) = z$

and $gof(x_2) = g(f(x_2)) = g(y) = z$

$\Rightarrow (gof)^{-1}(z) = \left\langle \begin{matrix} x_1 \\ x_2 \end{matrix} \right.$ does not exists

Hence f must be one-one

Again let g is not onto function and f is one-one then clearly $(gof)^{-1}$ does not exists.

75. If $P = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$, then P^{50} is:

(A) $\begin{bmatrix} 1 & 25 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 \\ 50 & 1 \end{bmatrix}$

Sol. (C)

$$P^2 = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 1 & 0 \\ \frac{2}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 1 & 0 \\ \frac{3}{2} & 1 \end{bmatrix}$$

Similarly

$$P^{50} = \begin{bmatrix} 1 & 0 \\ \frac{50}{2} & 1 \end{bmatrix}$$

$$P^{50} = \begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$$

76. Let a, b and c be distinct positive numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}, \hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ are co-planar, then c is equal to:

(A) $\sqrt{ab}$ (B) $\frac{a+b}{2}$ (C) $\frac{2}{\frac{1}{a}+\frac{1}{b}}$ (D) $\frac{1}{a}+\frac{1}{b}$

Sol. (A)

$$\begin{vmatrix} 1 & 0 & 1 \\ a & a & c \\ c & c & b \end{vmatrix} = 0$$

$$1(ab - c^2) + 1\,(ac - ac) = 0 \Rightarrow ab = c^2$$

$$c = \sqrt{ab}$$

77. Let the equation of the pair of lines, $y = px$ and $y = qx$, can be written as $(y - px)(y - qx) = 0$. Then the equation of the pair of the angle bisectors of the lines $x^2 - 4xy - 5y^2 = 0$ is:

(A) $x^2 + 4xy - y^2 = 0$ (B) $x^2 - 3xy - y^2 = 0$

(C) $x^2 + 3xy + y^2 = 0$ (D) $x^2 + 3xy - y^2 = 0$

Sol. (D)

Equation of angle bisector of homogeneous equation of pair of straight line $ax^2 + 2hxy + by^2$ is

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

for $x^2 - 4xy - 5y^2 = 0$

$a = 1, h = -2, b = -5$

so, equation of angle bisector is

$$\frac{x^2 - y^2}{1 - (-5)} = \frac{xy}{-2} \Rightarrow x^2 - y^2 + 3xy = 0$$

so, combined equation of angle bisector is $x^2 + 3xy - y^2 = 0$

78. Consider the statement "The match will be played only if the weather is good and ground is not wet". Select the correct negation from the following :

(A) The match will be played and weather is not good or ground is wet.
(B) The match will not be played or weather is good and ground is not wet.
(C) If the match will not be played, then either weather is not good or ground is wet.
(D) The match will not be played and weather is not good and ground is wet.

Sol. (D)

p : The match will be played
q : Weather is good
r : ground is not wet

$\sim[p \to (q \wedge r)] = p \wedge \sim(q \wedge r)$

$= p \wedge (\sim q \vee \sim r)$

The match will be played and weather is not good or ground is wet.

79. The lowest integer which is greater than $\left(1 + \frac{1}{10^{100}}\right)^{10^{100}}$ is ____________.

(A) 4 (B) 3 (C) 1 (D) 2

Sol. (B)

Let $10^{100} = n$

So, $\left(1 + \frac{1}{n}\right)^n = {}^nC_0 + {}^nC_1\left(\frac{1}{n}\right) + {}^nC_2\left(\frac{1}{n}\right)^2 + {}^nC_3\left(\frac{1}{n}\right)^3 + \ldots..$

$= 1 + 1 + \frac{n(n-1)}{2n^2} + \frac{n(n-1)(n-2)}{6n^3} + \ldots\ldots$

$\Rightarrow \quad \left(1 + \frac{1}{n}\right)^n > 2$

Also $\lim\limits_{n \to \infty}\left(1 + \frac{1}{n}\right)^n = e < 3.$

80. If [x] be the greatest integer less than or equal to x, then $\sum_{n=8}^{100}\left[\frac{(-1)^n n}{2}\right]$ is equal to:

(A) –2 (B) 2 (C) 4 (D) 0

Sol. (C)

$$\sum_{n=8}^{100}\left[(-1)^n \frac{n}{2}\right] \Rightarrow \underbrace{[4]+[-4.5]}_{-1}+\underbrace{[5]+[-5.5]}_{-1}+........[49]+[-49.5]+[50]$$

$$\Rightarrow -1 \times 46 + 50 = 4$$

Integer

81. A fair coin is tossed n – times such that the probability of getting at least one head is at least 0.9. Then the minimum value of n is________.

Sol. (4)

$$1-\left(\frac{1}{2}\right)^n > 0.9 \quad \Rightarrow 0.1 > \left(\frac{1}{2}\right)^n \quad \Rightarrow n = 4$$

82. If $\left(\vec{a}+3\vec{b}\right)$ is perpendicular to $\left(7\vec{a}-5\vec{b}\right)$ and $\left(\vec{a}-4\vec{b}\right)$ is perpendicular to $\left(7\vec{a}-2\vec{b}\right)$, then the angle between $\vec{a}$ and $\vec{b}$ (in degrees) is ___________.

