PCM 25 July, 2021 (SHIFT-1)
Physics

1. A monoatomic ideal gas, initially at temperature T_1 is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to temperature T_2 by releasing the piston suddenly. If l_1 and l_2 are the lengths of the gas column, before and after the expansion respectively, then the value of $\frac{T_1}{T_2}$ will be:

(A) $\frac{l_2}{l_1}$ (B) $\frac{l_1}{l_2}$ (C) $\left(\frac{l_2}{l_1}\right)^{\frac{2}{3}}$ (D) $\left(\frac{l_1}{l_2}\right)^{\frac{2}{3}}$

Sol. (C)

For adiabatic process

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$= \left(\frac{V_2}{V_1}\right)^{5/3-1} = \left(\frac{A\ell_2}{A\ell_1}\right)^{2/3} = \left(\frac{\ell_2}{\ell_1}\right)^{2/3}$$

2. Some nuclei of a radioactive material are undergoing radioactive decay. The time gap between the instances when a quarter of the nuclei have decayed and when half of the nuclei have decayed is given as: (where λ is the decay constant)

(A) $\frac{\ln\frac{3}{2}}{\lambda}$ (B) $\frac{1}{2}\frac{\ln 2}{\lambda}$ (C) $\frac{\ln 2}{\lambda}$ (D) $\frac{2\ln 2}{\lambda}$

Sol. (A)

$$N = N_0 e^{-\lambda t}$$

$$\frac{3N_0}{4} = N_0 e^{-\lambda t_1}$$

$$t_1 = \frac{\ell n\left(\frac{4}{3}\right)}{\lambda}$$

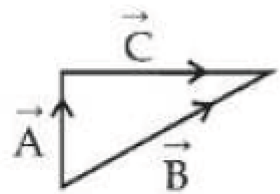
$$t_2 = \frac{\ell n 2}{\lambda}$$

$$t_2 - t_1 = \frac{\ell n 2}{\lambda} - \frac{\ell n\left(\frac{4}{3}\right)}{\lambda}$$

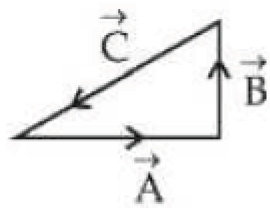
$$= \frac{\ell n\left(\frac{3}{2}\right)}{\lambda}$$

3. Match List-I with List-II

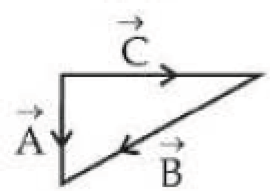
(a) $\vec{C} - \vec{A} - \vec{B} = 0$

(i) 

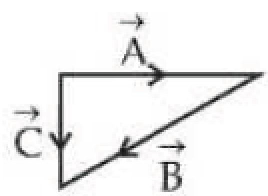
(b) $\vec{A} - \vec{C} - \vec{B} = 0$

(ii) 

(c) $\vec{B} - \vec{A} - \vec{C} = 0$

(iii) 

(d) $\vec{A} + \vec{B} = -\vec{C}$

(iv) 

Choose the correct answer from the options given below

(A) (a) $\to$ (iii), (b) $\to$ (ii), (c) $\to$ (iv), (d) $\to$ (i)
(B) (a) $\to$ (iv), (b) $\to$ (iii), (c) $\to$ (i), (d) $\to$ (ii)
(C) (a) $\to$ (i), (b) $\to$ (iv), (c) $\to$ (ii), (d) $\to$ (iii)
(D) (a) $\to$ (iv), (b) $\to$ (i), (c) $\to$ (iii), (d) $\to$ (ii)

Sol. (B)

(I) $\vec{A}+\vec{C}=\vec{B}$

(II) $\vec{A}+\vec{B}+\vec{C}=0$

(III) $\vec{A}-\vec{B}-\vec{C}=0$

(IV) $\vec{A}+\vec{B}-\vec{C}=0$

4. A particle of mass 4M at rest disintegrates into two particles of mass M and 3M respectively having non zero velocities. The ratio of de-Broglie wavelength of particle of mass M to that of mass 3M will be

(A) $1:\sqrt{3}$ (B) $3:1$ (C) 1 : 1 (D) 1 : 3

Sol. (C)

$$\lambda = \frac{h}{P}$$

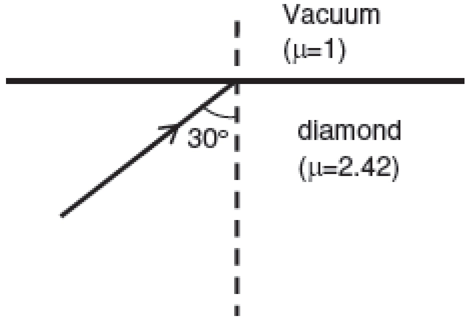
here momentum is same for both

$$\frac{\lambda_{2m}}{\lambda_m} = \frac{1}{1}$$

5. A ray of laser of a wavelength 630 nm is incident at an angle of $30°$ at the diamond-air interface. It is going from diamond to air. The refractive index of diamond is 2.42 and that of air is 1. Choose the correct option.

(A) angle of refraction is $30°$ (B) angle of refraction is $24.41°$
(C) refraction is not possible (D) angle of refraction is $53.4°$

Sol. (C)



Critical angle C = $\sin^{-1}\left(\frac{1}{2.42}\right) = 24.4^\circ$

Given Incident angle 30° > C

So there is TIR at interface

ray will not get refracted

6. In amplitude Modulation, the message signal

$V_m(t) = 10\sin\left(2\pi \times 10^5 t\right) volts\ and$

Carrier signal

$V_C(t) = 20\sin\left(2\pi \times 10^7 t\right) volts\ and$

The modulated signal now contains the message signal with lower side band and upper side band frequency, therefore the bandwidth of modulated signal is $\alpha\ kHz$. The value of is α is :

(A) 50 kHz (B) 200 kHz (C) 100 kHz (D) 0

Sol. (B)

Band width = $2f_m$

$= 2 \times 10^5$ Hz = 200 kHz

7. The half – life of ^{198}Au is 3 days. If atomic weight of ^{198}Au is 198 g/mol then the activity of 2 mg ^{198}Au is [in disintegration/second]

(A) 2.67×10^{12} (B) 6.06×10^{18}

(C) 32.36×10^{12} (D) 16.18×10^{12}

Sol. (D)

No. of Nuclei = $\frac{m}{M}.N_A = \frac{2 \times 10^{-3}}{198} \times 6.02 \times 10^{23}$

$A_0 = \lambda N_0 = \frac{0.693}{3 \times 24 \times 60 \times 60} \times \frac{2 \times 10^{-3}}{198} \times 6.02 \times 10^{23}$

$1618 \times 10^{-8} \times 10^{18} = 16.18 \times 10^{12}$

8. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A: Moment of inertia of a circular disc of mass 'M' and radius 'R' about X, Y axes (passing through its plane) and Z-axis which is perpendicular to its plane were found to be $l_x, l_y \& l_z$ respectively. The respective radii of gyration about all the three axes will be the same.

Reason R : A rigid body making rotational motion has fixed mass and shape.

In the light of the above statements, choose the most appropriate answer from the options given below :

(A) Both A and R are correct and R is the correct explanation of A.
(B) Both A and R are correct but R is NOT the correct explanation of A.
(C) A is not correct but R is correct.
(D) A is correct hut R is not correct.

Sol. (C)

9. A linearly polarized electromagnetic wave in vacuum is $E = 3.1\cos\left[(1.8)z - \left(5.4\times10^{6}\right)t\right]\hat{i}\ N/C$ is incident normally on a perfectly reflecting wall at z = a. Choose the correct option

(A) The frequency of electromagnetic wave is $54\times10^{4}\,Hz$
(B) The reflected wave will be $3.1\cos\left[(1.8)z + \left(5.4\times10^{6}\right)t\right]\hat{i}\ N/C$
(C) The transmitted wave will be $3.1\cos\left[(1.8)z - \left(5.4\times10^{6}\right)t\right]\hat{i}\ N/C$
(D) The wavelength is 5.4 m.

Sol. (B)

$\omega = 5.6 \times 10^3$

$2\pi f = 5.6 \times 10^3$

$$f = \frac{5.6\times10^3}{2\pi} = \frac{5.6\times10^3}{2\times3.14} = 891.7\,Hz$$

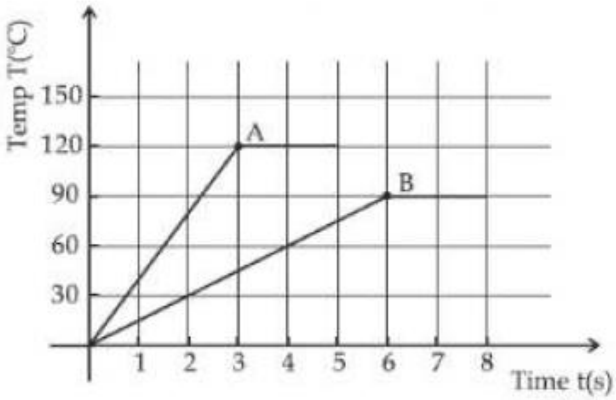
$C = f\lambda$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{891.7} = 3.36\times10^5\,m$$

Reflecting wave

$E = 3.1\cos[(1.8)z + (5.4\times10^6)t]\ \hat{i}\ N/C$

10. Two different metal bodies A and B of equal mass are heated at a uniform rate under similar conditions. The variation of temperature of the bodies is graphically represented as shown in the figure. The ratio of specific heat capacities is:



(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $\frac{8}{3}$ (D) $\frac{3}{8}$

Sol. (D)

$Q = nC\Delta T$

$$\frac{dQ}{dt} = nC\frac{dT}{dt}$$

Rate of heat is same for both gases

$$\text{So, } n_1C_1\left(\frac{dT}{dt}\right)_1 = n_2C_2\left(\frac{dT}{dt}\right)_2$$

$$\frac{C_1}{C_2} = \frac{\left(\frac{dT}{dt}\right)_2}{\left(\frac{dT}{dt}\right)_1} = \frac{\frac{90}{6}}{\frac{120}{3}} = \frac{90\times 3}{120\times 6} = \frac{3}{8}$$

11. Two wires of same length and radius are joined end to end and loaded. The Young's modulii of the materials of the two wires are Y_1 and Y_2. The combination behaves as a single wire then its Young's modulus is

(A) $Y = \frac{Y_1Y_2}{Y_1+Y_2}$ (B) $Y = \frac{Y_1Y_2}{2(Y_1+Y_2)}$

(C) $Y = \frac{2Y_1Y_2}{3(Y_1+Y_2)}$ (D) $Y = \frac{2Y_1Y_2}{Y_1+Y_2}$

Sol. (D)

$$K = \frac{K_1K_2}{K_1 + K_2}$$

$$\frac{YA}{2\ell} = \frac{\frac{Y_1A}{\ell} \times \frac{Y_2A}{\ell}}{\frac{Y_1A}{\ell} + \frac{Y_2A}{\ell}}$$

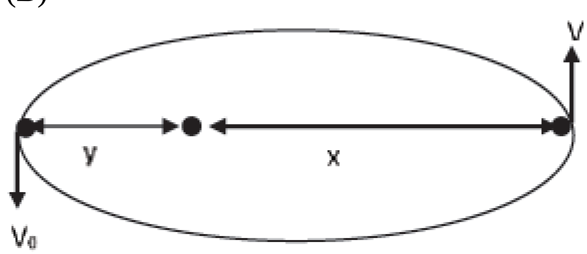
$$Y = \frac{2Y_1Y_2}{Y_1 + Y_2}$$

12. The minimum and maximum distances of a planet revolving around the Sun are x_1 and x_2. If the minimum speed of the planet on its trajectory is v_0 then its maximum speed will be:

(A) $\frac{v_0x_2^2}{x_1^2}$ (B) $\frac{v_0x_2}{x_1}$

(C) $\frac{v_0x_1}{x_2}$ (D) $\frac{v_0x_1^2}{x_2^2}$

Sol. (B)



By angular momentum conservation

$mv_0 x_2 = mvx_1$

$$v = \frac{v_0x_2}{x_1}$$

13. For a gas $C_P - C_V = R$ in s state P and $C_P - C_V = 1.10R$ in a state Q, T_P and T_Q are the temperature in two different state P and Q respectively. Then

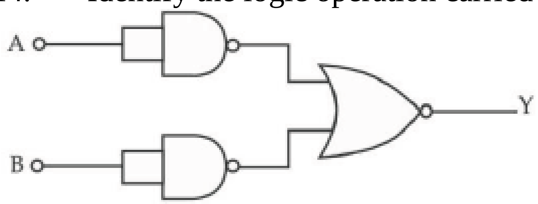
(A) $T_P > T_Q$ (B) $T_P = T_Q$

(C) $T_P = 0.9T_Q$ (D) $T_p < T_Q$

Sol. (A)

At high temperature gas behaves has ideal gas.

