JEE Advance-2025 Paper With Solutions

CatalyseR

Unparalleled Legacy

10 TIMES

JEE **INDORE TOPPERS**

in 13 Yrs

PHYSICS

SECTION-1 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : +3 If **ONLY** the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : −1 In all other cases.

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1. A temperature difference can generate e.m.f. in some materials. Let S be the e.m.f. produced per unit temperature difference between the ends of a wire, σ the electrical conductivity and κ the thermal conductivity of the material of the wire. Taking M, L, T, I and K as dimensions of mass, length, time, current and temperature, respectively, the dimensional formula of the quantity $Z = \frac{S^2\sigma}{\kappa}$ is :-

(A) $[M^0L^0T^0I^0K^0]$ (B) $[M^0L^0T^0I^0K^{-1}]$ (C) $[M^1L^2T^{-2}I^{-1}K^{-1}]$ (D) $[M^1L^2T^{-4}I^{-1}K^{-1}]$

Ans. **(B)**

Sol. S = emf per unit temperature difference

σ = Electrical conductivity

k = Thermal conductivity

$[S] = [ML^2T^{-3}I^{-1}K^{-1}]$

$[\sigma] = [M^{-1}L^{-3}T^3I^2]$

$[K] = [M^1L^1T^{-3}K^{-1}]$

$[Z] = \frac{S^2\sigma}{K} = \frac{\left[M^1L^1T^{-3}K^{-2}\right]}{\left[M^1L^1T^{-3}K^{-1}\right]}$

$[Z] = [K^{-1}]$

2. Two co-axial conducting cylinders of same length ℓ with radii $\sqrt{2}R$ and $2R$ are kept, as shown in Fig. 1. The charge on the inner cylinder is Q and the outer cylinder is grounded. The annular region between the cylinders is filled with a material of dielectric constant $\kappa = 5$. Consider an imaginary plane of the same length ℓ at a distance R from the common axis of the cylinders. This plane is parallel to the axis of the cylinders. The cross-sectional view of this arrangement is shown in Fig. 2. Ignoring edge effects, the flux of the electric field through the plane is ($\in_0$ is the permittivity of free space):

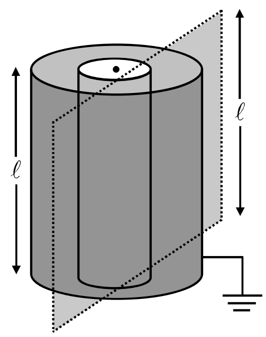


Fig.1

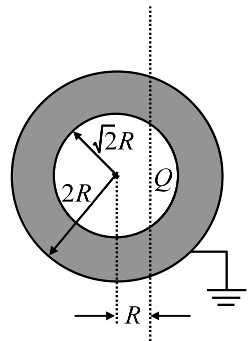
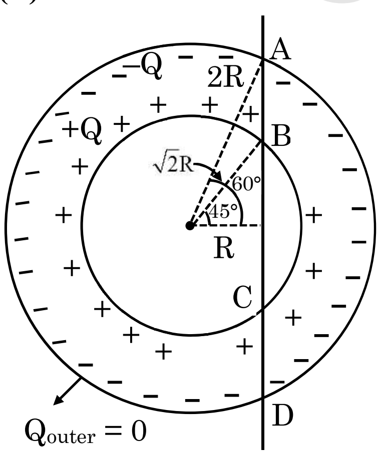


Fig.2

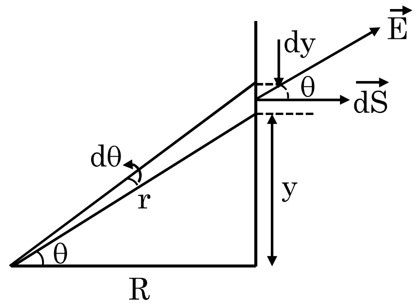
(A) $\dfrac{Q}{30 \in_0}$ (B) $\dfrac{Q}{15 \in_0}$ (C) $\dfrac{Q}{60 \in_0}$ (D) $\dfrac{Q}{120 \in_0}$

Ans. **(C)**

Sol.



Here we are assuming that "ℓ" is very large just for the sake of symmetry. Outside cylinder will have zero electric field inside, so the flux generated on the plate will be due to inner cylinder only in sections AB and CD, as section be will be at that place where electric field is zero.



flux through element will be

$$d\phi = \vec{E}.\vec{dS}$$

$$d\phi = \frac{2k\lambda}{r} dy.\ \ell.\ \cos\theta ....(1)$$

from figure we can say that

$$\cos\theta = \frac{R}{r} \Rightarrow r = R\sec\theta$$

$$\tan\theta = \frac{y}{R} \Rightarrow y = R\tan\theta$$

$$\Rightarrow dy = R\sec^2\theta d\theta$$

$$d\phi = \frac{2k\lambda}{R\sec\theta} R.\sec^2\theta.\ell.\cos\theta d\theta$$

$$d\phi = 2k\lambda\ell\ dq$$

$$\int_0^{\phi_{AB}} d\phi = 2k\lambda\ell \int_{\pi/4}^{\pi/3} d\theta$$

$$\phi_{AB} = 2k\lambda\ell\left[\frac{\pi}{3} - \frac{\pi}{4}\right]$$

$$\phi_{AB} = 2kQ\left[\frac{\pi}{12}\right]$$

$$\phi_{AB} = 2\times\frac{1}{4\pi\in_0\in_r}Q\left[\frac{\pi}{12}\right]$$

$$\phi_{AB} = \frac{Q}{120\in_0}$$

$$\phi_{plate} = \phi_{AB} + \phi_{BC} + \phi_{CD} \qquad (\phi_{CD} = \phi_{AB})$$

$$\phi_{plate} = \frac{Q}{60\in_0}$$

3. As shown in the figures, a uniform rod OO' of length l is hinged at the point O and held in place vertically between two walls using two massless springs of same spring constant. The springs are connected at the midpoint and at the top-end (O') of the rod, as shown in Fig. 1 and the rod is made to oscillate by a small angular displacement. The frequency of oscillation of the rod is f_1. On the other hand, if both the springs are connected at the midpoint of the rod, as shown in Fig. 2 and the rod is made to oscillate by a small angular displacement, then the frequency of oscillation is f_2. Ignoring gravity and assuming motion only in the plane of the diagram, the value of $\frac{f_1}{f_2}$ is:

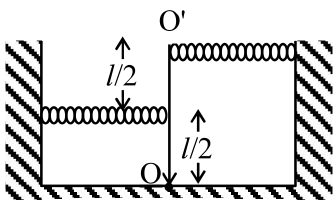


Fig. 1

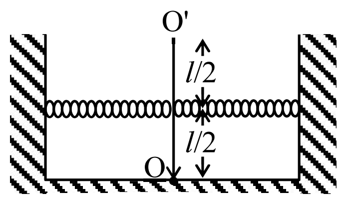


Fig. 2

(A) 2 (B) $\sqrt{2}$ (C) $\sqrt{\frac{5}{2}}$ (D) $\sqrt{\frac{2}{5}}$

Ans. **(C)**

Sol.

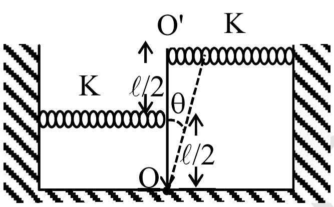


Fig. 1

$$\frac{ML^2}{3}\ddot{\theta}+K.\frac{\ell}{2}\theta.\frac{\ell}{2}+K.\ell.\theta.\ell=0$$

$$\ddot{\theta}+\frac{15}{4}\frac{k}{M}\theta=0$$

$$\ddot{\theta}=-\left(\frac{15}{4}\frac{k}{M}\right)\theta$$

$$\omega_1=\sqrt{\frac{15K}{4M}}$$

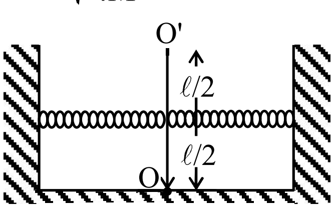


Fig. 2

$$\frac{1}{3}ML^2\ddot{\theta}+2K\frac{L}{2}\theta.\frac{L}{2}=0$$

$$\ddot{\theta}+\frac{3}{2}\frac{K}{M}\theta=0$$

$$\ddot{\theta}=-\frac{3}{2}\frac{K}{M}\theta$$

$$\omega_2=\sqrt{\frac{3}{2}\frac{K}{M}}$$

$$\frac{\omega_1}{\omega_2}=\sqrt{\frac{15}{4}\times\frac{2}{3}}=\sqrt{\frac{5}{2}}$$

4. Consider a star of mass m_2 kg revolving in a circular orbit around another star of mass m_1 kg with $m_1 >> m_2$. The heavier star slowly acquires mass from the lighter star at a constant rate of γ kg/s. In this transfer process, there is no other loss of mass. If the separation between the centers of the stars is r, then its relative rate of change $\frac{1}{r}\frac{dr}{dt}$ (in s^{-1}) is given by:

(A) $-\frac{3\gamma}{2m_2}$ (B) $-\frac{2\gamma}{m_2}$ (C) $-\frac{2\gamma}{m_1}$ (D) $-\frac{3\gamma}{2m_1}$

Ans. **(B)**

Sol. $$m_2\omega^2 r=\frac{Gm_1m_2}{r^2}$$

$$\omega=\sqrt{\frac{Gm_1}{r^3}}$$

$$L=m_2\omega r^2$$

$$=m_2\sqrt{\frac{Gm_1}{r^3}}\,r^2$$

$$L=m_2\sqrt{Gm_1r}=\text{const.}$$

$$\ell nL=\ell nm_2+\ell nG+\frac{1}{2}\ell nm_1+\frac{1}{2}\ell nr$$

$$0=\frac{dm_2}{m_2}+\frac{1}{2}\frac{dm_1}{m_1}+\frac{1}{2}\frac{dr}{r}$$

$$\frac{dr}{rdt}=-\frac{2dm_2}{m_2dt}-\frac{dm_1}{m_1dt}\approx-\frac{2\gamma}{m_2} \qquad \text{Here}\left(\frac{dm}{dt}=\gamma\right)$$

SECTION-2 : (Maximum Marks : 16)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks	: +4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	: +3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	: +2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	: +1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: −2	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 marks;
  choosing ONLY (B) will get +1 marks;
  choosing ONLY (D) will get +1 marks;
  choosing no option (i.e. the question is unanswered) will get 0 marks and
  choosing any other option(s) will get –2 marks.