Sol. (60)

$$\left(\vec{a}+3\vec{b}\right)\cdot\left(7\vec{a}-5\vec{b}\right)=0$$

$$7|\vec{a}|^2-15|\vec{b}|^2+16\ \vec{a}\cdot\vec{b}=0 \qquad \text{.......} (1)$$

$$\left(\vec{a}-4\vec{b}\right)\cdot\left(7\vec{a}-2\vec{b}\right)=0$$

$$7|\vec{a}|^2+8|\vec{b}|^2-30\vec{a}\cdot\vec{b}=0 \qquad \text{.......} (2)$$

Equation (1) × 30

$$210|\vec{a}|^2-450|\vec{b}|^2+480\ \vec{a}\cdot\vec{b}=0 \qquad \text{.......} (3)$$

Equation (2) × 16

$$112|\vec{a}|^2+128|\vec{b}|^2-480\ \vec{a}\cdot\vec{b}=0 \qquad \text{.......} (4)$$

(3) + (4)

$$332|\vec{a}|^2-332|\vec{b}|^2=0$$

$$|\vec{a}|^2=|\vec{b}|^2$$

$$|\vec{a}|=|\vec{b}|$$

From equation (2)

$$15|\vec{a}|^2=30\ \vec{a}\cdot\vec{b}$$

$$15|\vec{a}|^2=30\ |\vec{a}||\vec{b}|\ \cos\theta$$

$$\cos\theta=\frac{15}{30}=\frac{1}{2}$$

$$\theta=60^{\circ}$$

83. If the co-efficients of x^7 and x^8 in the expansion of $\left(2+\frac{x}{3}\right)^n$ are equal, then the value of n is equal to ______________.

Sol. (55)

$$\left(2+\frac{x}{3}\right)^n = \sum_{r=0}^{n} {}^nC_r 2^{n-r}\left(\frac{x}{3}\right)^r$$

Coefficient of $x^7 = {}^nC_7\ 2^{n-7}.\ \left(\frac{1}{3}\right)^7$

Coefficient of $x^8 = {}^nC_8\ 2^{n-8}.\ \left(\frac{1}{3}\right)^8$

$\therefore\ {}^nC_7\ \frac{2^{n-7}}{3^7} = {}^nC_8\ \frac{2^{n-8}}{3^8}$

$\Rightarrow {}^nC_7\ .\ 6 = {}^nC_8$

$\Rightarrow\ \frac{6.n!}{7!.(n-7)!} = \frac{n!}{8!.(n-8)!}$

$\Rightarrow 48 = n - 7 \Rightarrow n = 55$

84. If the lines $\frac{x-k}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x+1}{3} = \frac{y+2}{2} = \frac{z+3}{1}$ are co-planar, then the value of k is ____________.

Sol. (1)

$$\begin{vmatrix} k+1 & 2+2 & 3+3 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow (k + 1)\ (-4)\ -4(-8) + 6\ (2-6) = 0$

$\Rightarrow (k + 1)\ (-4) = -8$

$k = 1.$

85. Consider the function $f(x) = \frac{P(x)}{\sin(x-2)}$, $x \neq 2$

$= 7$, $x = 2$

where P(x) is a polynomial such that P''(x) is always a constant and P(3) = 9. If f(x) is continuous at x = 2, then P(5) is equal to _______.

Sol. (39)

$P(x) = K(x-2)(x-\beta)$

$$\lim_{x\to 2} f(x) = \lim_{x\to 2}\frac{K(x-2)(x-\beta)}{\sin(x-2)}$$

$\Rightarrow K(2-\beta) = 7$ ....(1)

and $P(3) = K(3-2)(3-\beta) = 9$
$K(3-\beta) = 9$ ......(2)

Divide equation (1) by (2)

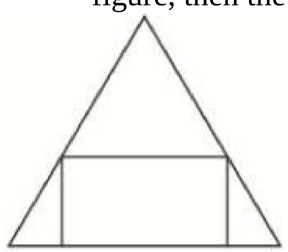
$$\frac{2-\beta}{3-\beta} = \frac{7}{9} \Rightarrow \beta = \frac{-3}{2}$$

So, $K = 2$

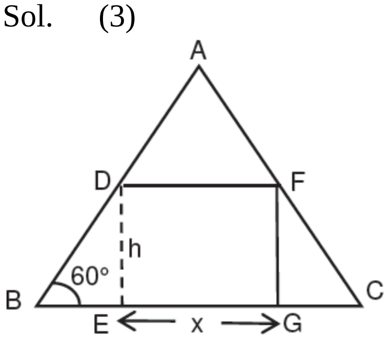
Then $P(x) = 2(x-2)\left(x+\frac{3}{2}\right)$

$P(5) = 2 \times (5-2) \times \left(5+\frac{3}{2}\right) = 39$

86. If a rectangle is inscribed in an equilateral triangle of side length $2\sqrt{2}$ as shown in the figure, then the square of the largest area of such a rectangle is ___________.