14. Identify the logic operation carried out



(A) OR (B) NOR (C) NAND (D) AND

Sol. (D)

15. What should be the order of arrangement of de-Broglie wavelength of electron (λ_e) an $\alpha-$ particle (λ_α) and proton (λ_p) given that all have the same kinetic energy?

(A) $\lambda_e = \lambda_p > \lambda_\alpha$ (B) $\lambda_e < \lambda_p < \lambda_\alpha$

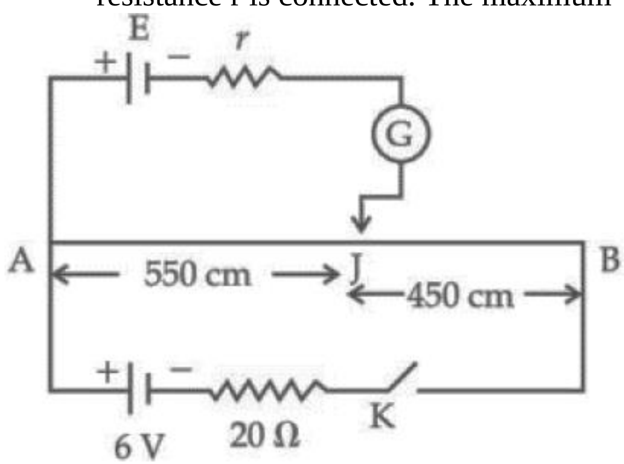
(C) $\lambda_e = \lambda_p = \lambda_\alpha$ (D) $\lambda_e > \lambda_p > \lambda_\alpha$

Sol. (D)

$$\lambda = \frac{h}{\sqrt{2mK.E.}} \Rightarrow \lambda \propto \frac{1}{\sqrt{m}}$$

$m_\alpha > m_p > m_e$, so $\lambda_e > \lambda_p > \lambda_\alpha$

16. In the given figure, there is a circuit of potentiometer of length AB = 10 m. The resistance per unit length of 0.1Ω per cm. Across AB, a battery of emf E and internal resistance r is connected. The maximum value of emf measured by this potentiometer is:



(A) 1.25 V (B) 6 V (C) 5 V (D) 2.75 V

Sol. (C)

At zero deflection

The maximum value of E that can be measured = V_{AB}

$$V_{AB} = \frac{6}{100+20} \times 100 = 5\text{ V}$$

17. Water droplets are coming from an open tap at a particular rate. The spacing between a droplets observed at 4th second after its fall to the next droplet is 34.3 m. At what rate the droplets are coming from the tap? $\left(Take\ g = 9.8\, m/s^2\right)$

(A) 1 drop/second (B) 1 drop/7seconds

(C) 2 drops/second (D) 3 drops/2seconds

Sol. (A)

Let next drop after t sec distance travelled by I[st] drop in 4 sec. is $S_1 = \frac{1}{2}at^2 = 78.4$ m (t should be less then 4 sec) distance travelled by succeeding drop in 4 – t sec

$S_2 = \frac{1}{2}a\,(4-t)^2$

$S_1 - S_2 = 34.3$

$78.4 - 4.9\,(4-t)^2 = 34.3$

$(4-t)^2 = 9\,;\quad 4 - t = 3$

$t = 1$ sec

18. A parallel plate capacitor with plate area 'A' and distance of separation d is filled with a dielectric. What is the capacity of the capacitor when permittivity of the dielectric varies as :

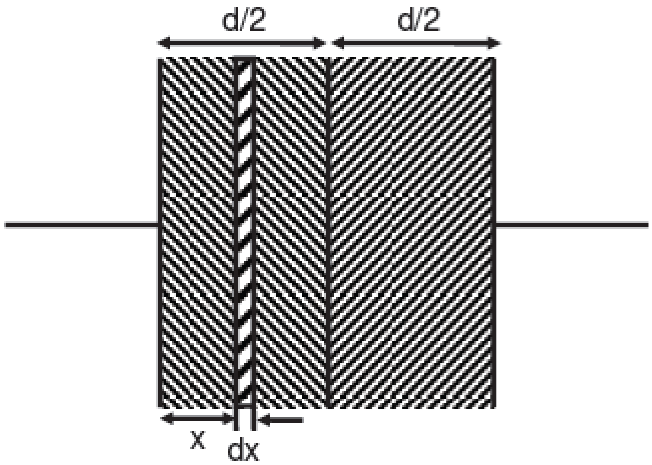
$$\varepsilon(x) = \varepsilon_0 + kx,\ \ for\left(0 < x \le \frac{d}{2}\right)$$

$$\varepsilon(x) = \varepsilon_0 + k(d-x),\ for\left(\frac{d}{2} \le x \le d\right)$$

(A) $\dfrac{kA}{2\ell n\left(\dfrac{2\varepsilon_0 + kd}{2\varepsilon_0}\right)}$ (B) $\left(\varepsilon_0 + \dfrac{kd}{2}\right)^{2/kA}$

(C) $\dfrac{kA}{2}\ell n\left(\dfrac{2\varepsilon_0}{2\varepsilon_0 - kd}\right)$ (D) 0

Sol. (A)



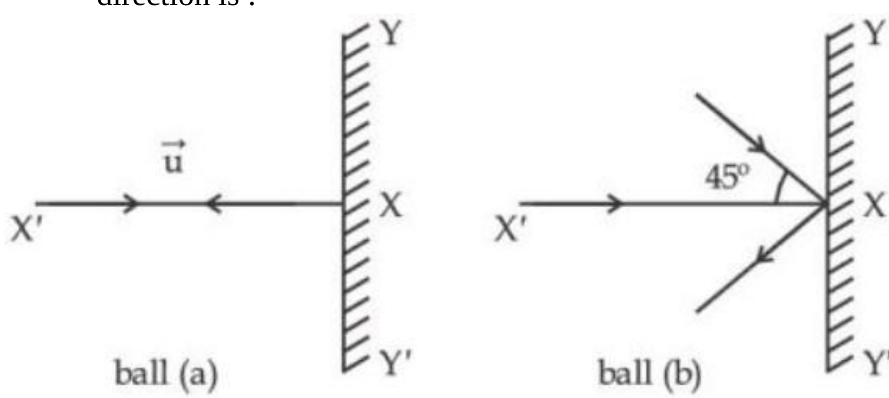
$$\frac{1}{C_{eq}} \Rightarrow \int \frac{dx}{A\varepsilon}$$

$$\Rightarrow \int_0^{d/2} \frac{dx}{A(\varepsilon_0 + kx)} + \int_{d/2}^{d} \frac{dx}{A[\varepsilon_0 + k(d-x)]}$$

$$\Rightarrow \frac{1}{A}\left\{\frac{1}{k}\left[\ell n(\varepsilon_0 + kx)\right]_0^{d/2} + \frac{1}{k}\left[\ell n(\varepsilon_0 + k(d-x))\right]_{d/2}^{d}\right\}$$

$$\Rightarrow \frac{1}{Ak} \times 2\ell n \frac{\varepsilon_0 + \frac{kd}{2}}{\varepsilon_0} \Rightarrow C_{eq} = \frac{kA}{2\ell n\left(\frac{2\varepsilon_0 + kd}{2\varepsilon_0}\right)}$$

19. Two billiard balls of equal mass 30 g strike a right wall with same speed of 108 kmph (as shown) but at different angles. If the balls get reflected with the same speed then the ratio of the magnitude of impulses imparted to ball ‘a’ and ball ‘b’ by the wall along ‘X’ direction is :



(A) $\sqrt{2}:1$ (B) $1:\sqrt{2}$ (C) 2 : 1 (D) 1 : 1

Sol. (A)

ratio of impulse

$$\frac{J_1}{J_2} = \frac{\Delta P_1}{\Delta P_2} = \frac{m(u+u)}{m\left(\frac{u}{\sqrt{2}} + \frac{u}{\sqrt{2}}\right)} = \sqrt{2}:1$$

20. In the Young's double slit experiment, the distance between the slits varies in time as $d(t) = d_0 + a_0 \sin \omega t$; where d_0, ω and a_0 are constants. The difference between the largest fringe width and the smallest fringe width obtained over time is given as:

(A) $\frac{2\lambda D a_0}{\left(d_0^2 - a_0^2\right)}$ (B) $\frac{2\lambda D\left(d_0\right)}{\left(d_0^2 - a_0^2\right)}$

(C) $\frac{\lambda D}{d_0 + a_0}$ (D) $\frac{\lambda D}{d_0^2} a_0$

Sol. (A)

$$\text{Fringe width} = \frac{D\lambda}{d}$$

$$\beta = \frac{D\lambda}{(d_0 + a_0 \sin wt)}$$

$\beta_{max} - \beta_{sin}$

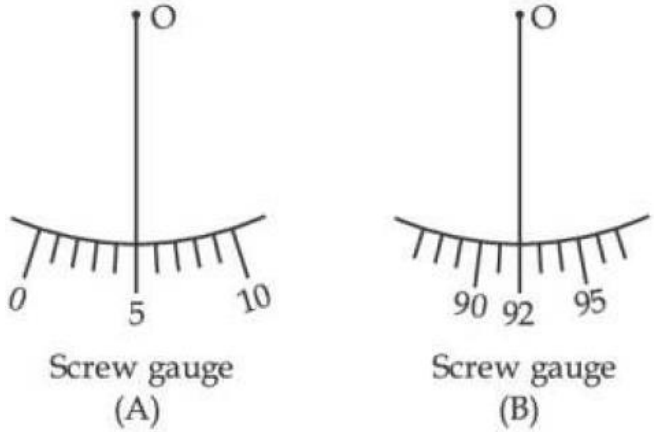
$$\Rightarrow \frac{D\lambda}{d_0 - a_0} - \frac{D\lambda}{d_0 + a_0}$$

$$\Rightarrow D\lambda \left[\frac{d_0 + a_0 - d_0 + a_0}{d_0^2 + a_0^2}\right]$$

$$= \frac{2\lambda D a_0}{(d_0^2 - a_0^2)}$$

Integer

21. Student A and student B used two screw gauges of equal pitch and 100 equal circular divisions to measure the radius of a given wire. The actual value of the radius of the wire is 0.322 cm. The absolute value of the difference between the final circular scale readings observed by the students A and B is _____. [Figures shows position of reference 'O' when jaws of screw gauge are closed] Given pitch = 0.1 cm.



Sol. (13)

The difference between the two student reading

= difference between zero error.

= 5 – (–8) = 13

22. The value of aluminum susceptibility is 2.2×10^{-5}. The percentage increase in the magnetic field if space within a current carrying toroid is filled with aluminum is $\frac{x}{10^4}$. Then the value of x is _______.

Sol. (2)

$\mu_r = 1 + \chi$

$\Delta\mu_r = \Delta\chi$

also $B \propto \mu_r$

$B = k\mu_r$ (k = constant)

$$\% \text{ change} = \frac{\Delta B}{B}\times100 = \frac{k(\Delta\mu_r)}{k\mu_r}\times100 = \frac{2.2\times10^{-5}}{1}\times100 = 0.022$$

x = 22

23. A body of mass 2 kg moving with a speed of 4 m/s, makes an elastic collision with another body at rest and continues to move in the original direction but with one fourth of its initial speed. The speed of the two body centre of mass is $\frac{x}{10} m/s$. Then the value of x is _____.

Sol. (25)

From linear momentum conservation

$2 \times 4 + 0 = 2 \times 1 + m_2 v_2$

From the definition of elastic collision

$v_2 - v_1 = e(u_1 - u_2)$

$v_2 - 1 = 1(4 - 0)$

$v_2 = 5$

$8 = 2 + m_2 \times 5$

$m_2 = 6/5$

$$V_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + v_2} = \frac{2 \times 4 + 0}{2 + \frac{6}{5}} = 2.5 \text{ m/s}$$

x = 25

24. A particle of mass 'm' is moving in time 't' on a trajectory given by $\vec{r} = 10\alpha t^2 \hat{i} + 2\beta(t-5)\hat{j}$ where a and b are dimensional constants. The angular momentum of the particle becomes the same as its was for t = 0 at time t = _______seconds.

Sol. (10)

$$\vec{r} = 10\alpha t^2 \hat{i} + 5\beta(t-5)\hat{j}$$

$$\vec{v} = 20\alpha t\,\hat{i} + 5\beta\hat{j}$$

$$\vec{L} = m(\vec{r} \times \vec{v})$$

$$\vec{L} = m\left(10\alpha t^2 \hat{i} + 5\beta(t-5)\hat{j}\right) \times \left(20\alpha t\hat{i} + 5\beta\hat{j}\right)$$

at t = 0, $\vec{L} = 0$

At any time t

$$\vec{L} = m\left(50\alpha\beta t\,\hat{k} - 100\alpha\beta(t-5)\right)\hat{k}$$

$0 = 50\ m\ \alpha\beta\ [t - 2\ (t - 5))\ \hat{k}$

$\Rightarrow t - 2t + 10 = 0$

$\Rightarrow t = 10$ sec

25. An inductor of 10mH is connected to a 20V battery through a resistor of $10k\Omega$ and a switch. After a long time, when maximum current is set up in the circuit, the current is

switched off. The current in the circuit after 1 ms is $\frac{x}{100} mA$. Then x is equal to ________. (Take $e^{-1} = 0.37$)

Sol. (74)

$$i_0 = \frac{20}{10} = 2A$$

$$i = i_0 e^{-Rt/L}$$

$$= 2 \times e^{-\frac{10\times10^{-3}}{10\times10^{-3}}} = \frac{2}{e} = 0.74A$$

$$x = 74$$

26. A pendulum bob ha a speed of 3 m/s at its lowest position. The pendulum is 50 cm long. The speed of bob, when the length makes an angle of 60° to the vertical will be $\left(g = 10 m/s^2\right)$ ____ m/s.