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5. A positive point charge of 10^{-8} C is kept at a distance of 20 cm from the center of a neutral conducting sphere of radius 10 cm. The sphere is then grounded and the charge on the sphere is measured. The grounding is then removed and subsequently the point charge is moved by a distance of 10 cm further away from the center of the sphere along the radial direction. Taking $\frac{1}{4\pi \epsilon_0} = 9\times10^9 \text{Nm}^2/\text{C}^2$ (where ϵ_0 is the permittivity of free space), which of the following statements is/are correct: :

(A) Before the grounding, the electrostatic potential of the sphere is 450 V.

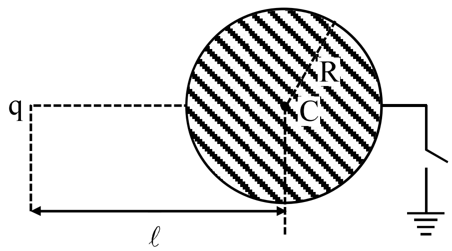
(B) Charge flowing from the sphere to the ground because of grounding is 5×10^{-9}C.

(C) After the grounding is removed, the charge on the sphere is -5×10^{-9}C.

(D) The final electrostatic potential of the sphere is 300 V.

Ans. **(A,B,C)**

Sol.

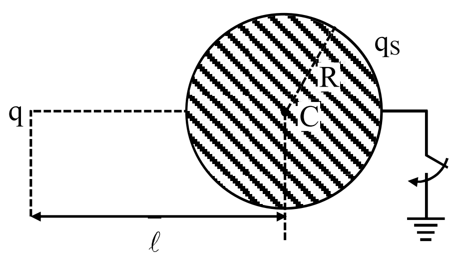


Before grounding

$V_{sphere} = (V_C)_{net} = (V_C)_q + (V_C)_{ind}$

$V_{sphere} = \frac{kq}{\ell} + 0 = \frac{9\times10^9\times10^{-8}}{0.2} = \frac{90}{0.2} = \frac{900}{2} = 450$ volt

After grounding



$\frac{kQ}{\ell} + \frac{kq_S}{R} = V'_{sphere} = 0$

$q_S = -\frac{R}{\ell}q = -\frac{1}{2}\times10^{-8} = -5\times10^{-9}$

$q_s = -5 \times 10^{-9}$ Coulomb

Charge flower from sphere to ground = 5×10^{-9} Coulomb

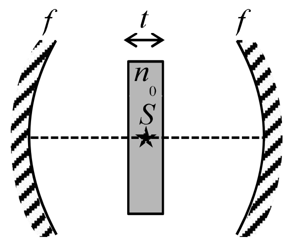
After grounding is removed

$\left(V_{sphere}\right)_{final} = \frac{kq}{\ell'} + \frac{kq_s}{R}$

$= \frac{9\times10^9\times10^2\times10^{-8}}{30} - \frac{9\times10^9\times5\times10^{-9}\times10^2}{10}$

$= \frac{9\times1000}{30} - 450 = 300$ volt $- 450$ volt $= -150$ volt

6. Two identical concave mirrors each of focal length *f* are facing each other as shown in the schematic diagram. The focal length *f* is much larger than the size of the mirrors. A glass slab of thickness t and refractive index n_0 is kept equidistant from the mirrors and perpendicular to their common principal axis. A monochromatic point light source *S* is embedded at the center of the slab on the principal axis, as shown in the schematic diagram. For the image to be formed on *S* itself, which of the following distances between the two mirrors is/are correct:



(A) $4f+\left(1-\frac{1}{n_0}\right)t$

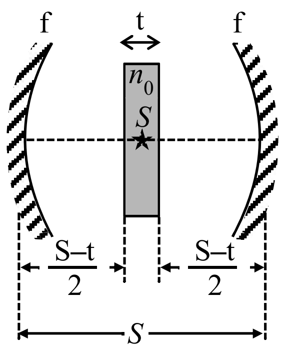
(B) $2f+\left(1-\frac{1}{n_0}\right)t$

(C) $4f+(n_0-1)t$

(D) $2f+(n_0-1)t$

Ans. **(A,B)**

Sol.



$$\frac{S-t}{2}+\frac{t}{2n_0}=2f$$

$$S+t\left(\frac{1}{n_0}-1\right)=4f$$

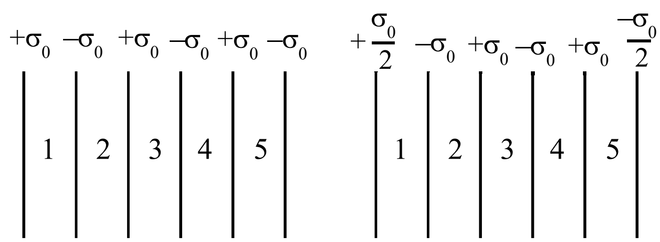
$$S=4f+\left(1-\frac{1}{n_0}\right)t$$

Also

$$\frac{S-t}{2}+\frac{t}{2n_0}=f$$

$$S=2f+\left(1-\frac{1}{n_0}\right)t$$

7. Six infinitely large and thin non-conducting sheets are fixed in configurations I and II. As shown in the figure, the sheets carry uniform surface charge densities which are indicated in terms of σ_0. The separation between any two consecutive sheets is 1μm. The various regions between the sheets are denoted as 1, 2, 3, 4 and 5. If $\sigma_0 = 9\ \mu C/m^2$, then which of the following statements is/are correct: (Take permittivity of free space $\epsilon_0 = 9 \times 10^{-12}$ F/m):



Configuration I   Configuration II

(A) In region 4 of the configuration I, the magnitude of the electric field is zero.

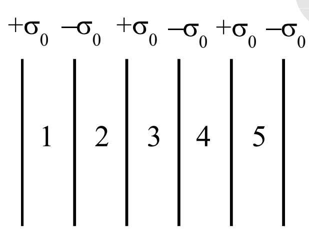
(B) In region 3 of the configuration II, the magnitude of the electric field is $\frac{\sigma_0}{\epsilon_0}$.

(C) Potential difference between the first and the last sheets of the configuration I is 5 V.

(D) Potential difference between the first and the last sheets of the configuration II is zero.

Ans. **(A)**

Sol.



Configuration I

$$(E_4)_I = \frac{\sigma_0}{2\epsilon_0}[1-1+1-1-1+1] = 0$$

$$(V_{First})_I = \frac{-\sigma_0}{2\epsilon_0}[-1+2-3+4-5]d$$

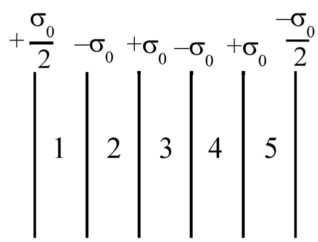
$$= \frac{-\sigma_0}{2\epsilon_0}[-3]d = \frac{\sigma_0 3d}{2\epsilon_0}$$

$$(V_{Last})_I = \frac{-\sigma_0}{2\in_0}[1-2+3-4+5]$$

$$= \frac{\sigma_0}{2\in_0}[-3d]$$

$$(V_{First} - V_{Last})_I = \frac{3\sigma_0 d}{\in_0}$$

$$= \frac{3\times 9\times 10^{-6}\times 1\times 10^{-6}}{9\times 10^{-12}} = 3 \text{ volt}$$



Configuration II

$$(E_3)_{II} = \frac{\sigma_0}{2\in_0}\left[\frac{1}{2}-1+1+1-1+\frac{1}{2}\right] = \frac{\sigma_0}{2\in_0}$$

$$= \frac{-\sigma_0}{2\in_0}\left[-1+2-3+4-\frac{5}{2}\right]d$$

$$= \frac{-\sigma_0}{2\in_0}\left[2-2.5\right]d = \frac{\sigma_0 d}{4\in_0}$$

$$(V_{Last})_{II} = \frac{-\sigma_0}{2\in_0}\left[1-2+3-4+\frac{5}{2}\right]d$$

$$= \frac{-\sigma_0}{2\in_0}\left[6.5-6\right]d = \frac{-\sigma_0 d}{4\in_0}$$

$$(V_{First} - V_{Last})_{II} = \frac{\sigma_0 d}{2\in_0} \neq 0$$

8. The efficiency of a Carnot engine operating with a hot reservoir kept at a temperature of 1000 K is 0.4. It extracts 150 J of heat per cycle from the hot reservoir. The work extracted from this engine is being fully used to run a heat pump which has a coefficient of performance 10. The hot reservoir of the heat pump is at a temperature of 300 K. Which of the following statements is/are correct:

(A) Work extracted from the Carnot engine in one cycle is 60 J.

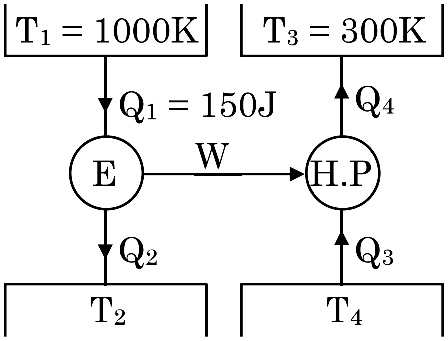
(B) Temperature of the cold reservoir of the Carnot engine is 600 K.

(C) Temperature of the cold reservoir of the heat pump is 270 K.

(D) Heat supplied to the hot reservoir of the heat pump in one cycle is 540 J.

Ans. (A,B,C)

Sol.



$\eta = 0.4$

$(COP)_{HP} = 10$

$W = \eta \times Q_1 = 0.4 \times 150 = 60$ J

$$COP = \frac{\text{Dissered Effect}}{\text{Work input}}$$

$$10 = \frac{Q_3}{W}$$

$Q_3 = 600$ J

Also

$$\eta = 0.4 = 1 - \frac{T_2}{T_1}$$

$T_2 = 600$ K

$$COP = \frac{T_3}{T_3 - T_4} = 10$$

$$= \frac{300}{300 - T_4} = 10$$

$T_4 = 270$ K

SECTION-3 : (Maximum Marks : 32)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : +4 If ONLY the correct numerical value is entered in the designated place;
  Zero Marks : 0 In all other cases.

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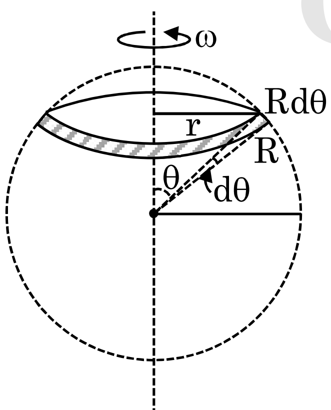
9. A conducting solid sphere of radius R and mass M carries a charge Q. The sphere is rotating about an axis passing through its center with a uniform angular speed ω. The ratio of the magnitudes of the magnetic dipole moment to the angular momentum about the same axis is given as $\alpha\frac{Q}{2M}$. The value of α is _______.