Sol. (3)



Area of rectangle = x. h ......(i)

from ΔBDE

h = BE tan 60

$$h = \frac{\left(2\sqrt{2} - x\right)}{2}.\sqrt{3} \quad ......(ii)$$

$$\text{So area, } A = \frac{\sqrt{3}}{2}\left(2\sqrt{2}\,x - x^2\right)$$

$$\text{For maxima } \frac{dA}{dx} = \frac{\sqrt{3}}{2}(2\sqrt{2} - 2x) = 0$$

$\Rightarrow x = \sqrt{2}$

$$\text{From (ii) } h = \sqrt{\frac{3}{2}}$$

Area = x. h = $\sqrt{3}$

$(\text{Area})^2 = 3$

87. If $a+b+c=1, ab+bc+ca=2$ and $abc=3$, then the value of $a^4+b^4+c^4$ is equal to _________.

Sol. (13)

$(a + b + c)^2 = 1$

$\Rightarrow a^2 + b^2 + c^2 + 2(ab + bc + ca) = 1$

$\Rightarrow a^2 + b^2 + c^2 = -3$ ….(i)

$\Rightarrow ab + bc + ca = 2$ ….(ii)

Squaring of equation (ii),

$\Rightarrow a^2b^2 + b^2c^2 + c^2a^2 + 2(ab^2c + bc^2a + ca^2b) = 4$

$\Rightarrow a^2b^2 + b^2c^2 + c^2a^2 + 2abc(a + b + c) = 4$

$\Rightarrow a^2b^2 + b^2c^2 + c^2a^2 + 6 = 4$

$\Rightarrow a^2b^2 + b^2c^2 + c^2a^2 = -2$ ….(iii)

Squaring of equation (i),

$\Rightarrow a^4 + b^4 + c^4 + 2(a^2b^2 + b^2c^2 + c^2a^2) = 9$

$\Rightarrow a^4 + b^4 + c^4 - 4 = 9$

$\Rightarrow a^4 + b^4 + c^4 = 13$

88. Let a curvy $y = f(x)$ pass through the point $\left(2, \left(\log_e 2\right)^2\right)$ and have slope $\frac{2y}{x \log_e x}$ for all positive real value of x. Then the value of f(e) is equal to ____________.

Sol. (1)

Slope of f(x) is $\frac{2y}{x\log_e x}$

$$\frac{dy}{dx}=\frac{2y}{x\log_e x}$$

$$\frac{dy}{2y}=\frac{2dx}{x\log_e x} \qquad \log_e x = t$$

$$\frac{1}{x}dx = dt$$

$\ell ny = 2\ell n(\ell nx) + \ell nC$

$$\ell ny = \ell n\frac{(\ell nx)^2}{C}$$

$$y = \frac{(\ell nx)^2}{C}$$

y(x) passes through the point $(2(\log e^2)^2$

$$(\ell n2)^2 = \frac{(\ell n2)^2}{C}$$

C = 1

$y = (\ell nx)^2$

$f(x) = (\ell nx)^2$

$f(e) = (\ell ne)^2 = 1$

89. Let $n \in N$ and [x] denote the greatest integer less than or equal to x. If the sum of $(n+1)$ terms ${}^nC_0, 3\cdot{}^nC_1, 5\cdot{}^nC_2, 7\cdot{}^nC_3, ...$ is equal to $2^{100}\cdot 101$, then $2\left[\frac{n-1}{2}\right]$ is equal to ________.

Sol. (98)

$^nC_0 + 3.\,^nC_1 + 5.\,^nC_2 + 7.\,^nC_3$ + ... (n + 1) terms = $\sum_{r=0}^{n}(2r+1).^nC_r$

$= 2\sum_{r=0}^{n} r.\,^nC_r + \sum_{r=0}^{n} {^nC_r}$

$= 2n \,.\, 2^{n-1} + 2^n = (n+1).2^n = (100+1)\,2^{100}$

$\Rightarrow n = 100$

$\Rightarrow 2\left[\frac{n-1}{2}\right] = 2\left[\frac{99}{2}\right] = 2 \times 49 = 98$

90. The equation of a circle is $\text{Re}\left(z^2\right)+2\left(lm\left(z\right)\right)^2+2\,\text{Re}\left(z\right)=0$, where $z = x+iy$. A line which passes through the centre of the given circle and the vertex of the parabola, $x^2-6x-y+13=0$, has y –intercept equal to ________.

Sol. (1)

$z = (x + iy)$

So, $z^2 = x^2 - y^2 + i2xy$

Now $x^2 - y^2 + 2y^2 + 2x = 0$

$x^2 + y^2 + 2x = 0 \Rightarrow$ centre = $(-1, 0)$ and $x^2 - 6x - y + 13 = 0$

$(x-3)^2 = (y-4)$

Vertex (3, 4)

$\therefore$ Equation of line is $(y-0) = \frac{4-0}{3+1}(x+1) \Rightarrow 4y = 4(x+1)$

$x - y + 1 = 0 \Rightarrow \frac{x}{-1} + \frac{y}{1} = 1$