Sol. (2)

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg\ell(1-\cos 60°)$$

$$u^2 = v^2 + 2g\ell(1-\cos 60°)$$

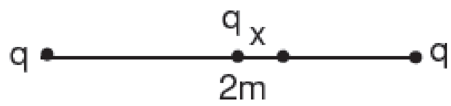
$$9 = v^2 + 20 \times 1/2 \times 1/2$$

$$9 = v^2 + 5$$

$$v = 2 \text{ m/s}$$

27. A particle of mass 1 mg and charge q is lying at the mid-point of two stationary particle kept at a distance 2m when each is carrying same charge q. If the free charged particle is displaced from its equilibrium position through distance x (x << 1m). The particle executes SHM. Its angular frequency of oscillation will be _____ $\times 10^5 \, rad/s$ if $q^2 = 10C^2$.

Sol. (6000)

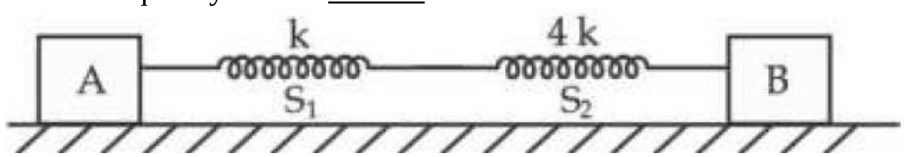


$$F_{net} = \frac{Kq^2}{(1-x)^2} - \frac{Kq^2}{(1+x)^2} = Kq^2\left[\frac{4x}{\left(1^2 - x^2\right)^2}\right]$$

$$a = \frac{4Kq^2x}{m}$$

$$\omega = 2\sqrt{\frac{Kq^2}{m}} = 2\times\sqrt{\frac{9\times10^9\times10}{10^{-6}}} = 6000$$

28. In the reported figure, two bodies A and B of masses 200 g and 800 g are attached with the system of springs. Springs are kept in a stretched position with some extension when the system is released. The horizontal surface is assumed to be frictionless. The angular frequency will be ______ rad/s when k = 20 N/m.



Sol. (10)

$$\mu = \frac{m_1m_2}{m_1+m_2} \Rightarrow \frac{200\times800}{200+800} = 160\,g = 0.16\,kg$$

$$K_{eq} = \frac{k_1k_2}{k_1+k_2} = \frac{4k\times k}{4k+k} = \frac{4}{5}k = \frac{4}{5}\times20 = 16\frac{N}{m}$$

$$\omega = \sqrt{\frac{K_{eq}}{\mu}} = \sqrt{\frac{16}{0.16}} = 10$$

29. A circular conducting coil of radius 1m is being heated by the change of magnetic field $\vec{B}$ passing perpendicular to the plane in which the coil is laid. The resistance of the coil is $2\mu\Omega$. The magnetic field is slowly switched off such that its magnitude changes in time as $B = \frac{4}{\pi}\times10^{-3}T\left(1-\frac{t}{100}\right)$. The energy dessipated by the coil before the magnetic field is switched off completely is E = ____ mJ.

Sol. (80)

$$\varepsilon = \left|-A\frac{dB}{dt}\right|$$

$$\varepsilon = \pi(1)^2 \times \frac{4}{\pi} \times \frac{10^{-3}}{100}$$

$\varepsilon = 4 \times 10^{-5}$ v

When B = 0, t = 100

$$\text{Energy} = \frac{\varepsilon^2}{R} \times t = \frac{16\times10^{-10}}{2\times10^{-6}} \times 100 = 8 \times 10^{-2} = 80 \text{ mJ}$$

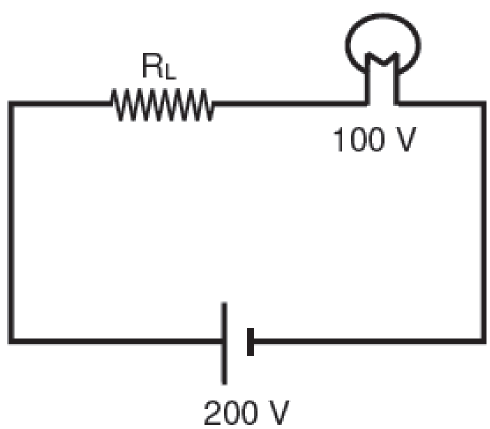
30. An electric bulb rated as 200 W at 100 V is used in a circuit having 200 V supply. The resistance R that must be put in series with the bulb so that the bulb delivers the same power is ______ Ω.

Sol. (50)

$$P = \frac{V^2}{R}$$

$$200 = \frac{(100)^2}{R}$$

$R = 50\ \Omega$



Current through bulb

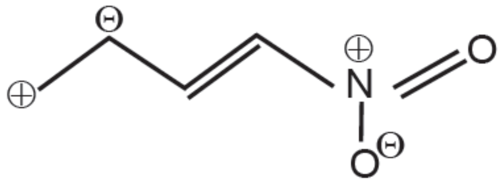
$$i = \frac{100}{50} \Rightarrow 2A$$

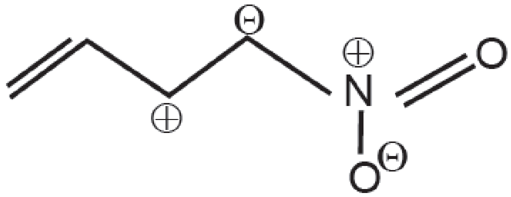
$V_L = 100$

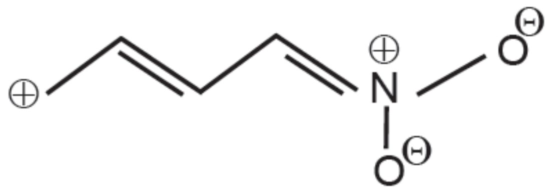
$$R_L = \frac{100}{2} = 50\ \Omega$$

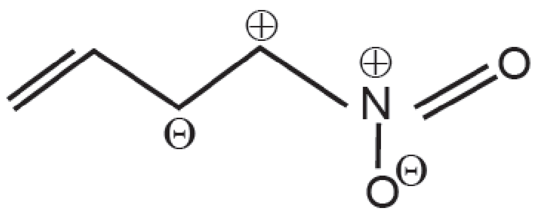
Chemistry

31. Which one among the following resonating structure is not correct ?

(A) 

(B) 

(C) 

(D) 

Sol. (C)

Same charge on adjacent atoms.

32. Given below are two statements, one is labelled as Assertion (A) and other is labelled as **Reason (R).**

Assertion (A) : Gabriel phthalimide synthesis cannot be used to prepare aromatic primary amines.

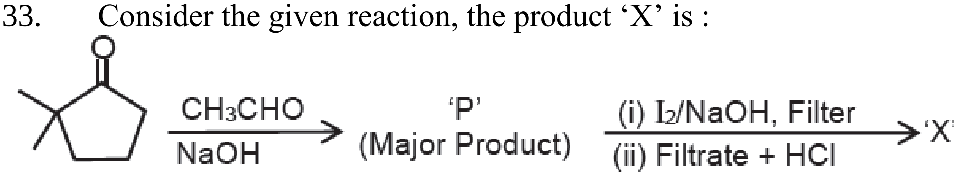
Reason (R) : Aryl halides do not undergo nucleophilic substitution reaction.

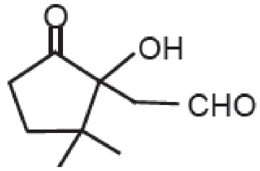
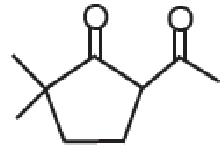
In the light of the above statements, choose the correct answer from the options given below:

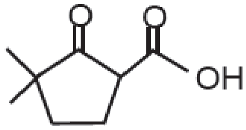
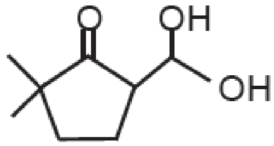
(A) (A) is true but (R) is false
(B) (A) is false but (R) is true
(C) Both (A) and (R) are true but (R) is not the correct explanation of (A)
(D) Both (A) and (R) are true but (R) is correct explanation of (A)

Sol. (D)

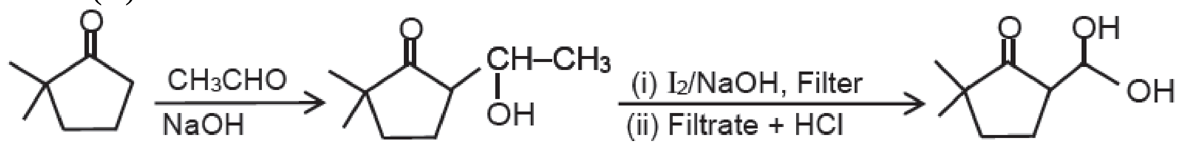
33. Consider the given reaction, the product 'X' is :



(A) 
(B) 

(C) 
(D) 

Sol. (D)



34. The ionic radii of K^+, Na^+, Al^{3+} and Mg^{2+} are in the order:

(A) $Al^{3+} < Mg^{2+} < Na^+ < K^+$

(B) $Al^{3+} < Mg^{2+} < K^+ < Na^+$

(C) $K^+ < Al^{3+} < Mg^{2+} < Na^+$

(D) $Na^+ < K^+ < Mg^{2+} < Al^{3+}$

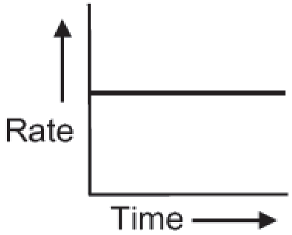
Sol. (A)

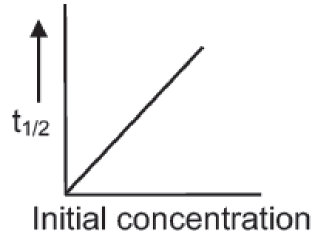
$K^+ > Na^+$ {moving down the group size increases}

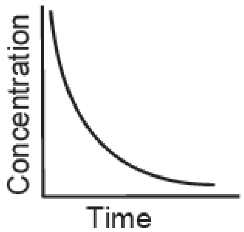
$Na^+ > Mg^{+2} > A\ell^{3+}$ {Isoelectronic species}

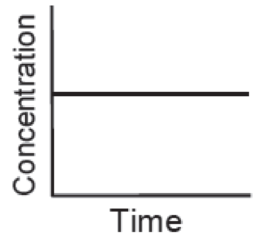
Z	11	12	13
E	10	10	10

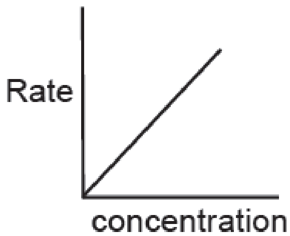
35. For the following graphs,

(a) 

(b) 

(c) 

(d) 

(e) 

Choose from the options given below, the correct one regarding order of reaction is :

(A) (b) Zero order (c) and (e) First order

(B) (b) and (d) Zero order (e) First order

(C) (a) and (b) Zero order (e) First order

(D) (a) and (b) Zero order (c) and (e) First order

Sol. (A)

$A_t = (A)_0 - Kt$. : Zero Oder

$$t_{1/2} = \left(\frac{A_0}{2K}\right)$$

$A = A_0 e^{-Kt}$; First oder

$$t_{1/2} = \frac{\ell n2}{K}$$

36. Given below are two statements :

Assertion I : None of the alkaline earth metal hydroxides dissolve in alkali.

Reason II : Solubility of alkaline earth metal hydroxide in water increases down the group.

In the light of the above statements, choose the most appropriate answer from the options given below:

(A) Statement I is incorrect but Statement II is correct.

(B) Statement I is correct but Statement II is incorrect.

(C) Statement I and Statement II both are incorrect.

(D) Statement I and Statement II both are correct.

Sol. (A)

Solubility of alkaline earth metal hydroxide in water increases down the group.

37. Which one of the following compounds of Group-14 elements is not known ?

(A) $[SiF_6]^{2-}$ (B) $[GeCl_6]^{2-}$

(C) $[SiCl_6]^{2-}$ (D) $[Sn(OH)_6]^{2-}$

Sol. (C)

The main reasons are :

(i) six large chloride ions cannot be accommodated around Si^{4+} due to limitation of its size.