Ans. **(1.66 to 1.67)**

Sol.



dM = dIA

$A = \pi r^2 = \pi(R\sin\theta)^2$

$$dI = \frac{da}{T} = \frac{\sigma(2\pi r)(Rd\theta)\omega}{2\pi}$$

$$dI = \frac{\sigma 2\pi R^2 \omega \sin\theta d\theta}{2\pi}$$

$dI = \sigma R^2 \omega \sin\theta \, d\theta$

Magnetic dipole moment :

$$M = \int dM = \int_0^{\pi} \sigma R^2 \omega \pi R^2 \sin^3\theta d\theta$$

$$M = \sigma R^4 \omega \pi \int_0^{\pi} \sin^3\theta \, d\theta \qquad \left(\because \int_0^{\pi} \sin^3\theta \, d\theta = \frac{4}{3} \right)$$

$$M = \left(\frac{Q}{4\pi R^2} \right) R^4 \omega \pi \left(\frac{4}{3} \right)$$

Magnetic dipole moment

$$M = \frac{QR^2\omega}{3}$$

Angular momentum

$$L = \left(\frac{2}{5} MR^2 \right) \omega$$

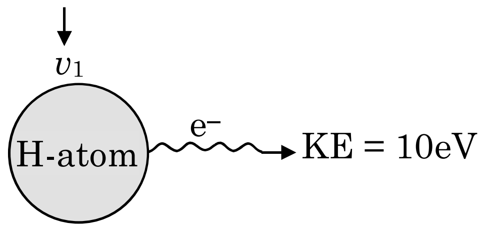
$$\frac{M}{L} = \frac{QR^2\omega}{3 \times \frac{2}{5} MR^2 \omega} = \frac{Q}{2M} \left(\frac{5}{3} \right)$$

$$\alpha = \frac{5}{3} = 1.67$$

10. A hydrogen atom, initially at rest in its ground state, absorbs a photon of frequency ν_1 and ejects the electron with a kinetic energy of 10 eV. The electron then combines with a positron at rest to form a positronium atom in its ground state and simultaneously emits a photon of frequency ν_2. The center of mass of the resulting positronium atom moves with a kinetic energy of 5 eV. It is given that positron has the same mass as that of electron and the positronium atom can be considered as a Bohr atom, in which the electron and the positron orbit around their center of mass. Considering no other energy loss during the whole process, the difference between the two photon energies (in eV) is ____

Ans. **(11.80)**

Sol.



$h\nu_1 = 13.6 + 10 = 23.6$ eV ....(1)

Energy of positronium in ground state

$= -13.6 \frac{\mu}{m}\left(\frac{z}{n}\right)^2$ eV

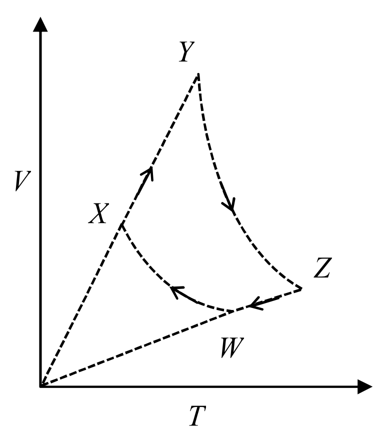
$= -13.6 \times \frac{1}{2}$ eV $= -6.8$ eV

So to make positronium 6.8 eV must release & 5 eV is the KE of COM. So total energy of photon released ($h\nu_2$) will be :

$h\nu_2 = (10 - 5) + 6.8 = 11.8$ eV ...(2)

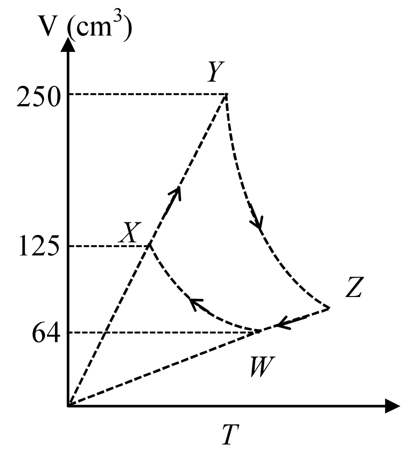
$\therefore$ Difference in energy = 23.6 – 11.8 = 11.8 eV

11. An ideal monatomic gas of n moles is taken through a cycle *WXYZW* consisting of consecutive adiabatic and isobaric quasi-static processes, as shown in the schematic *V-T* diagram. The volume of the gas at *W*, *X* and *Y* points are, 64 cm^3, 125 cm^3 and 250 cm^3, respectively. If the absolute temperature of the gas T_W at the point *W* is such that $nRT_W = 1$ J (R is the universal gas constant), then the amount of heat absorbed (in J) by the gas along the path *XY* is ____



Ans. (1.60)

Sol.



$nRT_w = P_w V_w = 1\ J$

$P_W = \frac{1}{64} \times 10^6$ Pa

For WX process

$P_X V_X^y = P_W V_W^y$

$\Rightarrow P_X = P_W \left(\frac{V_W}{V_X}\right)^y$

amount of heat absorbed in XY process

$Q = nCP\Delta T = n \times \frac{5}{2} R \times [T_Y - T_X]$ [For monoatomic gas $C_P = \frac{5R}{2}$]

$Q = \frac{5}{2}[nRT_Y - nRT_X]$

$= \frac{5}{2}[P_Y V_Y - P_X V_X]$

$= \frac{5}{2} P_X [V_Y - V_X]$ [$\because$ $P_X = P_Y$; Isobaric process]

$= \frac{5}{2} \times P_W \times \left[\frac{V_W}{V_X}\right]^y [V_Y - V_X]$

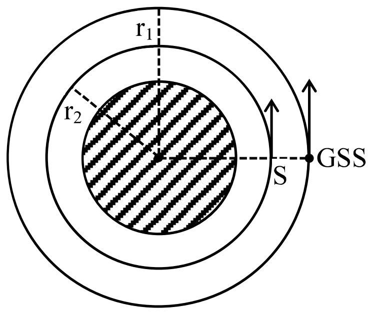
Putting values :

Q = 1.6 Joule

12. A geostationary satellite above the equator is orbiting around the earth at a fixed distance r_1 from the center of the earth. A second satellite is orbiting in the equatorial plane in the opposite direction to the earth's rotation, at a distance r_2 from the center of the earth, such that $r_1 = 1.21\ r_2$. The time period of the second satellite as measured from the geostationary satellite is $\frac{24}{p}$ hours. The value of p is _____

Ans. **(2.33)**

Sol.



$$T \propto r^{3/2}$$

$$\frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2}$$

$$\frac{\omega_2}{\omega_1} = \left(\frac{r_1}{r_2}\right)^{3/2} = (1.21)^{3/2}$$

$$\omega_2 = \omega_1\ (1.331) \qquad \ldots(i)$$

$$(\omega_2 + \omega_1)\ t_0 = 2\pi \qquad \ldots(ii)$$

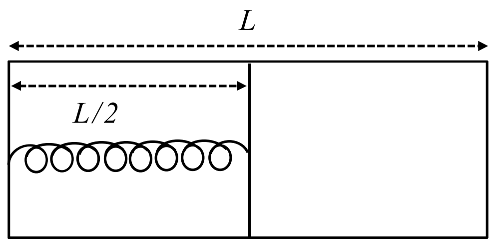
$$t_0 = \frac{2\pi}{\omega_2 + \omega_1} = \frac{2\pi}{\left(\frac{4}{3}+1\right)\omega_1} = \frac{6\pi}{7\omega_1}$$

$$t_0 = \frac{6\pi}{2\pi}\frac{(T_{GSS})}{(7)}$$

$$t_0 = \frac{3 \times 24 \text{ hours}}{7} = \frac{24}{p}\text{hours}$$

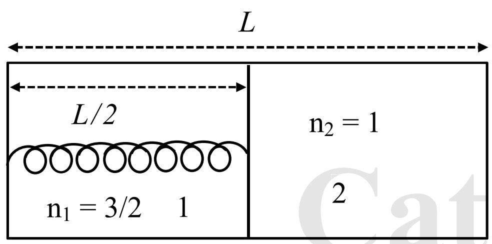
$$p = \frac{7}{3} = 2.33$$

13. The left and right compartments of a thermally isolated container of length L are separated by a thermally conducting, movable piston of area A. The left and right compartments are filled with $\frac{3}{2}$ and 1 moles of an ideal gas, respectively. In the left compartment the piston is attached by a spring with spring constant k and natural length $\frac{2L}{5}$. In thermodynamic equilibrium, the piston is at a distance $\frac{L}{2}$ from the left and right edges of the container as shown in the figure. Under the above conditions, if the pressure in the right compartment is $P = \frac{kL}{A}\alpha$, then the value of α is _____



Ans. **(0.20)**

Sol.

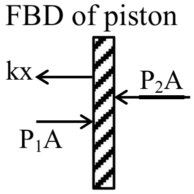


Extension in spring

$x = 0.5\,L - 0.4L$

$= 0.1\,L$

FBD of piston



$kx + P_2A = P_1A$

$P_2A = P_1A - kx$

$P_2 = P_1 - \frac{kL}{A(10)}$ ….(i)

$P_1V = n_1RT$

$P_2V = n_2RT$

$\frac{P_1}{P_2} = \frac{n_1}{n_2} = \frac{3}{2}$

$P_1 = \frac{3}{2}P_2$ ....(ii)

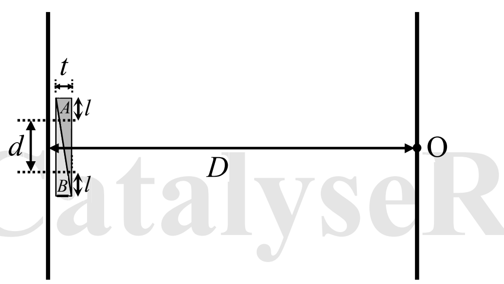
$P_2 = \frac{3}{2}P_2 - \frac{kL}{10A}$

$\frac{P_2}{2} = \frac{kL}{10A}$

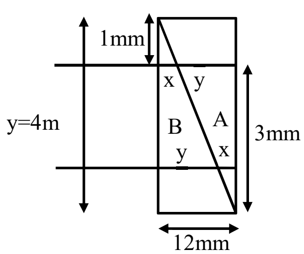
$P_2 = \frac{kL}{5A} = \frac{kL}{A}\alpha$

$\alpha = \frac{1}{5} = 0.2$

14. In a Young's double slit experiment, a combination of two glass wedges A and B, having refractive indices 1.7 and 1.5, respectively, are placed in front of the slits, as shown in the figure. The separation between the slits is d = 2 mm and the shortest distance between the slits and the screen is D = 2 m. Thickness of the combination of the wedges is t = 12 μm. The value of l as shown in the figure is 1 mm. Neglect any refraction effect at the slanted interface of the wedges. Due to the combination of the wedges, the central maximum shifts (in mm) with respect to O by ____



Ans. **(1.20)**



Sol.

x + y = 12μm

$\frac{4}{12} = \frac{1}{x}$

x = 3μm

y = 6μm

$$\therefore \Delta = \frac{yd}{D} - (\mu_B - 1)x - (\mu_A - 1)y + (\mu_B - 1)y + (\mu_A - 1)x$$

$$\frac{-yd}{D} = -0.5\times 3 - 0.7\times 9 + 0.5\times 9 + 0.7\times 3$$

$$\frac{-yd}{D} = -0.5\times 6 - 0.7\times 6$$

$$\Rightarrow \frac{-yd}{D} = -1.2\mu m$$

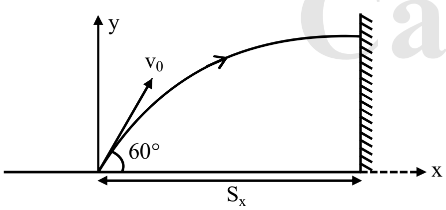
$$\Rightarrow y = \frac{1.2\times D}{d} = \frac{1.2\times 2}{2\times 10^{-3}}\times 10^{-6}$$

= 1.2 mm

15. A projectile of mass 200 g is launched in a viscous medium at an angle 60° with the horizontal, with an initial velocity of 270 m/s. It experiences a viscous drag force $\vec{F} = -c\vec{v}$ where the drag coefficient c = 0.1 kg/s and $\vec{v}$ is the instantaneous velocity of the projectile. The projectile hits a vertical wall after 2 s. Taking e = 2.7, the horizontal distance of the wall from the point of projection (in m) is ____

Ans. **(170.00)**

Sol.



$$\vec{F}_{net} = m\frac{d\vec{v}}{dt}$$

$$m\vec{g} + \vec{F} = \frac{md\vec{v}}{dt}$$

$$m\vec{g} - C\vec{v} = \frac{md\vec{v}}{dt}$$

Horizontal direction

$$-Cv_x = \frac{mdv_x}{dt}$$

$$-\frac{C}{m}\int_0^t dt = \int_{v_{0x}}^{v_x} \frac{dv_x}{v_x}$$

$$-\frac{t}{2} = \ell n \frac{v_x}{v_{0x}}$$

$$\frac{dx}{dt} = v_x = v_{0x}e^{-t/2}$$

$$\int_0^{S_x} dx = v_{0x}\int_0^t e^{-t/2}dt$$

$S_x = 2v_{0x}\,(1 - e^{-t/2})$

at t = 2 sec

$S_x = 2 \times 270 \times \cos 60° \,[1 - \frac{1}{e}]$

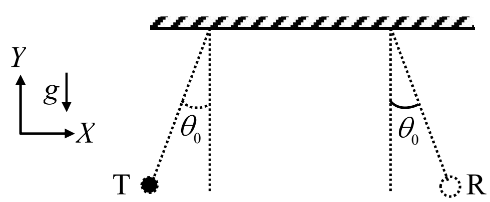
$S_x = 270 \left(1 - \frac{1}{2.7}\right)$

$= \frac{270}{2.7} \times (1.7)$

= 170 m

$S_x = 170\text{m}$

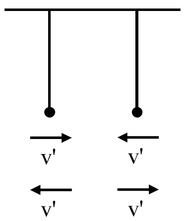
16. An audio transmitter (T) and a receiver (R) are hung vertically from two identical massless strings of length 8 m with their pivots well separated along the X axis. They are pulled from the equilibrium position in opposite directions along the X axis by a small angular amplitude $\theta_0 = \cos^{-1}(0.9)$ and released simultaneously. If the natural frequency of the transmitter is 660 Hz and the speed of sound in air is 330 m/s, the maximum variation in the frequency (in Hz) as measured by the receiver (Take the acceleration due to gravity $g = 10$ m/s^2) is ___



Ans. (31.19 to 32.27)

Sol. $\cos\theta_0 = 1 - \frac{\theta_0^2}{2} = 0.9$

$\frac{\theta_0^2}{2} = 0.1 \Rightarrow \theta_0 = 10.2 = \frac{1}{\sqrt{5}}$



$f_{max} = \frac{v+v'}{v-v'}f$

$f_{min} = \frac{v-v'}{v+v'}f$

$\Delta f_{max} = f_{max} - f_{min} = \frac{v+v'}{v-v'}f - \frac{v-v'}{v+v'}f$

$= \frac{(v+v')^2 - (v-v')^2}{v^2 - v'^2}f$

$\Delta f_{max} = \frac{4vv'}{v^2 - v'^2}f$ …. (i)

Here, $v' = \ell\Omega_{max}$

$= \ell\cdot\theta_0\cdot\omega$ (ω = angular frequency)

$= \ell\theta_0\sqrt{\frac{g}{\ell}}$

$v' = \theta_0\sqrt{g\ell}$

$v' = \frac{1}{\sqrt{5}}\sqrt{10\times 8}$

$v' = 4$

Put in equation (i)

$\Delta f_{max} = \frac{4\times 330\times 4\times 660}{330^2 - 4^2}$

$\approx \frac{16\times 330\times 660}{330} \approx 32$

CHEMISTRY
PAPER-2

SECTION-1 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

---

1. During sodium nitroprusside test of sulphide ion in an aqueous solution, one of the ligands coordinated to the metal ion is converted to

(A) NOS^- (B) SCN^- (C) SNO^- (D) NCS^-

Ans. (A)

Sol. $\mathbf{Na_2[Fe(CN)_5(NO)] + Na_2S \rightarrow Na_4[Fe(CN)_5(NOS)]}$
Sodium nitroprusside purple solution

2. The complete hydrolysis of ICl, ClF_3 and BrF_5 , respectively, gives

(A) IO^-, ClO_2^- and BrO_3^-
(B) IO_3^-, ClO_2^- and BrO_3^-
(C) IO^-, ClO^- and BrO_2^-
(D) IO_3^-, ClO_4^- and BrO_2^-

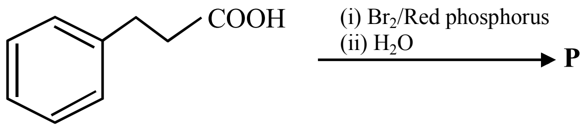
Ans. (A)

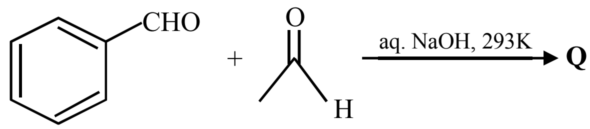
Sol. $ICl \xrightarrow{H_2O} HIO + HCl$, $HIO \downarrow IO^{\ominus}$

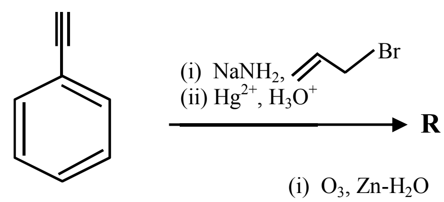
$ClF_3 \xrightarrow{H_2O} HClO_2 + 3HF$, $HClO_2 \downarrow ClO_2^{\ominus}$

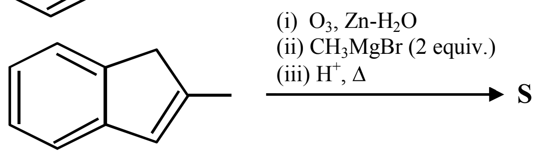
$BrF_5 \xrightarrow{H_2O} HBrO_3 + 5HF$, $HBrO_3 \downarrow BrO_3^{\ominus}$

3. Monocyclic compounds **P**, **Q**, **R** and **S** are the major products formed in the reaction sequences given below.







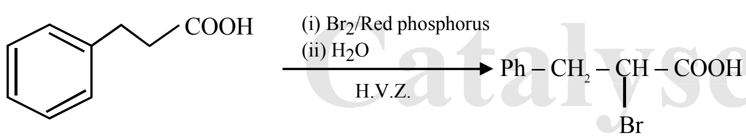


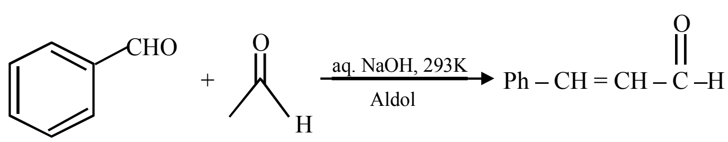
The product having the highest number of unsaturated carbon atom(s) is-

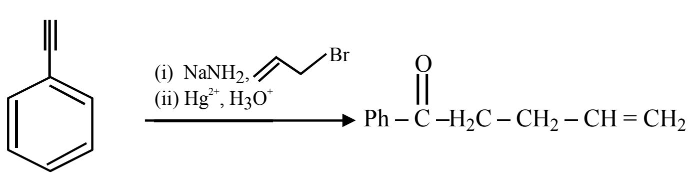
(A) **P** (B) **Q** (C) **R** (D) **S**

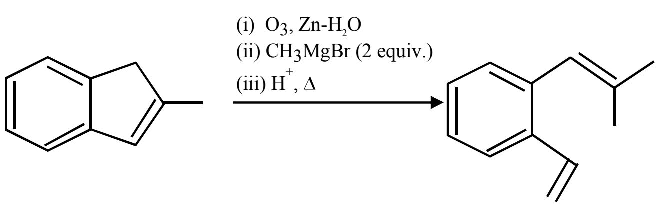
Ans. **(D)**

Sol.