(ii) interaction between lone pair of chloride ion and Si^{4+} is not very strong.

The species like, SiF_6^{2-}, $[GeCl_6]^{2-}$, $[Sn(OH)_6]^{2-}$ exist where the hybridisation of the central atom is sp^3d^2.

38. The water soluble protein is :

(1) Myosin (2) Collagen (3) Fibrin (4) Albumin

Sol. (D)

39. The correct order of following 3d metal oxides, according to their oxidation number is :

(a) CrO_3 (b) Fe_2O_3 (c) MnO_2 (d) V_2O_5

(e) Cu_2O

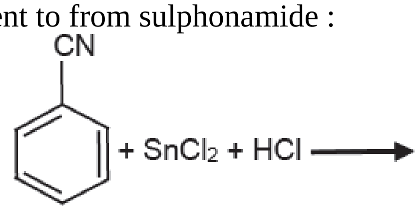
(A) (c) > (a) > (d) > (e) > (b) (B) (a) > (d) > (c) > (b) > (e)

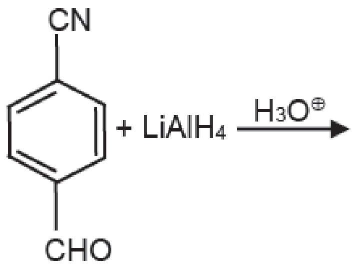
(C) (a) > (c) > (d) > (b) > (e) (D) (d) > (a) > (b) > (c) > (e)

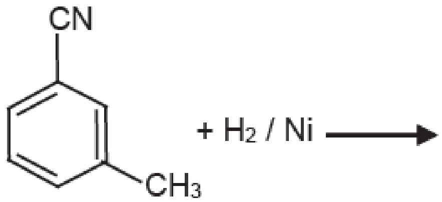
Sol. (B)

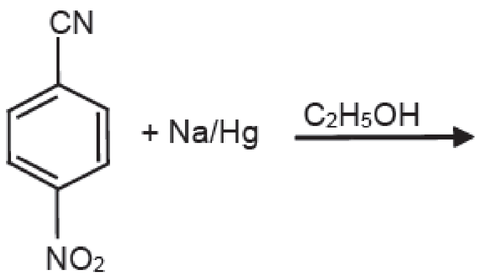
	Compound	Oxidation state of metal
(a)	CrO_3	+6
(b)	Fe_2O_3	+3
(c)	MnO_2	+4
(d)	V_2O_5	+5
(e)	Cu_2O	+1

40. Which one of the products of the following reactions does not react with Hinsberg reagent to from sulphonamide :

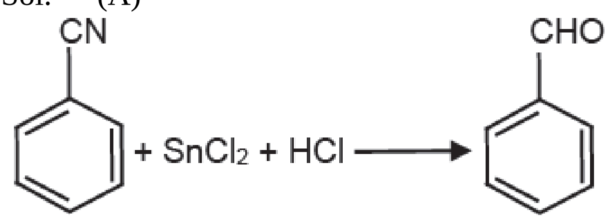
(A) 

(B) 

(C) 

(D) 

Sol. (A)



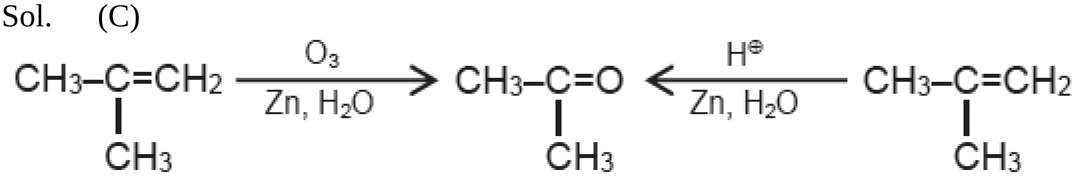
(does not react with Hinsberg reagent)

41. An organic compound 'A' C_4H_8 on treatment with $KMnO_4 / H^+$ yields compound 'B' C_3H_6O , Compound 'A' also yields compound 'B' an ozonolysis. Compound 'A' is :

(A) But-2-ene (B) Cyclobutane

(C) 2-Methylpropenene (D) 1-Methylcyclopropane

Sol. (C)



42. In the leaching of alumina from bauxite, the ore expected to leach out in the process by :

(A) SiO_2 (B) TiO_2 (C) Fe_2O_3 (D) ZnO

Sol. (A)

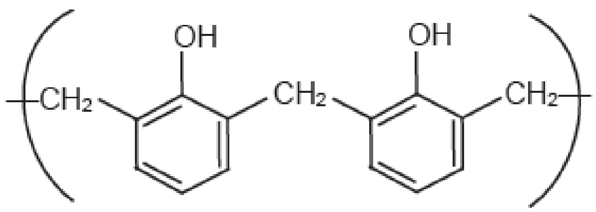
Bauxite, usually contains SiO_2, iron oxide and titanium oxide (TiO_2) as impurities.

Concentration is carried out by digesting the powdered ore with a concentrated solution of NaOH at 473-523 K and 35-36 bar pressure. This way, Al_2O_3 is leached out as sodium aluminate (and also SiO_2 as sodium silicate) leaving behind the impurities, iron oxide and titanium oxide.

$$Al_2O_3.xH_2O + NaOH \longrightarrow NaAlO_2$$

$$NaAlO_2 \xrightarrow{H_2O} Al(OH)_3 + NaOH$$

$$\underset{}{Al(OH)_3} \xrightarrow{1200K} \underset{\text{(pure)}}{Al_2O_3}$$

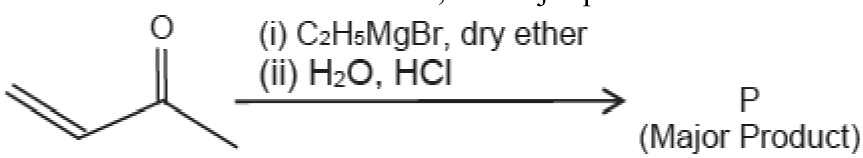
43. 

(A) Buna-N (B) Acrilan (C) Novolac (D) Neoprene

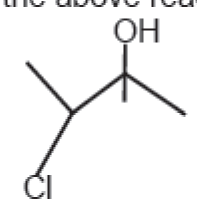
Sol. (C)

Novolac is linear polymer.

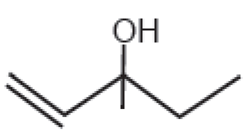
44. Consider the below reaction, the major product 'P' is :

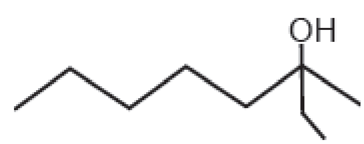


Consider the above reaction, the major product 'P' is :

(A) 

(B) 

(C) 

(D) 

Sol. (A)

45. Which one of the following species responds to an external magnetic field ?

(A) $\left[Co(CN)_6\right]^{3-}$ (B) $\left[Fe(H_2O)_6\right]^{3+}$

(C) $\left[Ni(CO)_4\right]$ (D) $\left[Ni(CO)_4\right]^{2-}$

Sol. (B)

Fe^{3+}; $3d^5$

If will contain 5 unpaired electrons.

Thus it is paramagnetic and attracted in external magnetic field.

46. At 298.2 K the relationship between enthalpy of bond dissociation $\left(in\, kJ\ mol^{-1}\right)$ for hydrogen $\left(E_H\right)$ and its isotope, deuterium $\left(E_D\right)$ is best described by:

(A) $E_H = \frac{1}{2}E_D$ (B) $E_H = 2E_D$

(C) $E_H = E_D$ (D) $E_H \simeq E_D - 7.5$

Sol. (D)

H–H bond desociation energy 435.90 KJ/mol

D–D bond desociation energy 443.40 KJ/mol

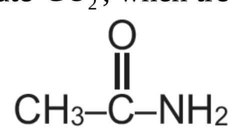
$E_H \simeq E_D - 7.5$

47. Which one of the following chemical agent is not being used for dry-cleaning of clothes ?

(A) H_2O_2 (B) CCl_4 (C) $Cl_2C = CCl_2$ (D) Liquid CO_2

Sol. (B)

48. Which one of the following compounds will liberate CO_2, when treated with $NaHCO_3$?

(A) $(CH_3)_3\overset{\oplus}{N}H\overset{-}{Cl}$ (B) 

(C) CH_3NH_2 (D) $(CH_3)_4\overset{\oplus}{N}\overset{-}{O}H$

Sol. (A)

HCl is stronger acid than $H_2CO_3\left(H_2O + CO_2\right)$.

49. Sodium stearate $CH_3\left(CH_2\right)_{16}COO^-Na^+$ is an anionic surfactant which forms micelles in oil. Choose the correct statement for it from the following:

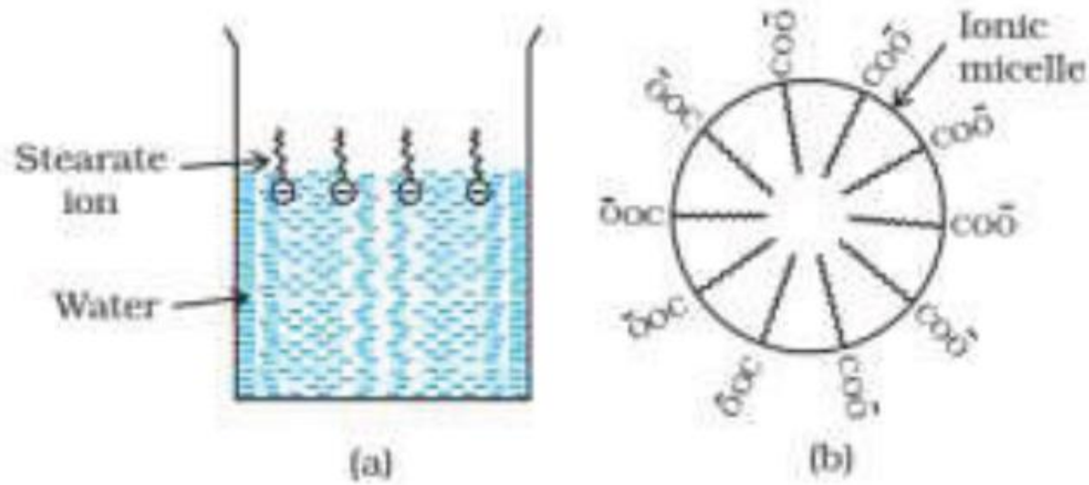
(A) It forms non-spherical micelles with $–COO^-$ group pointing outwards on the surface.

(B) It forms non-spherical micelles with $CH_3\left(CH_2\right)_{16}$ – group pointing towards the centre.

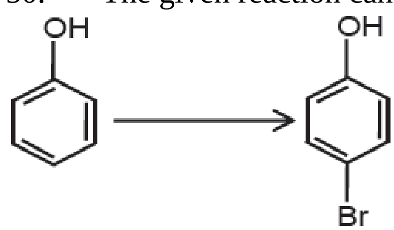
(C) It forms spherical micelles with $CH_3\left(CH_2\right)_{16}$ – group pointing towards the centre of sphere.

(D) It forms spherical micelles with $CH_3\left(CH_2\right)_{16}$ – group pointing outwards on the surface of sphere.

Sol. (C)



50. The given reaction can occur in the presence of :



(Major Product)

The given reaction can occur in the presence of :

(1) Bromine water (2) Br_2 in CS_2, 273 K (3) $Br_2 / FeBr_3$ (4) Br_2 in $CHCl_3$, 273 K

(A) (1), (2) and (4) only (B) (2) and (4) only

(C) (b), (c) and (d) only (D) (a) and (c) only

Sol. (C)

Phenol with bromine water give 2,4,6-Tribromo phenol while rest are give ortho or para bromophenols.

Integer

51. When 10 mL of an aqueous solution of Fe^{2+} ions was titrated in the presence of dil H_2SO_4 using diphenylamine indicator, 15 mL of 0.02 M solution of $K_2Cr_2O_7$ was required to get the end point. The molarity of the solution containing Fe^{2+} ions is $x \times 10^{-2} M$. The value of x is ________. (Nearest Integer)

Sol. (18

$Cr_2O_7^{2-} + Fe^{2+} \rightarrow Fe^{3+} + Cr^{3+}$

15 ml   10 ml

0.1 M

$N_1V_1 = N_2V_2$

$15 \times 0.02 \times 6 = 10 \times M \times 1$

$$M = \frac{15 \times 0.02 \times 6}{10} = 0.18 = 18 \times 10^{-2}\,M$$

52. CO_2 gas is bubbled through water during a soft drink manufacturing process at 298 K. If CO_2 exerts a partial pressure of 0.835 bar then x m mol of CO_2 would dissolve in 0.9 L of water. The value of x is ……………… (Nearest integer)
(Henry's law constant for CO_2 at 298 K is 1.67×10^3 bar)

Sol. (25)

$$P_{CO_2} = K_H X_{CO_2}$$

$$X_{CO_2} = \frac{P_{CO_2}}{K_H} = \frac{0.835}{0.835 \times 2 \times 10^3}$$

$$X_{CO_2} = 0.5 \times 10^{-3}$$

$$\text{Number of moles of water} = \frac{900}{18} = 50$$

$$\frac{n_{CO_2}}{n_{CO_2} + nH_2O} = 0.5 \times 10^{-3}$$

(n_{CO_2} in denominator is neglected as it is << 50)

$n_{CO_2} = 0.5 \times 10^{-3} \times 50 = 25 \times 10^{-3}$ moles

= 25 milimoles

53. Consider the complete combustion of butane, the amount of butane utilized to produce 72.0 g of water is………..$\times 10^{-1} g$ (in nearest integer)

Sol. (464)

$$C_4H_{10} + \frac{13}{2}O_2 \longrightarrow 4CO_2 + 5H_2O$$

$$C_nH_{2n+2} + \left(\frac{3n+1}{2}\right)O_2 \rightarrow n\ CO_2 + (n+1)\ H_2O$$

$\because$ 1 mole C_4H10 produces 5 mole H_2O.