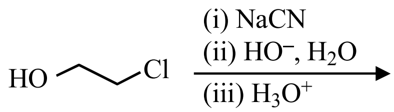




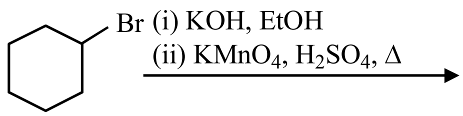


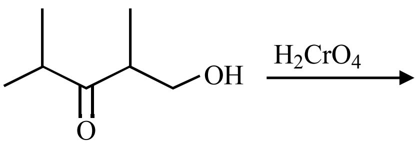


4. The correct reaction/reaction sequence that would produce a dicarboxylic acid as the major product is

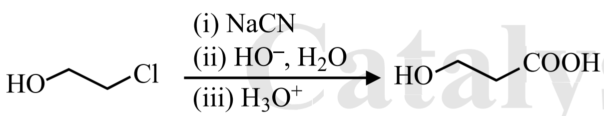
(A) 

(B) CHO–$(CHOH)_4$–CH_2OH $\xrightarrow{Br_2,\ H_2O}$

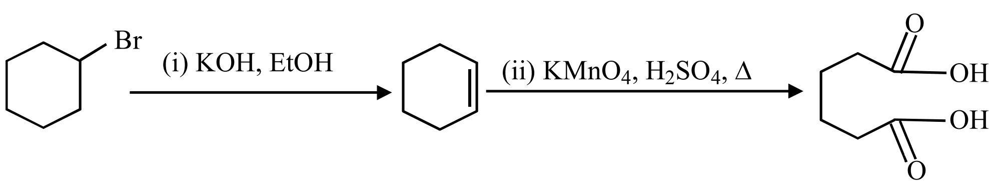
(C) 

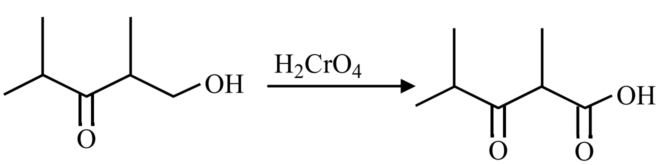
(D) 

Ans. **(C)**

Sol. (A) 

(B) CHO–$(CHOH)_4$–CH_2OH $\xrightarrow{Br_2,\ H_2O}$ COOH–$(CHOH)_4$–CH_2–OH

(C) 

(D) 

SECTION-2 : (Maximum Marks : 16)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks	: +4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	: +3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	: +2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	: +1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: −2	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 marks;
  choosing ONLY (B) will get +1 marks;
  choosing ONLY (D) will get +1 marks;
  choosing no option (i.e. the question is unanswered) will get 0 marks and
  choosing any other option(s) will get –2 marks.

---

5. The correct statements (s) about intermolecular forces is(are)

(A) The potential energy between two point charges approaches zero more rapidly than the potential energy between a point dipole and a point charge as the distance between them approaches infinity.

(B) The average potential energy of two rotating polar molecules that are separated by a distance r has $1/r^3$ dependence.

(C) The dipole-induced dipole average interaction energy is independent of temperature.

(D) Nonpolar molecules attract one another even though neither has a permanent dipole moment.

Ans. **(C, D)**

Sol. **(i)** Ion – Ion → Interaction energy $\propto \frac{1}{r}$

Ion – dipole → Interaction energy $\propto \frac{1}{r^2}$

Ion – dipole Interaction energy approaches zero more rapidly as ‘r’ increases

(ii) Rotating Polar molecules → Interaction energy $\propto \frac{1}{r^6}$.

(iii) Dipole – induced dipole forces are independent of temperature.

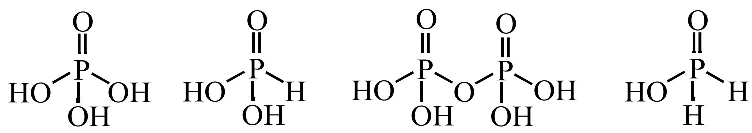
(iv) Non-polar species show London dispersion forces.

6. The compound(s) with P–H bond(s) is(are)

(A) H_3PO_4 (B) H_3PO_3 (C) $H_4P_2O_7$ (D) H_3PO_2

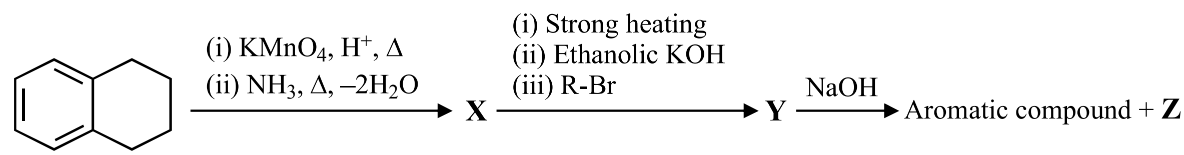
Ans. **(B, D)**

Sol.



H_3PO_4 (A) H_3PO_3 (B) $H_4P_2O_7$ (C) H_3PO_2 (D)

7. For the reaction sequence given below, the correct statement(s) is(are)



(A) Both **X** and **Y** are oxygen containing compounds.

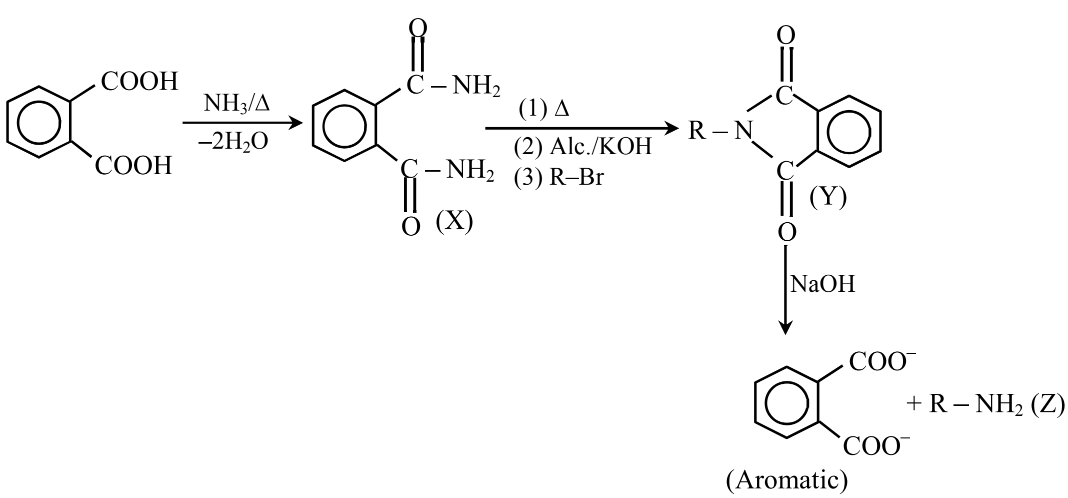
(B) **Y** on heating with $CHCl_3$/KOH forms isocyanide.

(C) **Z** reacts with Hinsberg's reagent.

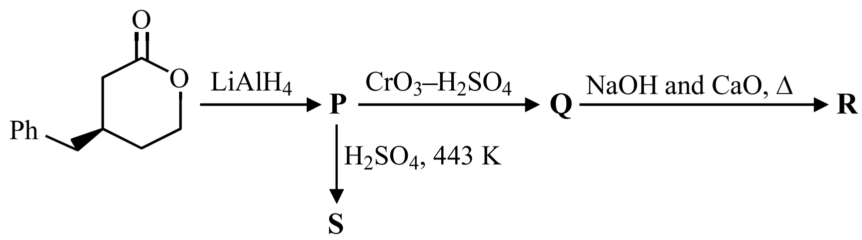
(D) **Z** is an aromatic primary amine.

Ans. **(A, C)**

Sol.



8. For the reaction sequence given below, the correct statement(s) is(are)



(A) **P** is optically active.

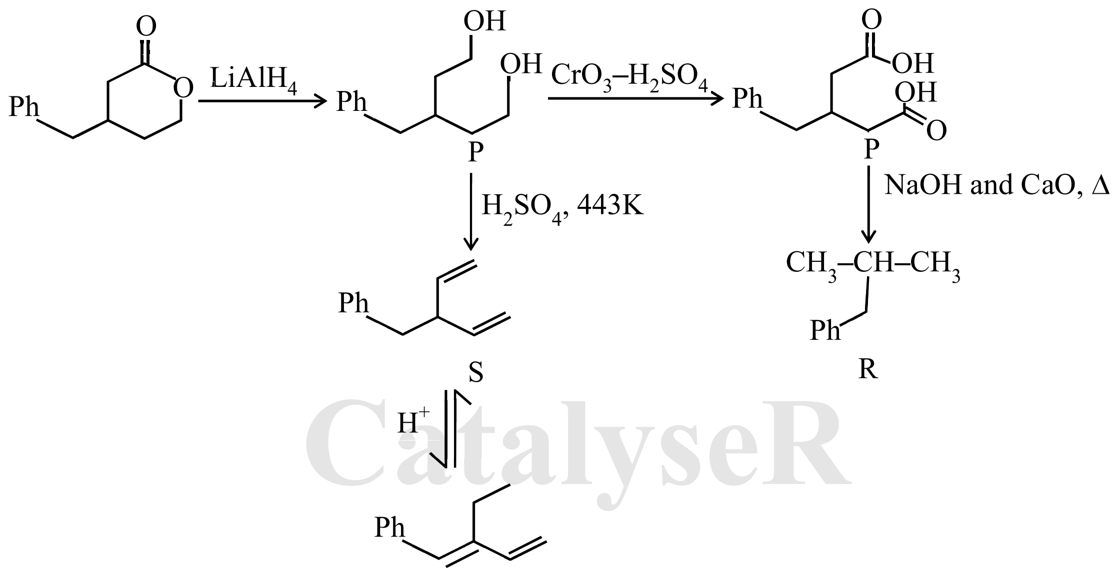
(B) **S** gives Bayer's test.

(C) **Q** gives effervescence with aq. $NaHCO_3$.

(D) **R** is an alkyne.

Ans. **(B,C)**

Sol.



SECTION-3 : (Maximum Marks : 32)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : +4 If ONLY the correct numerical value is entered in the designated place;
  Zero Marks : 0 In all other cases.

---

9. The density (in g cm^{-3}) of the metal which forms a cubic close packed (ccp) lattice with an axial distance (edge length) equal to 400 pm is ______.

Use: Atomic mass of metal = 105.6 amu and Avogadro's constant = 6×10^{23} mol^{-1}

Ans. **(11.00)**

Sol. CCP :

(Z = 4, a = 400 pm, M = 105.6 g/mol)

$$\text{Density} = \frac{Z \times M}{a^3 \times N_A}$$

$$= \frac{4 \times 105.6}{(400 \times 10^{-10})^3 \times 6 \times 10^{23}} = 11.00 \text{ g/cm}^3$$

10. The solubility of barium iodate in an aqueous solution prepared by mixing 200 mL of 0.010 M barium nitrate with 100 mL of 0.10 M sodium iodate is $\boldsymbol{X} \times 10^{-6}$ mol dm^{-3}. The value of $\boldsymbol{X}$ is _______.

Use: Solubility product constant (K_{sp}) of barium iodate = 1.58×10^{-9}

Ans. **(3.95)**

Sol. $Ba(NO_3)_2(aq) + 2NaIO_3(aq) \rightarrow Ba(IO_3)_2(s) + 2NaNO_3(aq)$

2mmol 10 mmol

LR

– 6 mmol – 4 mmol

$$[NaIO_3] = \frac{6}{300} = 2 \times 10^{-2} \text{ M}$$

$$Ba(IO_3)_2(s) \rightleftharpoons Ba^{2+} + 2IO_3^-$$

s $(2 \times 10^{-2} + 2s)$

$K_{sp} = [Ba^{+2}] [IO_3^-]^2$

$1.58 \times 10^{-9} = [Ba^{+2}] \times (2 \times 10^{-2})^2$

$[Ba^{+2}] = s = 3.95 \times 10^{-6}$ M

X = 3.95

11. Adsorption of phenol from its aqueous solution on to fly ash obeys Freundlich isotherm. At a given temperature, from 10 mg g^{-1} and 16 mg g^{-1} aqueous phenol solutions, the concentrations of adsorbed phenol are measured to be 4 mg g^{-1} and 10 mg g^{-1}, respectively. At this temperature, the concentration (in mg g^{-1}) of adsorbed phenol from 20 mg g^{-1} aqueous solution of phenol will be ___.

Use : $\log_{10} 2 = 0.3$

Ans. **(15.62 or 16.00)**

Sol. $\frac{x}{m} = K \times C^{1/n}$

$$\log\left(\frac{x}{m}\right) = \log K + \frac{1}{n}\log C$$

$$\log 4 = \log K + \frac{1}{n}\log 10$$

$$0.6 = \log K + \frac{1}{n} \qquad \text{.........} (1)$$

$$\log 10 = \log K + \frac{1}{n}\log 16$$

$$1 = \log K + \frac{1}{n} \times 1.2 \qquad \text{........}(2)$$

Equation (2) – equation (1)

$$0.4 = \frac{1}{n} \times (0.2) \Rightarrow n = 0.5$$

and log K = – 1.4

$$\log \frac{x}{m} = \log K + \frac{1}{n} \times \log C$$

$= -1.4 + 2 \times \log 20$

$= -1.4 + 2.6 = 1.2$

$$\frac{x}{m} = 10^{+1.2} = 16$$

($\log 2 = 0.3$, $4 \log 2 = 1.2$, $16 = 10^{+1.2}$)

OR

$\frac{x}{m} = K \times C^{1/n}$

$4 = K(10)^{1/n}$ ……….. (1)

$10 = K(16)^{1/n}$ ……….. (2)

$X = K(20)^{1/n}$ ……….. (3)

On solving equation (1) and (2)

$\frac{1}{n} = 2$

On solving equation (1) and (3)

$\frac{4}{X} = \left(\frac{10}{20}\right)^2$

X = 16

On solving equation (2) and (3)

$\frac{10}{X} = \left(\frac{16}{20}\right)^2$

X = 15.625

12. Consider a reaction *A + R → Product*. The rate of this reaction is measured to be $k[A][R]$. At the start of the reaction, the concentration of R, $[R]_0$, is 10-times the concentration of A, $[A]_0$. The reaction can be considered to be a pseudo first order reaction with assumption that $k[R] = k'$ is constant. Due to this assumption, the relative error (in %) in the rate when this reaction is 40% complete, is ______.

[k and k' represent corresponding rate constants]

Ans. **(4.16 or 4.17)**

Sol.

	A	+	R	→	Product
t = 0	A_0		$10A_0$		
t = t	$0.6A_0$		$9.6A_0$		

Rate = k[A] [R]

$Rate_1 = k(0.6\ A_0) \times 9.6A_0$

	A	+	R	→	Product
t = 0	A_0		$10A_0$ (excess)		
t = t	$0.6A_0$		$10A_0$		

Rate = k'[A], k' = k[R]

$Rate_2 = (k \times 10A_0) \times (0.6\ A_0)$

$$100 \times \frac{\Delta Rate}{Rate_1} = \frac{(0.6\times10 - 0.6\times9.6}{0.6\times9.6}\times100 = 4.1666$$

13. At 300 K, an ideal dilute solution of a macromolecule exerts osmotic pressure that is expressed in terms of the height (h) of the solution (density = 1.00 g cm^{-3}) where h is equal to 2.00 cm. If the concentration of the dilute solution of the macromolecule is 2.00 g dm^{-3}, the molar mass of the macromolecule is calculated to be $\boldsymbol{X} \times 10^4$ g mol^{-1}. The value of ***X*** is ______.

Use : Universal gas constant (R) = 8.3 J K^{-1} mol^{-1} and acceleration due to gravity (g) = 10 m s^{-2}

Ans. **(2.49)**

Sol. $\pi = \rho gh = 10^3 \times 10 \times 2 \times 10^{-2}$ Pascal = 200 Pascal

$\pi = CRT$

$$200 = \frac{2}{M} \times 1000 \times 8.3 \times 300$$

$M = 24900 = 2.49 \times 10^4$ g/mol

X = 2.49

14. An electrochemical cell is fueled by the combustion of butane at 1 bar and 298 K. Its cell potential is $\frac{\boldsymbol{X}}{\boldsymbol{F}} \times 10^3$ volts, where ***F*** is the Faraday constant. The value of ***X*** is ______.

Use : Standard Gibbs energies of formation at 298 K are : $\Delta_f G^o_{CO_2} = -394\,\text{kJ mol}^{-1}$;

$\Delta_f G^o_{\text{water}} = -237\,\text{kJ mol}^{-1}$; $\Delta_f G^o_{\text{butane}} = -18\,\text{kJ mol}^{-1}$

Ans. **(105.50)**

Sol. $C_4H_{10}(g) + \frac{13}{2} O_2(g) \rightarrow 4CO_2(g) + 5H_2O(l)$

$\Delta_r G^o = 4\Delta_f G^o_{CO_2} + 5\Delta_f G^o_{H_2O} - \Delta_f G^o_{C_4H_{10}}$

= 4 × (–394) + 5 (–237) + 18

= – 2743 kJ/mol

$\Delta_r G° = -nFE°$

$-2743 \times 1000 = -26 \times FE°$

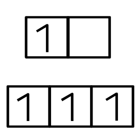
$$E^o = \frac{105.5}{F} \times 10^3 = 105.50$$

15. The sum of the spin only magnetic moment values (in B.M.) of $[Mn(Br)_6]^{3-}$ and $[Mn(CN)_6]^{3-}$ is ______.

Ans. **(7.70 to 7.73)**

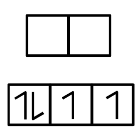
Sol.

$[MnBr_6]^{3-}$



μ = 4.89 B.M.

$[Mn(CN)_6]^{3-}$



μ = 2.84 B.M.

Sum of spin magnetic moments of both complexes is 7.70 to 7.73 B.M.

16. A linear octasaccharide (molar mass = 1024 g mol^{-1}) on complete hydrolysis produces three monosaccharides: ribose, 2-deoxyribose and glucose. The amount of 2-deoxyribose formed is 58.26% (w/w) of the total amount of the monosaccharides produced in the hydrolyzed products. The number of ribose unit(s) present in one molecule of octasaccharide is _______.

Use : Molar mass (in g mol^{-1}): ribose = 150, 2-deoxyribose = 134, glucose = 180;

Atomic mass (in amu): H = 1, O = 16

Ans. **(2.00)**

Sol. $\underset{\text{M.M.}=1024}{\text{Octasaccharide}} + \underset{\text{M.M.}=126}{7H_2O} \longrightarrow$ Ribose + 2deoxyribose + glucose

Total mass = 1024 + 126 = 1150

$$58.26 = \frac{134 \times n}{1150} \times 100$$

$$\frac{66.999}{100} = 134n \qquad n = 4.99 = 5$$

5 units of 2-Deoxyribose

1150 = (5 × 150) + (x × 150) + (y × 180)

$$1150 = \underset{5\,\text{unit}}{\underbrace{750}} + \underset{\substack{300 \\ 2\,\text{unit}}}{\underbrace{150x}} + \underset{\substack{180 \\ 1\,\text{unit}}}{\underbrace{180y}}$$

n = 2.00

MATHEMATICS

PAPER-2

SECTION-1 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

---

1. Let x_0 be the real number such that $e^{x_0} + x_0 = 0$. For a given real number α, define

$$g(x) = \frac{3xe^x + 3x - \alpha e^x - \alpha x}{3(e^x + 1)}$$

for all real numbers x.

Then which one of the following statements is TRUE?