5 mole (5 × 18) = 90 g.

90 g $\rightarrow$ 58 g

$$72\text{ g} \rightarrow \left(\frac{58 \times 72}{90}\right) = 46.4\text{ g}$$

$= 464 \times 10^{-1}$ g

54. A source of monochromatic radiation of wavelength 400 nm provides 1000 J of energy in 10 seconds. When this radiation falls on the surface of sodium, $x \times 10^{20}$ electrons are ejected per second. Assume that wavelength 400 nm is sufficient for ejection of electron from the surface of sodium metal. The value of x is………. (Nearest integer) $\left(h = 6.626 \times 10^{-34}\ Js\right)$

Sol. (2)

Energy per second = 100 J

$$E = E_0 + KE \qquad \text{so } E = E_0 = \left[\frac{hc}{\lambda}\right] = \left\{\frac{6.62 \times 10^{-32} \times 3 \times 10^8}{400 \times 10^{-9}}\right\}$$

$= 0.04965 \times 10^{-17}$ J $= 4.965 \times 10^{-19}$ J

Number of of electron ejected by 100 J energy $= \frac{100}{4.965 \times 10^{-19}} = 20.140 \times 10^{19} = 2.01 \times 10^{20}$

55. For the reaction

$A + B \rightleftharpoons 2C$

the value of equilibrium constant is 100 at 298 K. If the initial concentration of all the three species is 1 M each, then the equilibrium concentration of C is $x \times 10^{-1} M$. The value of x is…………(Nearest Integer)

Sol. (25)

$$A + B \rightleftharpoons 2C \quad K_C = 100$$

	A	B	2C
t = 0	1	1	1
t = $t_{eq.}$	1–x	1–x	1 + 2x

$$Kc = \frac{(1+2x)^2}{(1-x)^2}$$

$$100 = \frac{(1+2x)^2}{(1-x)^2} \text{ of [C]}$$

$$10 = \frac{(1+2x)}{(1-x)}$$

$$X = \frac{3}{4}$$

Concentration of [C] = $1 + 2\left(\frac{3}{4}\right) = 25 \times 10^{-1}$

56. At 298 K, the enthalpy of a solid (X) is $2.8\,kJ\,mol^{-1}$ and the enthalpy of vaporisation of the liquid (X) is $98.2\,kJ\,mol^{-1}$. The enthalpy of sublimation of the substance (X) in $kJ\,mol^{-1}$ is……… (in nearest integer)

Sol. (101)

$$\Delta H_{Sublimation} = \Delta H_{vap} + \Delta H_{fusion}$$
$$= 98.6 + 2.4$$
$$= 101 \text{ K.Cal mol}^{-1}$$

57. Consider the cell at $25°C$

$Zn\left|Zn^{2+}(aq),(1M)\right|\left|Fe^{3+}(aq), Fe^{2+}(aq)\right|Pt(s)$

The fraction of total iron present as Fe^{3+} ion at the cell potential of 1.500 V is $x \times 10^{-2}$. The value of x is _____________(Nearest integer)

(Given $E^0_{Fe^{3+}/Fe^{2+}} = 0.77V, E^0_{Zn^{2+}/Zn} = -0.76V$)

Sol. (24)

Cathode = $Fe^{3+} + e^- \longrightarrow Fe^{2+}$ $\qquad E^0_{Fe^{3+}|Fe^{2+}} = 0.77\ V$

Anode $\Rightarrow Zn(s) \longrightarrow Zn^{2+}(aq) + 2e^-$ $\qquad E^0_{Zn^{2+}|Zn} = -0.76\ V$

$$E_{cell} = E^0_{cell}\ \frac{-0.059}{2}\log\frac{[Zn^{2+}][Fe^{2+}]^2}{[Fe^{3+}]^2}$$

$$1.5 = 1.53 - \frac{-0.06}{2}\log\left(\frac{[Fe^{2+}]}{[Fe^{3+}]}\right)^2$$

$$10 = \left(\frac{[Fe^{2+}]}{[Fe^{3+}]}\right)^2$$

$$\frac{[Fe^{3+}]}{[Fe^{2+}]} = \frac{1}{\sqrt{10}} = 0.316$$

$$\text{Fraction of } Fe^{3+} = \frac{[Fe^{3+}]}{[Fe^{3+}]+[Fe^{2+}]} = \frac{0.316}{1.316} = 0.24 = 24\times 10^{-2}$$

58. The number of sigma bonds in

$$H_3C-\underset{\underset{H}{|}}{C}=CH-C\equiv C-H$$

Sol. (10)

$10\sigma, 3\pi$.

59. Three moles of $AgCl$ get precipitated when one mole of an octahedral co-ordination compound with empirical formula $CrCl_3.3NH_3.3H_2O$ reacts with excess of silver nitrate. The number of chloride ions satisfying the secondary valency of the metal ion is _______.

Sol. (0)

$$[Cr(NH_3)_3(H_2O)]Cl_3 + AgNO_3 \longrightarrow \underset{\text{White ppt.}}{3AgCl\downarrow}$$

The three chloride ions satisfies only primary valency.

60. A home owner uses $4.00\times10^3 m^3$ of methane (CH_4) gas, (assume CH_4 is an ideal gas) in a year to heat his home. Under the pressure of 1.0 atm and 300 K, mass of gas used is $x\times10^5 g$. The value of x is _____________ (Nearest integer) (Given $R = 0.083\,L\,atm\,K^{-1}mol^{-1}$)

Sol. (26)

Volume of methane gas (V) = 4×10^3 m^3 = 4×10^6 Lt

Pressure of the gas (P) = 1.0 atm

Temperature (T) = 300 K

PV = nRT

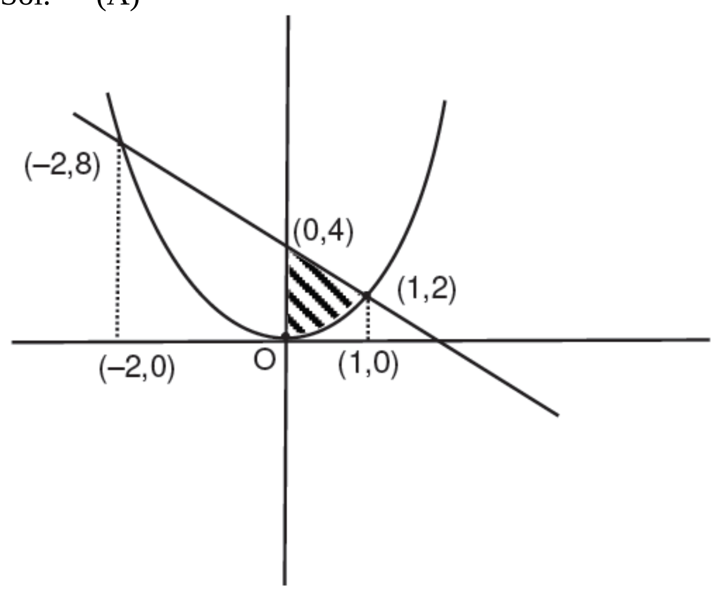
$$W = \frac{PVM}{RT} = \frac{1 \times 4 \times 10^6 \times 64}{0.083 \times 300} = 25.7 \times 10^5 \text{ g} \approx 26 \times 10^5 \text{ g}.$$

Mathematics

61. The area (in sq. units) of the region, given by the set $\left\{(x, y) \in R \times R \middle| x \geq 0, 2x^2 \leq y \leq 4-2x\right\}$ is:

(A) $\frac{7}{3}$ (B) $\frac{17}{3}$ (C) $\frac{8}{3}$ (D) $\frac{13}{3}$

Sol. (A)



$2x^2 = 4 - 2x$

$x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$x = -2, x = 1$

Required area = $\frac{1}{2}(2+4) \times 1 - \int_0^1 2x^2 dx = 3 - \frac{2}{3} = \frac{7}{3}$ square units

62. The locus of the centroid of the triangle formed by any point P on the hyperbola $16x^2 - 9y^2 + 32x + 36y - 164 = 0$ and its foci is:

(A) $16x^2 - 9y^2 + 32x + 36y - 36 = 0$

(B) $9x^2 - 16y^2 + 36x + 32y - 36 = 0$

(C) $16x^2 - 9y^2 + 32x + 36y - 144 = 0$

(D) $9x^2 - 16y^2 + 36x + 32y - 144 = 0$

Sol. (A)

Given $\frac{(x+1)^2}{9} - \frac{(y-2)^2}{16} = 1$

Let $x + 1 = X$

$y - 2 = Y$

$\frac{X^2}{9} - \frac{Y^2}{16} = 1$

$a = 3,\ b = 4$

$b^2 = a^2(e^2 - 1) \Rightarrow e = \frac{5}{3}$

Focus $(\pm ae, 0) \Rightarrow X = \pm ae,\quad Y = 0$

$x + 1 = \pm 5,\ y - 2 = 0$

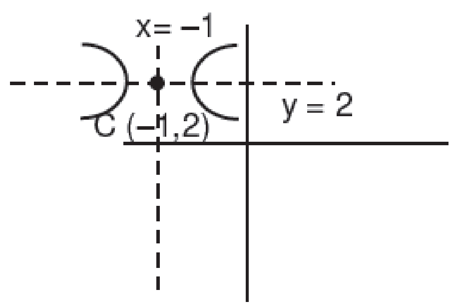
$x = -6, 4,\ y = 2$

Hence focus $S(-6, 2)$, $S'(4, 2)$

Let any point on hyperbola $x + 1 = 3\sec\theta$, $y - 2 = 4\tan\theta \Rightarrow P(-1 + 3\sec\theta, 2 + 4\tan\theta)$

Hence centroid is $\equiv \left(\frac{-6+4-1+3\sec\theta}{3}, \frac{2+2+2+4\tan\theta}{3}\right)$

$h = \frac{-3+3\sec\theta}{3} \Rightarrow \sec\theta = \frac{3h+3}{3}$



$\Rightarrow \tan\theta = \frac{3k-6}{4}$

$\sec^2\theta - \tan^2\theta = 1$

$\left(\frac{3h+3}{3}\right)^2 - \frac{(3k-6)^2}{16} = 1$

Locus is $\frac{9(x+1)^2}{9} - \frac{9(y-2)^2}{16} = 1$

$\Rightarrow 16x^2 - 9y^2 + 32x + 36y - 36 = 0$

63. Let S_n be the sum of the first n terms of an arithmetic progression. If $S_{3n} = 3S_{2n}$, then the value of $\frac{S_{4n}}{S_{2n}}$ is:

(A) 6 (B) 8 (C) 4 (D) 2

Sol. (A)

$$\frac{S_{3n}}{S_{2n}} = \frac{\frac{3n}{2}\left[2a+(3n-1)d\right]}{\frac{2n}{2}\left[2a+(2n-1)d\right]} = 3$$

$\Rightarrow$ 2a+(3n–1)d = 2[2a + (2n–1)d]

$\Rightarrow$ 2a+(n–1)d = 0 ……(1)

$$\text{Now } \frac{S_{4n}}{S_{2n}} = \frac{\frac{4n}{2}\left[2a+(4n-1)d\right]}{\frac{2n}{2}\left[2a+(2n-1)d\right]}$$

$$= \frac{2\left[2a+(4n-1)d\right]}{\left[2a+(2n-1)d\right]}$$

Put, $2a = -(n-1)d$, we have, $\frac{S_{4n}}{S_{2n}} = \frac{2\left[3nd\right]}{nd} = 6$

64. Let 9 distinct balls distributed among 4 boxes, B_1, B_2, B_3 and B_4. If the probability that B_3 contains exactly 3 balls is $k\left(\frac{3}{4}\right)^9$ then k lies in the set

(A) $\{x \in R : |x-1| < 1\}$

(B) $\{x \in R : |x-2| \le 1\}$

(C) $\{x \in R : |x-3| < 1\}$

(D) $\{x \in R : |x-5| \le 1\}$

Sol. (C)