(A) For $\alpha = 2$, $\lim_{x\to x_0}\left|\frac{g(x)+e^{x_0}}{x-x_0}\right| = 0$

(B) For $\alpha = 2$, $\lim_{x\to x_0}\left|\frac{g(x)+e^{x_0}}{x-x_0}\right| = 1$

(C) For $\alpha = 3$, $\lim_{x\to x_0}\left|\frac{g(x)+e^{x_0}}{x-x_0}\right| = 0$

(D) For $\alpha = 3$, $\lim_{x\to x_0}\left|\frac{g(x)+e^{x_0}}{x-x_0}\right| = \frac{2}{3}$

Ans. (C)

Sol. $e^{x_0} + x_0 = 0$

$$g(x) = \frac{3xe^x + 3x - \alpha e^x - \alpha x}{3(e^x + 1)}$$

$$g(x) = x - \frac{\alpha(e^x + x)}{3(e^x + 1)}, \; g(x) + e^{x_0} = x + e^{x_0} - \frac{\alpha}{3}\left(\frac{e^x + x}{e^x + 1}\right)$$

$$g(x) + e^{x_0} = (x - x_0) - \frac{\alpha}{3}\left(\frac{e^x + x}{e^x + 1}\right)$$

⇒ As very clear $g(x_0) + x_0 \Rightarrow 0$

$$\lim_{x\to x_0}\left|\frac{g(x)+e^{x_0}}{x-x_0}\right|\begin{matrix}0\uparrow\\0\uparrow\end{matrix}=\lim_{x\to x_0}\left|\frac{g'(x)}{1}\right|=\left|g'(x_0)\right|$$

$$g'(x)=1-\frac{\alpha}{3}\left(\frac{(e^x+1)(e^x+1)-(e^x+x)e^x}{(e^x+1)^2}\right)$$

$$g'(x_0)=1-\frac{\alpha}{3}\left(\frac{(e^{x_0}+1)^2}{(e^{x_0}+1)^2}\right)=1-\frac{\alpha}{3}$$

$$\Rightarrow \lim_{x\to x_0}\left|\frac{g(x)+e^{x_0}}{x-x_0}\right|=\left|g'(x_0)\right|=\left|1-\frac{\alpha}{3}\right|$$

2. Let $\mathbb{R}$ denote the set of all real numbers. Then the area of the region

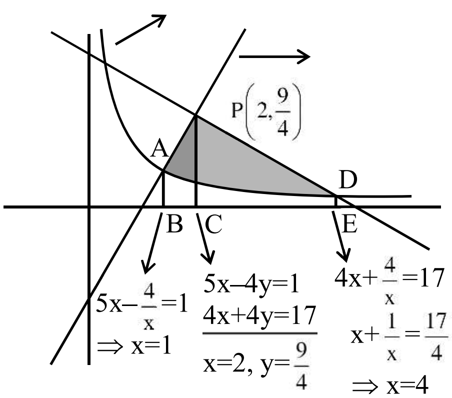
$$\left\{(x,y)\in\mathbb{R}\times\mathbb{R}: x>0,\ y>\frac{1}{x}, 5x-4y-1>0,\ 4x+4y-17<0\right\}$$

is

(A) $\frac{17}{16}-\log_e 4$ (B) $\frac{33}{8}-\log_e 4$ (C) $\frac{57}{8}-\log_e 4$ (D) $\frac{17}{2}-\log_e 4$

Ans. **(B)**

Sol.



$$\text{Area}=\underset{\text{Trapezium APBC}}{\underbrace{\frac{1}{2}\left(1+\frac{9}{4}\right)}}+\underset{\text{Trapezium PDCE}}{\underbrace{\frac{1}{2}\left(\frac{9}{4}+\frac{1}{4}\right)}}.2-\int_1^4\frac{dx}{x}$$

$$\text{Area}=\frac{13}{8}+\frac{20}{8}-\log_e 4=\left(\frac{33}{8}-\log_e 4\right)$$

3. The total number of real solutions of the equation

$$\theta = \tan^{-1}(2\tan\theta) - \frac{1}{2}\sin^{-1}\left(\frac{6\tan\theta}{9+\tan^2\theta}\right)$$

is

(Here, the inverse trigonometric functions $\sin^{-1}x$ and $\tan^{-1}x$ assume values in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, respectively.)

(A) 1 (B) 2 (C) 3 (D) 5

Ans. **(C)**

Sol. Let $\alpha = \frac{1}{2}\sin^{-1}\left(\frac{6\tan\theta}{9+\tan^2\theta}\right)$

$\theta = \tan^{-1}(2\tan\theta) - \alpha$

$\theta + \alpha = \tan^{-1}(2\tan\theta)$

$\tan(\theta + \alpha) = 2\tan\theta$

$\frac{\tan\theta + \tan\alpha}{1 - \tan\alpha\tan\theta} = 2\tan\theta$ ....(1)

$\sin 2\alpha = \frac{6\tan\theta}{9+\tan^2\theta} = \frac{2\tan\alpha}{1+\tan^2\alpha}$

$3\tan\theta + 3\tan\theta\tan^2\alpha = 9\tan\alpha + \tan\alpha\tan^2\theta$

$3(\tan\theta - 3\tan\alpha) = \tan\alpha\tan\theta(\tan\theta - 3\tan\alpha)$

$\tan\theta = \frac{3}{\tan\alpha}$ or $\tan\theta = 3\tan\alpha$

Case-I : $\tan\theta = 3\tan\alpha$

$$\frac{\tan\theta + \frac{\tan\theta}{3}}{1 - \frac{\tan^2\theta}{3}} = 2\tan\theta$$

$\tan\theta = 0$, $\frac{2}{3} = 1 - \frac{\tan^2\theta}{3}$ $\Rightarrow \tan\theta = 1, -1$

$\tan\theta = 0, -1, 1$ $\Rightarrow$ $\boxed{\theta = \frac{\pi}{4}, -\frac{\pi}{4}, 0}$

Case-II : $\tan\theta = \frac{3}{\tan\alpha}$

$$\frac{\tan\theta + \frac{3}{\tan\theta}}{-2} = 2\tan\theta$$

$\tan\theta + \frac{3}{\tan\theta} = -4\tan\theta$ $\Rightarrow$ $\frac{3}{\tan\theta} = -5\tan\theta$

↓ No Solution

4. Let S denote the locus of the point of intersection of the pair of lines

$$4x - 3y = 12\alpha,$$

$$4\alpha x + 3\alpha y = 12,$$

where α varies over the set of non-zero real numbers. Let T be the tangent to S passing through the points $(p, 0)$ and $(0, q)$, $q > 0$, and parallel to the line $4x - \frac{3}{\sqrt{2}}y = 0$.

Then the value of pq is

(A) $-6\sqrt{2}$ (B) $-3\sqrt{2}$ (C) $-9\sqrt{2}$ (D) $-12\sqrt{2}$

Ans. (A)

Sol. $\left.\begin{matrix} 4x - 3y = 12\alpha \\ 4x + 3y = \frac{12}{\alpha} \end{matrix}\right\} \rightarrow 16x^2 - 9y^2 = 144$

$\frac{x^2}{9} - \frac{y^2}{16} = 1 \rightarrow$ Curve : S

$T : y = mx \pm \sqrt{9m^2 - 16}$

$m = \frac{4\sqrt{2}}{3}$

$y = \frac{4\sqrt{2}x}{3} \pm \sqrt{32 - 16}$

$3y = 4\sqrt{2}\,x \pm 12$

as $q > 0$

$3y = 4\sqrt{2}\,x + 12$

$p = -\frac{3}{\sqrt{2}}$ & $q = 4$

$pq = -6\sqrt{2}$

SECTION-2 : (Maximum Marks : 16)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2 marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 marks;
choosing ONLY (B) will get +1 marks;
choosing ONLY (D) will get +1 marks;
choosing no option (i.e. the question is unanswered) will get 0 marks and
choosing any other option(s) will get –2 marks.

---

5. Let $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $P = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$. Let $Q = \begin{pmatrix} x & y \\ z & 4 \end{pmatrix}$ for some non-zero real numbers x, y, and z, for which there is 2×2 matrix R with all entries being non-zero real numbers, such that $QR = RP$.

Then which of the following statements is (are) TRUE?

(A) The determinant of $Q - 2I$ is zero

(B) The determinant of $Q - 6I$ is 12

(C) The determinant of $Q - 3I$ is 15

(D) $yz = 2$

Ans. **(A,B)**

Sol. QR = RP

$$P=\begin{pmatrix}2 & 0\\ 0 & 3\end{pmatrix} \quad Q=\begin{pmatrix}x & y\\ z & 4\end{pmatrix}$$

$$\begin{pmatrix}x & y\\ z & 4\end{pmatrix}\begin{pmatrix}r_1 & r_2\\ r_3 & r_4\end{pmatrix}=\begin{pmatrix}r_1 & r_2\\ r_3 & r_4\end{pmatrix}\begin{pmatrix}2 & 0\\ 0 & 3\end{pmatrix}$$

$$\left.\begin{matrix}xr_1+yr_3=2r_1\\ zr_1+4r_3=2r_3\end{matrix}\right\}\rightarrow \frac{x-2}{-y}=\frac{r_3}{r_1}=\frac{z}{-2}$$

$$\boxed{2x-4=yz}$$

$$\left.\begin{matrix}xr_2+yr_4=3r_2\\ zr_2+4r_4=3r_4\end{matrix}\right\}\rightarrow \frac{x-3}{-y}=\frac{r_4}{r_2}=-z$$

$$\boxed{x-3=yz}$$

$$\Rightarrow 2x-4=x-3 \qquad \Rightarrow \boxed{x=1\ \&\ yz=-2}$$

$$\Rightarrow Q-\lambda I=\begin{pmatrix}x-\lambda & y\\ z & 4-\lambda\end{pmatrix}$$

$|Q-\lambda I| = (\lambda - x)(\lambda - 4) - yz$

$= \lambda^2 - (x+4)\lambda + 4x - yz$

$|Q-\lambda I| = \lambda^2 - 5\lambda + 6$

Now verify the option

6. Let S denote the locus of the mid-points of those chords of the parabola $y^2 = x$, such that the area of the region enclosed between the parabola and the chord is $\frac{4}{3}$. Let R denote the region lying in the first quadrant, enclosed by the parabola $y^2 = x$, the curve S, and the lines $x = 1$ and $x = 4$.

Then which of the following statements is (are) TRUE?

(A) $(4,\sqrt{3})\in S$

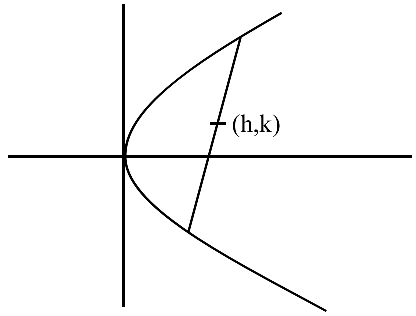
(B) $(5,\sqrt{2})\in S$

(C) Area of R is $\frac{14}{3}-2\sqrt{3}$

(D) Area of R is $\frac{14}{3}-\sqrt{3}$

Ans. **(A,C)**

Sol.