The numbers of ways of distributing 9 distinct balls in 4 boxes B_1, B_2, B_3, and B_4, is = 4^9

When box B_3, contains exactly 3 balls then number of ways = ${}^9C_3 \times (3)^6$

$$\text{Probability} = {}^9C_3 \times \frac{3^6}{4^9} = \frac{28}{9}\left(\frac{3}{4}\right)^9$$

$$\text{Hence} \quad k = \frac{28}{9} = 3\frac{1}{9}$$

Clearly $2<k<4 \Rightarrow k \in \{x \in R : |x-3| < 1\}$

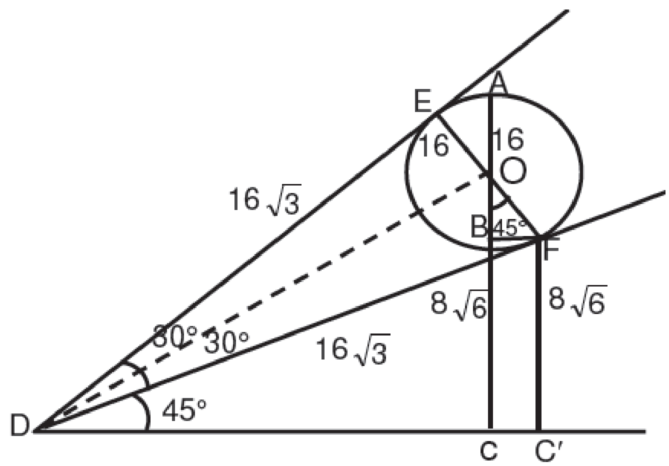
65. A spherical gas balloon of radius 16 meter subtends an angle $60°$ at the eye of the observer A while the angle of elevation of its center from the eyes of A is $75°$. Then the height (in meter) of the top most point of the balloon from the level of the observer's eye is:

(A) $8\left(\sqrt{2}+2+\sqrt{3}\right)$ (B) $8\left(\sqrt{6}+\sqrt{2}+2\right)$

(C) $8\left(2+2\sqrt{3}+\sqrt{2}\right)$ (D) $8\left(\sqrt{6}-\sqrt{2}+2\right)$

Sol. (B)

$ED = \dfrac{16}{\tan 30°} = 16\sqrt{3}$



$DF = 16\sqrt{3}$

Now in Δ DFC′

$C'F = 16\sqrt{3}.\sin 45°$

$= 16\sqrt{3}.\dfrac{1}{\sqrt{2}} = 8\sqrt{6}$

$CB = 8\sqrt{6}$

In Δ^{le} OBF

$OB = 16\cos 45°$

$= \dfrac{16}{\sqrt{2}} = 8\sqrt{2}$

Height of top most point $= 8\sqrt{6}+8\sqrt{2}+16 = 8\left(\sqrt{6}+\sqrt{2}+2\right)$

66. The values of a and b, for which the system of equations

$2x+3y+6z=8$

$x+2y+az=5$

$3x+5y+9z=b$

has no solution, are:

(A) $a=3, b=13$ (B) $a=3, b\neq 13$

(C) $a\neq 3, b=3$ (D) $a\neq 3, b\neq 13$

Sol. (B)

$\Delta = 0$ and any one of Δ_1 , Δ_2 and Δ_3 should not be equal to zero

$$\Delta = \begin{vmatrix} 2 & 3 & 6 \\ 1 & 2 & a \\ 3 & 5 & 9 \end{vmatrix} = 0$$

$\Delta = 2[18 - 5a] - 3[9 - 3a] + 6[5 - 6]$
$\Delta = 3 - a = 0$
$a = 3$

$$\Delta_1 = \begin{vmatrix} 8 & 3 & 6 \\ 5 & 2 & 3 \\ b & 5 & 9 \end{vmatrix} = 8[18 - 15] - 3[45 - 3b] + 6[25 - 2b] = 3(13 - b)$$

$$\Delta_2 = \begin{vmatrix} 2 & 8 & 6 \\ 1 & 5 & 3 \\ 3 & b & 9 \end{vmatrix} = 3\begin{vmatrix} 2 & 8 & 2 \\ 1 & 5 & 1 \\ 3 & b & 3 \end{vmatrix} = 0$$

$$\Delta_3 = \begin{vmatrix} 2 & 3 & 8 \\ 1 & 2 & 5 \\ 3 & 5 & b \end{vmatrix} = 2[2b - 25] - 3[b - 15] + 8[5 - 6] = b - 13$$

If $b = 13$, then Δ_1, Δ_2, and Δ_3 all will be zero.

$\therefore b \neq 13$

67. Let $f : R \to R$ be defined as

$$f(x) = \begin{cases} \dfrac{\lambda\left|x^2 - 5x + 6\right|}{\mu\left(5x - x^2 - 6\right)} & , \quad x < 2 \\ e^{\frac{\tan(x-2)}{x-[x]}} & , \quad x > 2 \\ \mu & , \quad x = 2 \end{cases}$$

where [x] is the greatest integer less than or equal to x. If f is continuous at x = 2, then $\lambda + \mu$ is equal to:

(A) $e(e-2)$ (B) $2e-1$ (C) 1 (D) $e(-e+1)$

Sol. (D)

$$\text{RHL} = \lim_{x \to 2^+} e^{\frac{\tan(x-2)}{x-[x]}} = \lim_{x \to 2^+} e^{\frac{\tan(x-2)}{(x-2)}} = e$$

$$\text{LHL} = \lim_{x \to 2^-} \frac{\lambda}{\mu} \frac{|x^2 - 5x + 6|}{5x - 6 - x^2}$$

For $x < 2$, $|x^2 - 5x + 6| = x^2 - 5x + 6$

$$\therefore \text{LHL} = \lim_{x \to 2^-} \frac{\lambda}{\mu} \frac{|x^2 - 5x + 6|}{(5x - 6 - x^2)} = \frac{-\lambda}{\mu}$$

Also, $f(2) = \mu$

For $f(x)$ to be continuous at $x = 2$,

RHL = LHL = f(2)

$$\therefore e = \frac{-\lambda}{\mu} = \mu$$

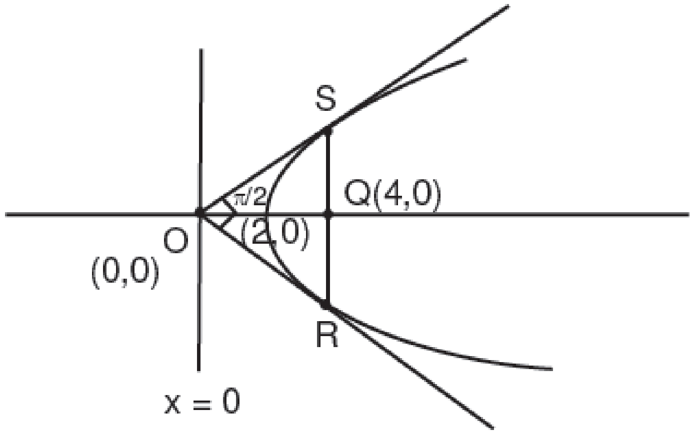
$$\Rightarrow \mu = e \text{ and } \lambda = -e^2$$

$$\therefore \lambda + \mu = e - e^2$$

68. Let a parabola P be such that its vertex and focus lie on the positive x-axis at a distance 2 and 4 units from the origin, respectively. If tangents are drawn from O(0,0) to the parabola P which meet P at S and R, then the area (in sq. units) of ΔSOR is equal to:

(A) $8\sqrt{2}$ (B) 32 (C) 16 (D) $16\sqrt{2}$

Sol. (C)



Equation of parabola

$(y-0)^2 = 4(2)(x-2)$

origin lies on the directrix x = 0

so ΔSOR is right angle triangle.

Equation of chord of contact SR is

T = 0

x = 4

It is latus rectum of parabola

so area of SOR = $\frac{1}{2} \times 4 \times 8 = 16$ square units

69. The sum of all values of x in $[0, 2\pi]$, for which $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$, is equal to:

(A) 12π (B) 8π (C) 11π (D) 9π

Sol. (D)

$(\sin x + \sin 4x) + (\sin 2x + \sin 3x) = 0$

$\Rightarrow 2\sin\frac{5x}{2}\cos\frac{3x}{2} + 2\sin\frac{5x}{2}\cos\frac{x}{2} = 0$

$\Rightarrow 2\sin\frac{5x}{2}\left(\cos\frac{3x}{2} + \cos\frac{x}{2}\right) = 0$

$\Rightarrow 4\sin\frac{5x}{2}\cos x\cos\frac{x}{2} = 0$

$\Rightarrow \sin\frac{5x}{2} = 0$ or $\cos x = 0$ or $\cos\frac{x}{2} = 0$

$\Rightarrow \frac{5x}{2} = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$ or $x = \frac{\pi}{2}, \frac{3\pi}{2}$ or $\frac{x}{2} = \frac{\pi}{2}$

$\Rightarrow x = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, \frac{10\pi}{5}$ or $x = \frac{\pi}{2}, \frac{3\pi}{2}$ or $x = \pi$

$\Rightarrow x = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, 2\pi, \frac{\pi}{2}, \frac{3\pi}{2}, \pi$

Hence sum of all solutions $= 9\pi$

70. The value of the definite integral $\int_{\pi/24}^{5\pi/24} \frac{dx}{1+\sqrt[3]{\tan 2x}}$ is:

(A) $\frac{\pi}{12}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{18}$

Sol. (A)

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{dx}{1+\sqrt[3]{\tan 2x}}$ ......(1)

By property

$$I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{dx}{1+\sqrt[3]{\cot 2x}}$$

$$I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{\sqrt[3]{\tan 2x}\ dx}{1+\sqrt[3]{\tan 2x}} \quad ......(2)$$

By adding (1) & (2)

$$2I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{(1+\sqrt[3]{\tan 2x})\ dx}{1+\sqrt[3]{\tan 2x}}$$

$$2I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} dx = \frac{\pi}{6}$$

$$\therefore\ I = \frac{\pi}{12}$$

71. Let the vectors $(2+a+b)\hat{i}+(a+2b+c)\hat{j}-(b+c)\hat{k}, (1+b)\hat{i}+2b\hat{j}-b\hat{k}$ and $(2+b)\hat{i}+2b\hat{j}+(1-b)\hat{k}, a,b,c \in R$ be co-planar. Then which of the following is true?

(A) $3c = a+b$ (B) $2b = a+c$

(C) $a = b+2c$ (D) $2a = b+c$

Sol. (B)

Three vectors are coplanar

$$\Rightarrow \quad \begin{vmatrix} 2+a+b & a+2b+c & -(b+c) \\ 1+b & 2b & -b \\ 2+b & 2b & 1-b \end{vmatrix} = 0$$

Apply $C_2 \to C_2 + C_3$

$$\Rightarrow \quad \begin{vmatrix} 2+a+b & a+b & -(b+c) \\ 1+b & b & -b \\ 2+b & 1+b & 1-b \end{vmatrix} = 0$$

Apply $C_1 \to C_1 - C_2$

$$\Rightarrow \quad \begin{vmatrix} 2 & a+b & -(b+c) \\ 1 & b & -b \\ 1 & 1+b & 1-b \end{vmatrix} = 0$$

$\Rightarrow \quad 2[b - b^2 + b + b^2] - (a + b)(1 - b + b) - (b + c)(1 + b - b) = 0$

$\Rightarrow \quad 2b = a + c$ Ans.

72. Let $g : N \to N$ defined as

$g(3n+1) = 3n+2,$

$g(3n+2) = 3n+3,$

$g(3n+3) = 3n+1,$ for all $n \geq 0$.

Then which of the following statements is true?

(A) gogog = g

(B) There exists a function $f : N \to N$ such that gof = f

(C) There exists a one – one function $f : N \to N$ such that fog = f

(D) There exists an onto function $f : N \to N$ such that fog = f

Sol. (D)

g(3n + 1) = 3n + 2
g(3n + 2) = 3n + 3
g(3n + 3) = 3n +1, $n \geq 0$
For x = 3n + 1
(1) gogog (3n + 1) = gog(3n + 2) = g(3n + 3) = 3n + 1
Similarly
gogog (3n + 2) = 3n + 2
gogog (3n + 3) = 3n + 3
So gogog (x) = x $\forall\ x \in N$
(2) As f : N $\rightarrow$ N, f = 3n + 1
= 3n + 2
= 3n + 3
So, g(3n + 1) = 3n + 2, g(3n + 2) = 3n + 3, g(3n + 3) = 3n + 1
So g(f(x)) $\neq$ f(x)
(3) If f : N $\rightarrow$ N and f is a one-one function such that f(g(x)) = f(x) then
g(x) = x

but g(x) $\neq$ x

(4) If f : N $\rightarrow$ N and f is an onto function such that f(g(x)) = f(x) then

One of its possibilities is by taking f(x) as onto function

$$f(x) = \begin{cases} a & x = 3n+1 \\ a & x = 3n+2, \quad a \in N \\ a & x = 3n+3 \end{cases}$$

$\Rightarrow$ f(g(x)) = f(x) $\forall\ x \in N$

73. Let ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a^2 > b^2$ passes through $\left(\sqrt{\frac{3}{2}}, 1\right)$ and the has eccentricity $\frac{1}{\sqrt{3}}$.

If a circle. Centered at focus $F(\alpha, 0), \alpha > 0$ of E and radius $\frac{2}{\sqrt{3}}$, intersects E at two points P and Q, then PQ^2 is equal to:

(A) 3 (B) $\frac{16}{3}$ (C) $\frac{8}{3}$ (D) $\frac{4}{3}$

Sol. (B)

Let equation of ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (a > b)

it passes through $\left(\sqrt{\frac{3}{2}}, 1\right) \Rightarrow \frac{3}{2a^2}+\frac{1}{b^2}=1$ ....(1)

Given $e = \frac{1}{\sqrt{3}} \Rightarrow b^2 = a^2(1-e^2) = \frac{2}{3}a^2$ ....(2)

Solve (1) & (2) we get $a^2 = 3, b^2 = 2$

$\therefore$ Ellipse is $\frac{x^2}{3}+\frac{y^2}{2}=1$ .... (3)

Focus $(\pm ae, 0) \equiv \left(\pm\sqrt{3}.\frac{1}{\sqrt{3}}, 0\right) \equiv (\pm 1, 0)$

Hence circle is $(x-1)^2 + y^2 = \left(\frac{2}{\sqrt{3}}\right)^2 = \frac{4}{3}$ ....(4)

Solve (3) & (4)

$2x^2 + 3\left(\frac{4}{3}-(x-1)^2\right) = 6$

$2x^2 + 4 - 3(x^2 + 1 - 2x) = 6$

$-x^2 + 6x - 5 = 0$

$x = 1, 5$

When $x = 1 \Rightarrow \frac{1}{3}+\frac{y^2}{2}=1 \Rightarrow y^2 = \frac{4}{3} \Rightarrow y = \pm\frac{2}{\sqrt{3}}$

Hence $P((1, \frac{2}{\sqrt{3}}), Q(1, -\frac{2}{\sqrt{3}}) \Rightarrow PQ^2 = \frac{16}{3}$

When $x = 5 \Rightarrow \frac{y^2}{2} = 1 - \frac{25}{3} = -\frac{22}{3} \Rightarrow$ not possible

$PQ^2 = \frac{16}{3}$

74. Let $f:[0,\infty)\rightarrow[0,\infty)$ be defined as

$f(x)=\int_0^x [y]dy$.

Where [x] is the greatest integer less than or equal to x. which of the following is true?

(A) f is continuous at every point in $[0,\infty)$ and differentiable except at the integer points

(B) f is differentiable at every point in $[0,\infty)$.

(C) f is continuous everywhere except at the integer points in $[0,\infty)$

(D) f is both continuous and differentiable except at the integer points in $[0,\infty)$.

Sol. (A)

$$f(x) = \int_0^x [y]dy$$

Let $n \le x < n+1,\ n \in I$

$$= \int_0^1 (0)dy + \int_1^2 (1)dy + \int_2^3 (2)dy + \ldots\ldots + \int_{n-1}^{n} (n-1)dy + \int_n^x (n)dy$$

$$= 0 + 1 + 2 + 3 + \ldots\ldots. + (n-1) + n(x-n)$$

$$= \frac{n(n-1)}{2} - n^2 + nx$$

$$\Rightarrow f(x) = \frac{-n-n^2}{2} + nx = nx - \frac{n(n+1)}{2} \quad \Rightarrow f(x) = n\left(x - \frac{n+1}{2}\right)$$

$$\Rightarrow f(x) = \begin{cases} x\left(\dfrac{x-1}{2}\right) & ;\ x = n(\text{Integer}) \\ [x]\left(\dfrac{2x-[x]-1}{2}\right) & ;\ x \notin \text{Integer} \end{cases}$$

Since $\lim_{x\to n} f(x) = \frac{n(n-1)}{2} = f(n)$

Hence f(x) is continuous at all integers.

But f'(x) = [x] is discontinuous at integers.

So, f(x) is non derivable at $x \in$ integers.

75. Let $f(x) = 3\sin^4 x + 10\sin^3 x - 3,\ x \in \left[-\frac{\pi}{6}, \frac{\pi}{2}\right]$. Then, f is:

(A) Increasing in $\left(-\frac{\pi}{6}, \frac{\pi}{2}\right)$

(B) Decreasing in $\left(-\frac{\pi}{6}, 0\right)$

(C) Increasing in $\left(-\frac{\pi}{6}, 0\right)$

(D) Decreasing in $\left(0, \frac{\pi}{6}\right)$

Sol. (B)

$f(x) = 3 \sin^4 x + 10 \sin^3 x + 6 \sin^2 x - 3$

$f'(x) = 12 \sin^3 x \cos x + 30 \sin^2 x \cos x + 12 \sin x \cos x$

$f'(x) = 0$

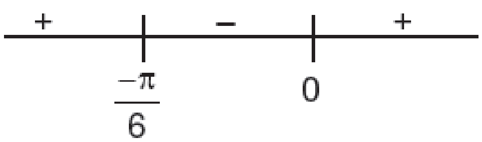
$\Rightarrow 6 \sin x \cos x (2 \sin^2 x + 5 \sin x + 2) = 0$

$\sin 2x = 0$ or $2 \sin^2 x + 5 \sin x + 2 = 0$

$x = 0$ $(2 \sin x + 1)$ [$(\sin x + 2) = 0$ is not possible]

$\sin x = \frac{-1}{2}$, $(\sin x + 2) = 0$

$x = \frac{-\pi}{6}$ $x \in \left(\frac{-\pi}{6}, \frac{\pi}{2}\right)$



Decreasing in $x \in \left(\frac{-\pi}{6}, 0\right)$

76. The number of real roots of the equation $e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^x + 1 = 0$ is

(A) 2 (B) 1 (C) 6 (D) 4

Sol. (A)

$e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^x + 1 = 0$

$\Rightarrow (e^{3x} - 1)^2 - e^x(e^{3x} - 1) - 12e^{2x} = 0$

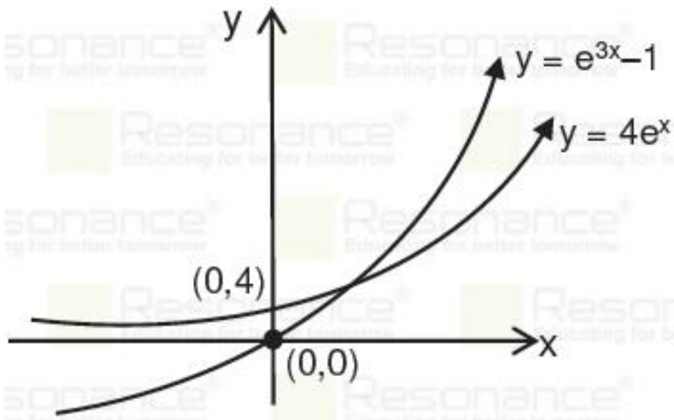
$\Rightarrow m^2 - mn - 12n^2 = 0$ where $m = e^{3x}-1$, $n = e^x$

$\Rightarrow (m-4n)(m+3n) = 0$

$\Rightarrow m = 4n$ or $m + 3n = 0$

$\Rightarrow e^{3x} - 1 = 4e^x$ or $(e^{3x}-1) \cdot e^x = 0$

Case-I $e^{3x}-1 = 4e^x$



One solution

Case –II $(e^{3x}-1)\, e^x = 0$

$\Rightarrow e^{3x} = 1$

$\Rightarrow x = 0$ (one solution)

$\therefore$ Total 2 roots

77. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = 1 + xe^{y-x}, -\sqrt{2} < x < \sqrt{2}, y(0) = 0$ then, the minimum value of $y(x), x \in \left(-\sqrt{2}, \sqrt{2}\right)$ is equal to:

(A) $\left(2-\sqrt{3}\right) - \log_e 2$

(B) $\left(1-\sqrt{3}\right) - \log_e\left(\sqrt{3}-1\right)$

(C) $\left(1+\sqrt{3}\right) - \log_e\left(\sqrt{3}-1\right)$

(D) $\left(2+\sqrt{3}\right) + \log_e 2$

Sol. (B)

$$\frac{dy}{dx} = 1 + xe^{y-x} \quad ....(1)$$

$$e^{-y}\frac{dy}{dx} = e^{-y} + xe^{-x}$$

Put $e^{-y} = t \Rightarrow e^{-y}\frac{dy}{dx} = -\frac{dt}{dx}$

$$-\frac{dt}{dx} = t + xe^{-x}$$

$$\frac{dt}{dx} + t = -xe^{-x} \quad ....(2)$$

I.F. = $e^{\int 1.dx} = e^{x}$

Solution of equation (2) is

$$te^{x} = \int (-xe^{-x}).\, e^{x}dx + c$$

$$te^{x} = -\frac{x^2}{2} + c$$

$$e^{x-y} = -\frac{x^2}{2} + c \quad ....(3)$$

$\therefore$ $y(0) = 0 \;\Rightarrow\; 1 = c \Rightarrow e^{x-y} = \left(\frac{2-x^2}{2}\right)$

$$x - y = \ell n\left(\frac{2-x^2}{2}\right)$$

$$y = x - \ell n\left(\frac{2-x^2}{2}\right)$$

Now, $\frac{dy}{dx} = 1 + x\left(\frac{2}{2-x^2}\right)$

$$\Rightarrow \left(\frac{2-x^2+2x}{2-x^2}\right) = 0$$

$$\Rightarrow -\left(\frac{x^2-2x-2}{2-x^2}\right) = 0$$

$x = 1 \pm \sqrt{3}$

–   +   –

$1-\sqrt{3}$ $1+\sqrt{3}$

$\Rightarrow$ y_{min} at $x = 1 - \sqrt{3}$ $\quad\Rightarrow$ $y_{min} = (1-\sqrt{3}) - \ell n(\sqrt{3}-1)$

78. If b is very small as compared to the value of a, so that the cube and other higher powers of $\frac{b}{a}$ can be neglected in the identity $\frac{1}{a-b}+\frac{1}{a-2b}+\frac{1}{a-3b}+...+\frac{1}{a-nb}=\alpha n+\beta n^{2}+\gamma n^{3}$ then the value of γ is:

(A) $\frac{a+b}{3a^{2}}$ (B) $\frac{a^{2}+b}{3a^{3}}$

(C) $\frac{b^{2}}{3a^{3}}$ (D) $\frac{a+b^{2}}{3a^{3}}$

Sol. (C)

$$\frac{1}{a-b}+\frac{1}{a-2b}+\frac{1}{a-3b}+........+\frac{1}{a-nb}$$

$$=\frac{1}{a}\left[\left(1-\frac{b}{a}\right)^{-1}+\left(1-\frac{2b}{a}\right)^{-1}+\left(1-\frac{3b}{a}\right)^{-1}+.......+\left(1-\frac{nb}{a}\right)^{-1}\right]$$

$$=\frac{1}{a}\left[\left\{1+\left(\frac{b}{a}\right)+\left(\frac{b}{a}\right)^{2}+......\right\}+\left\{1+\left(\frac{2b}{a}\right)+\left(\frac{2b}{a}\right)^{2}+........\right\}+\left\{1+\left(\frac{nb}{a}\right)+\left(\frac{nb}{a}\right)^{2}+......\right\}\right]$$

$$=\frac{1}{a}\left[n+\frac{b}{a}(1+2+......+n)+\frac{b^{2}}{a^{2}}\left(1^{2}+2^{2}+......n^{2}\right)\right]$$

$$=\frac{1}{a}\left[n+\frac{n(n+1)}{2}\frac{b}{a}+\frac{n(n+1)(2n+1)}{6}\frac{b^{2}}{a^{2}}\right]$$

$$=\frac{1}{a}\left[n+\frac{n^{2}}{2}\frac{b}{a}+\frac{n}{2}\frac{b}{a}+\frac{2n^{3}+3n^{2}+n}{6}\left(\frac{b^{2}}{a^{2}}\right)\right]$$

$$=n\left(\frac{1}{a}+\frac{b}{2a^{2}}+\frac{b^{2}}{6a^{3}}\right)+\left(\frac{b}{2a^{2}}+\frac{b^{2}}{2a^{3}}\right)n^{2}+\frac{b^{2}}{3a^{3}}n^{3}$$

by comparing this result to $\alpha n+\beta n^{2}+\gamma n^{3}$

we get $\gamma=\frac{b^{2}}{3a^{3}}$

79. The Boolean expression $(p\Rightarrow q)\wedge(q\Rightarrow\sim p)$ is equivalent to:

(A) $\sim p$ (B) p (C) $\sim q$ (D) q

Sol. (A)

$= (p \to q) \wedge (p \to \sim q)$
$= (\sim p \vee q) \wedge (\sim p \vee \sim q)$
$= \sim p \vee (q \wedge \sim q)$
$= \sim p \vee f$
$= \sim p$

80. Let the foot of perpendicular from a point P(1, 2, –1) to the straight line $L = \frac{x}{1} = \frac{y}{0} = \frac{z}{-1}$ be N. Let a line be drawn from P parallel to the plane $x + y + 2z = 0$ which meets L at point Q. if α is the acute angle between the lines PN and PQ, then $\cos\alpha$ is equal to ________

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{1}{2\sqrt{3}}$ (D) $\frac{1}{\sqrt{5}}$

Sol. (B)

Let foot of $\perp$ from P(1, 2, – 1) on line $\frac{x}{1} = \frac{y}{0} = \frac{z}{-1}$ is N $(\lambda, 0, -\lambda)$

$\Rightarrow 1(\lambda - 1) + 0(0 - 2) - 1(-\lambda + 1) = 0$

$\Rightarrow \lambda = 1$

$\Rightarrow$ N (1, 0, – 1)

Now equation of line passes through P(1, 2, –1) is

$$\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+1}{c} \quad \ldots\ldots (1)$$

This line is parallel to plane $x + y + 2z = 0$

$\Rightarrow \quad a + b + 2c = 0 \quad \ldots\ldots (2)$

Any point on the line $L : \frac{x}{1} = \frac{y}{0} = \frac{z}{-1}$ is

Q(r, 0 , – r) lies on line (1)

$$\Rightarrow \quad \frac{r-1}{a} = \frac{0-2}{b} = \frac{1-r}{c}$$

$\Rightarrow \quad c = -a$

By (2) we have $b = a$

Hence $a : b : c = a : a : -a$

So, line PQ is $\frac{x-1}{a} = \frac{y-2}{a} = \frac{z+1}{-a}$

And directions line PN is (0, 2, 0)

Acute angle between PQ and PN is 'α' then

$$\cos\alpha = \left|\frac{0 + 2a + 0}{\sqrt{3}a \cdot 2}\right| = \frac{1}{\sqrt{3}}$$

Integer

81. If the value $\left(1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+...upto\ \infty\right)^{\log_{(0.25)}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+...upto\ \infty\right)}$ is ℓ, then ℓ^2 is equal to _______.

Sol. (3.00)

$$\left(1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+......\right)^{\log_{0.25}\left(\frac{1}{3}\left(\frac{1}{\frac{2}{3}}\right)\right)}=\ell$$

$$\left(1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+......\right)^{\log_{\frac{1}{4}}\frac{1}{2}}=\ell$$

Let $1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+.....=x$

$$(x-1) = \frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+.... \quad .... (1)$$

$$\frac{1}{3}(x-1)=\frac{2}{3^2}+\frac{6}{3^3}+..... \quad .... (2)$$

From (1) – (2), we get

$$\frac{2}{3}(x-1)=\frac{2}{3}+\frac{4}{3^2}+\frac{4}{3^3}+.....$$

$$\frac{2}{3}(x-1)=\frac{2}{3}+\frac{4}{3^2}\left(\frac{1}{1-\frac{1}{3}}\right)$$

$$\frac{2}{3}(x-1)=\frac{2}{3}+\frac{4}{3^2}\frac{3}{2}.$$

$$\frac{2}{3}(x-1)=\frac{2}{3}+\frac{2}{3}$$

$x - 1 = 2$ & $x = 3$

So , $\quad 3^{\log_{\frac{1}{4}}\frac{1}{2}} = \ell$

$$3^{\frac{1}{2}} = \ell$$

$$\ell^2 = 3$$

82. There are 5 students in class 10, 6 students in class 11 and 8 students in class 12. If the number of ways, in which 10 students can be selected from them so as to include at least 2 students from each class and at most 5 students from the total 11 students of class 10 and 11 is 100 k, then k is equal to ________

Sol. (238)

Total student	(5)	(6)	(8)		
Class	10^{th}	11^{th}	12^{th}		
	2	2	6	$\rightarrow$	$5_{C_2} \times 6_{C_2} \times 8_{C_6}$
	2	3	5	$\rightarrow$	$5_{C_2} \times 6_{C_3} \times 8_{C_5}$
	3	2	5	$\rightarrow$	$5_{C_3} \times 6_{C_2} \times 8_{C_5}$

Total number of ways $= 5_{C_2} \times 8_{C_3}\left(6_{C_3} + 6_{C_2}\right) + 5_{C_2} \times 6_{C_2} \times 8_{C_6}$

$= 23800$

83. If α, β are roots of the equation $x^2+5\left(\sqrt{2}\right)x+10=0, \alpha>\beta$ and $P_n=\alpha^n-\beta^n$ and for each positive integer n, then the value of $\left(\dfrac{P_{17}P_{20}+5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19}+5\sqrt{2}P_{18}^2}\right)$ is equal to _____________.

Sol. (1.00)

$x^2+5\sqrt{2}x+10=0$; $\alpha>\beta$

$P_n=\alpha^n-\beta^n$

$P_{17}=\alpha^{17}-\beta^{17}$, $P_{18}=\alpha^{18}-\beta^{18}$

$\alpha^2+10=5\sqrt{2}\alpha$ ... (1)

$\beta^2+10=-5\sqrt{2}\beta$ ... (2)

$$\Rightarrow \frac{P_{17}\left(\alpha^{20}-\beta^{20}+5\sqrt{2}\left(\alpha^{19}-\beta^{19}\right)\right)}{P_{18}\left(\left(\alpha^{15}-\beta^{19}\right)+5\sqrt{2}\left(\alpha^{18}-\beta^{18}\right)\right)}=\frac{P_{17}\left(\alpha^{18}\left(\alpha^2+5\sqrt{2}\alpha\right)-\beta^{18}\left(5\sqrt{2}\beta+\beta^2\right)\right)}{P_{18}\left(\alpha^{17}\left(\alpha^2+5\sqrt{2}\alpha\right)-\beta^{17}\left(5\sqrt{2}\beta+\beta^2\right)\right)};$$

using eq (1) & (2) = $\dfrac{P_{17}\cdot P_{18}(-10)}{P_{18}\cdot P_{17}(-10)}=1$

84. Let $y=y(x)$ be solution of the following differential equation $e^y\dfrac{dy}{dx}-2e^y\sin x+\sin x\cos^2 x=0, y\left(\dfrac{\pi}{2}\right)=0$. If $y(0)=\log_e\left(\alpha+\beta e^{-2}\right)$, then $4(\alpha+\beta)$ is equal to ______.

Sol. (4)

$$e^y \frac{dy}{dx} - 2e^y \sin x = -\sin x \cos^2 x$$

Put $e^y = t$

$$e^y \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} - 2t \sin x = -\sin x \cos^2 x$$

$$\text{I.F} = e^{-\int 2\sin x dx} = e^{2\cos x}$$

$$e^y \cdot e^{2\cos x} = \int e^{2\cos x}(-\sin x\_\cos^2 x dx$$

$\cos x = z$

$$= \int p^{2z} z^2 dz$$

$$\frac{e^{2z}}{2} z^2 - \int e^{2z}.zdz$$

$$= \frac{e^{2z}}{2} z^2 - \left[\frac{e^{2z}}{2}.z - \frac{e^{2z}}{4}\right] + c$$

$$\Rightarrow e^y e^{2\cos x} = \frac{e^{2z}}{4}(2z^2 - 2z + 1) + c$$

$$\Rightarrow e^y e^{2\cos x} = \frac{e^{2\cos x}}{4}(2\cos^2 x - 2\cos x + 1) + c$$

$$\Rightarrow e^y = \frac{1}{4}(2\cos^2 x - 2\cos x + 1) + c$$

At $x = \frac{\pi}{2}$ $y = 0$,

$$\Rightarrow 1 = \frac{1}{4} + c \Rightarrow c = \frac{3}{4}$$

$$\Rightarrow e^y = \frac{1}{4}\left(2\cos^2 x - 2\cos x + 1\right) + \frac{3}{4} e^{-2\cos x}$$

$$\Rightarrow y(0) = \ell n\left(\frac{1}{4} + \frac{3}{4}e^{-2}\right)$$

$$\Rightarrow y(0) = \ell n\left(\alpha + \beta e^{-2}\right)$$

$$\Rightarrow \alpha = \frac{1}{4}, \beta = \frac{3}{4}$$

$$\Rightarrow 4(\alpha + \beta) = 4$$

85. Let $\vec{p} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{q} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors. If a vector $\vec{r} = \left(\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}\right)$ is perpendicular to each of the vectors $(\vec{p}+\vec{q})$ and $(\vec{p}-\vec{q})$, and $|\vec{r}| = \sqrt{3}$, then $|\alpha|+|\beta|+|\gamma|$ is equal to ____________.

Sol. (3)

$$(\vec{p}+\vec{q})\times(\vec{p}-\vec{q}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 2 \\ 1 & 1 & 0 \end{vmatrix} = -2\hat{i} + 2\hat{j} - 2\hat{k}$$

$$\vec{r} = \pm\sqrt{3}\,\frac{((\vec{p}+\vec{q})\times(\vec{p}-\vec{q}))}{|(\vec{p}+\vec{q})\times(\vec{p}-\vec{q})|}$$

$$= \pm\left(-\hat{i} + \hat{j} - \hat{k}\right)$$

so $|\alpha| = 1, |\beta| = 1, |\gamma| = 1$

$$|\alpha| + |\beta| + |\gamma| = 3$$

86. Let $S = \left\{ n \in N \left| \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^n \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \forall a,b,c,d \in R \right. \right\}$, where $i = \sqrt{-1}$, then the numbers of 2 – digit number in the set S is __________.

Sol. (11)

Let A = $\begin{bmatrix} 0 & i \\ 1 & 0 \end{bmatrix}^n$ and B = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

AB = IB

(A − I)B = 0

A = I

$\begin{bmatrix} 0 & i \\ 1 & 0 \end{bmatrix}^n = I$

$A^8 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

n = multiple of 8

Number of two digit numbers is S = 11 (16, 24,......96)

87. The ratio of the coefficient of the middle term in the expansion of $(1+x)^{20}$ and the sum of the coefficients of two middle terms in expansion of $(1+x)^{19}$ is ___________.

Sol. (1)

$$\frac{^{20}C_{10}}{^{19}C_9 + ^{19}C_{10}} = \frac{^{20}C_{10}}{^{20}C_{10}} = 1$$

88. The term independent of 'x' in the expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}}\right)^{10}$, where $x \neq 0,1$ is equal to _________.

Sol. (210)

$$\left((x^{1/3}+1)-\left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10}$$

$(x^{1/3}-x^{-1/2})^{10}$

$T_{r+1} = {}^{10}c_r\,(x^{1/3})^{10-r}\,(-x^{-1/2})^r$

$$\frac{10-r}{3}-\frac{r}{2}=0 \quad\Rightarrow\quad 20-2r-3r=0$$

$\Rightarrow \quad r = 4$

$$T_5 = {}^{10}C_4 = \frac{10\times9\times8\times7}{4\times3\times2\times1} = 210$$

89. Let $M=\left\{A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}: a,b,c,d \in \{\pm3,\pm2,\pm1,0\}\right\}$. Define $f: M \to Z$, as $f(A)=\det(A)$, for all $A \in M$, where Z is set of all integers. Then the number of $A \in M$ such that $f(A)=15$ is equal to _______.

Sol. (16)

$|A| = (ad - bc) = 15$

where a, b, c, d $\in \{\pm1, \pm2, \pm3\}$

Case – I ad = 9 & bc = – 6

ad $\equiv$ (3,3) or (– 3, –3) bc $\equiv$ (2, – 3), (–2, 3), (–3,2), (3, –2)

Total = 2 × 4 = 8 matrix

Case – II ad = 6 and bc = – 9

Similarly, Total = 4 × 2 = 8 matrix

Total such matrix = 8 + 8 = 16 matrix

90. Consider the following frequency distribution

Class :	10-20	20-30	30-40	40-50	50-60
Frequency	α	110	54	30	β

If the sum of all frequencies is 584 and median is 45, then $|\alpha-\beta|$ is equal to _______.

Sol. (164)

Class	Frequency	C.F
10-20	α	α
20-30	110	$\alpha + 110$
30-40	54	$\alpha + 164$
40-50	30	$\alpha + 194$
50-60	β	$\alpha + \beta + 194 = 584$

$N = \sum f = 584$

$\alpha + \beta = 390$

$$\text{Median (m)} = \ell + = \left[\frac{\left(\frac{N}{2}\right) - c}{f}\right] \times h$$

$$N = \frac{584}{2} = 292$$

$$m = 45 = 40 + \left[\frac{292 - (\alpha + 164)}{30}\right] \times 10$$

$$45 = 40 + \left(\frac{128 - \alpha}{3}\right)$$

$$5 = \frac{128 - \alpha}{3}$$

$15 = 128 - \alpha$

$\alpha = 113$

$\beta = 277$

$|\alpha - \beta| = |113 - 277| = 164$