$T = S_1$

$ky - \left(\frac{x+h}{2}\right) = k^2 - h$

$x - 2ky + 2k^2 - h = 0$

$k^2 - h < 0 \quad \Rightarrow h - k^2 > 0$

For Area : interchange x & y

$y - 2kx + 2k^2 - h = 0$ & $y = x^2$

$x^2 - 2kx + (2k^2 - h) = 0 < \begin{matrix}\alpha \\ \beta\end{matrix}$

$|\alpha - \beta| = \sqrt{4k^2 - 4(2k^2 - h)} = \sqrt{4h - 4k^2}$

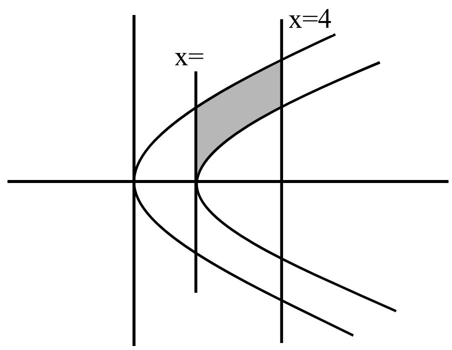
$A = \int_{\alpha}^{\beta}\left(\left(2kx + h - 2k^2\right) - x^2\right) dx$

$A = \frac{\left(4h - 4k^2\right)^{3/2}}{6} = \frac{4}{3}$

$(4h - 4k^2)^{3/2} = 8 \quad \Rightarrow (4h - 4k^2) = 4$

$\boxed{h - k^2 = 1}$

$\left(4, \sqrt{3}\right) \in S$



$A = \int_{1}^{4}\left(\sqrt{x} - \sqrt{x-1}\right)dx = \frac{2}{3}\left(x^{3/2} - (x-1)^{3/2}\right)_1^4$

$A = \frac{2}{3}\left(8 - 3\sqrt{3} - 1\right) = \frac{2}{3}\left(7 - 3\sqrt{3}\right) = \frac{14}{3} - 2\sqrt{3}$

7. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two distinct points on the ellipse

$$\frac{x^2}{9}+\frac{y^2}{4}=1$$

such that $y_1 > 0$, and $y_2 > 0$. Let C denote the circle $x^2 + y^2 = 9$, and M be the point (3, 0). Suppose the line $x = x_1$ intersects C at R, and the line $x = x_2$ intersects C at S, such that the y-coordinates of R and S are positive. Let $\angle ROM = \frac{\pi}{6}$ and $\angle SOM = \frac{\pi}{3}$, where O denotes the origin (0, 0). Let $|XY|$ denote the length of the line segment XY.

Then which of the following statements is (are) TRUE?

(A) The equation of the line joining P and Q is $2x + 3y = 3(1+\sqrt{3})$

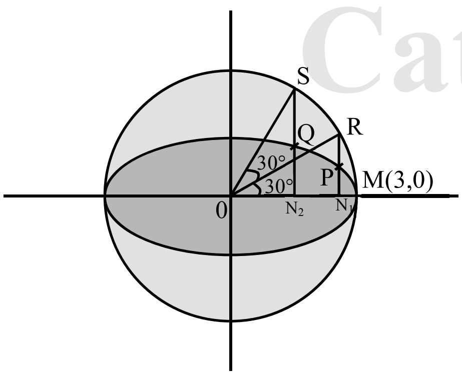
(B) The equation of the line joining P and Q is $2x + y = 3(1+\sqrt{3})$

(C) If $N_2 = (x_2, 0)$, then $3|N_2Q| = 2|N_2S|$

(D) If $N_1 = (x_1, 0)$, then $9|N_1P| = 4|N_1R|$

Ans. **(A,C)**

Sol.



$P \equiv (3\cos30°, 2\sin30°) \equiv \left(\frac{3\sqrt{3}}{2}, 1\right)$

$Q \equiv (3\cos60°, 2\sin60°) \equiv \left(\frac{3}{2}, \sqrt{3}\right)$

$R\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$, $S\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$

Slope of PQ = $m_{PQ} = \dfrac{\sqrt{3}-1}{\dfrac{3}{2}-\dfrac{3\sqrt{3}}{2}} = -\dfrac{2}{3}$

Equation of line PQ

$$y-\sqrt{3} = -\frac{2}{3}\left(x-\frac{3}{2}\right)$$

$\Rightarrow 2x+3y = 3\left(\sqrt{3}+1\right)$ option (A) is correct

Now

if $N_2 = \left(x_2, 0\right) = \left(\dfrac{3}{2}, 0\right)$

$\left|N_2Q\right| = \sqrt{3}$ and $\left|N_2S\right| = \dfrac{3\sqrt{3}}{2}$

$\Rightarrow 3\left|N_2Q\right| = 2\left|N_2S\right|$ option (C) is correct

Now, if $N_1 = (x_1, 0) \Rightarrow N_1 = \left(\dfrac{3\sqrt{3}}{2}, 0\right)$

$\Rightarrow \left|N_1P\right| = 1, \left|N_1R\right| = \dfrac{3}{2}$ option (D) is incorrect

8. Let $\mathbb{R}$ denote the set of all real numbers. Let $f:\mathbb{R}\to\mathbb{R}$ be defined by

$$f(x)=\begin{cases}\dfrac{6x+\sin x}{2x+\sin x} & \text{if } x\neq 0,\\[2mm] \dfrac{7}{3} & \text{if } x=0.\end{cases}$$

Then which of the following statements is (are) TRUE?

(A) The point $x = 0$ is a point of local maxima of f

(B) The point $x = 0$ is a point of local minima of f

(C) Number of points of local maxima of f in the interval $[\pi, 6\pi]$ is 3

(D) Number of points of local minima of f in the interval $[2\pi, 4\pi]$ is 1

Ans. **(B,C,D)**

Sol. $\lim_{x\to 0^+} f(x)=\lim_{x\to 0^-} f(x)=3$

since $3>\dfrac{7}{3}\Rightarrow 3>f(0)$

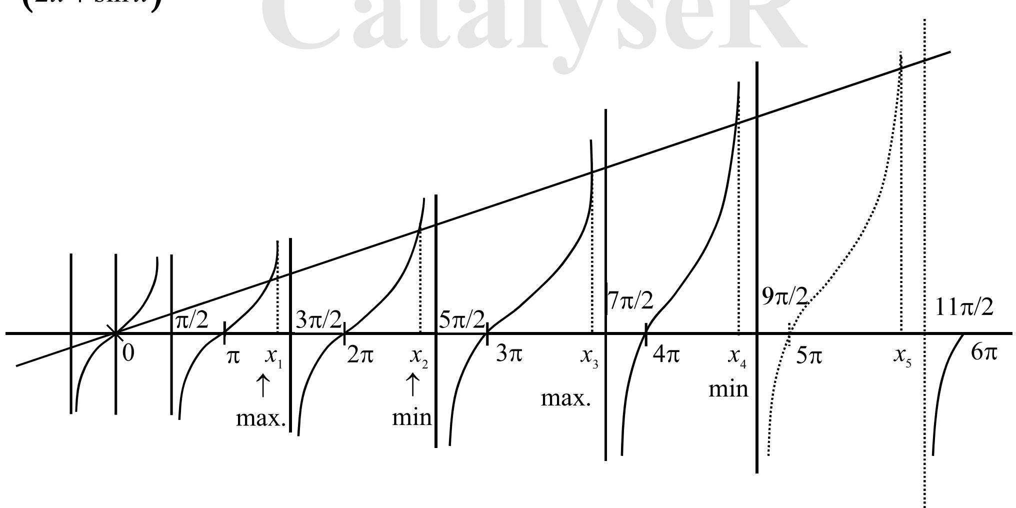
$\Rightarrow$ x = 0 is local minima ….option (B) is correct

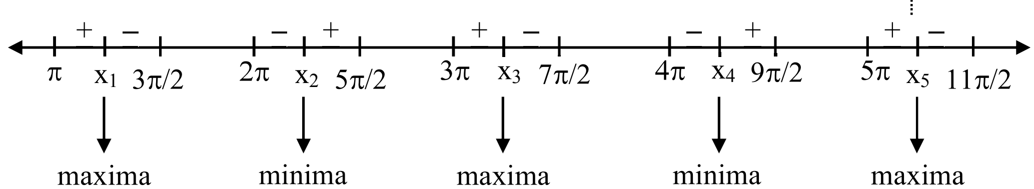
Now,

$$f(x)=\frac{6x+\sin x}{2x+\sin x}=1+\frac{4x}{2x+\sin x}$$

$$f'(x)=\frac{4\left[(2x+\sin x).1-x(2+\cos x)\right]}{(2x+\sin x)^2}=\frac{4(\sin x-x\cos x)}{(2x+\sin x)^2}$$

$$=\frac{4\cos x(\tan x-x)}{(2x+\sin x)^2}$$





Sign of f '(x)

SECTION-3 : (Maximum Marks : 32)

- This section contains **EIGHT (08)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : +4 If ONLY the correct numerical value is entered in the designated place;
  Zero Marks : 0 In all other cases.

---

9. Let *y(x)* be the solution of the differential equation

$$x^2\frac{dy}{dx}+xy=x^2+y^2, x>\frac{1}{e},$$

satisfying $y(1) = 0$. Then the value of $2\frac{\left(y(e)\right)^2}{y(e^2)}$ is ________.

Ans. (0.75)

9. Put y = vx $\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

D.E.

$$x^2\left(v+x\frac{dv}{dx}\right)+x^2v=x^2\left(1+v^2\right)$$

$$\Rightarrow v+x\frac{dv}{dx}+v=1+v^2$$

$$\Rightarrow x\frac{dv}{dx}=1+v^2-2v$$

$$\Rightarrow \int\frac{dv}{(v-1)^2}=\int\frac{dx}{x}$$

$$\Rightarrow -\frac{1}{v-1}=\ell n\,|x|+C$$

$$\Rightarrow \frac{x}{x-y}=\ell n\,|x|+C=\ell n x+C \qquad \left(\text{Since } x>\frac{1}{e}\right)$$

Given y(1) = 0

$\Rightarrow$ C = 1

So $\frac{x}{x-y}=\ell n\left(ex\right)$

Now $y=(e)=\frac{e}{2}$ and $y(e^2)=\frac{2e^2}{3}$

$$\therefore \frac{2\left(y(e)\right)^2}{y(e^2)}=\frac{2.\frac{e^2}{4}}{\frac{2e^2}{3}}=\frac{3}{4}=0.75$$

10. Let $a_0, a_1, ..., a_{23}$ be real numbers such that

$$\left(1+\frac{2}{5}x\right)^{23} = \sum_{i=0}^{23} a_i x^i$$

for every real number x. let a_r be the largest among the numbers a_j for $0 \le j \le 23$.
The the value of r is________.

Ans. **(6.00)**

Sol. For x = 1

$$\left(1+\frac{2}{5}\right)^{23} = a_0 + a_1 + a_2 + ... + a_{23}$$

for numerically greatest term

$$\frac{n+1}{1+\left|\frac{a}{b}\right|} = \frac{23+1}{1+\frac{5}{2}} = \frac{48}{7}$$

$$\Rightarrow \left[\frac{48}{7}\right] = 6 = m \quad \text{(where [.] greatest integer function)}$$

so, T_7 is numerical greatest term

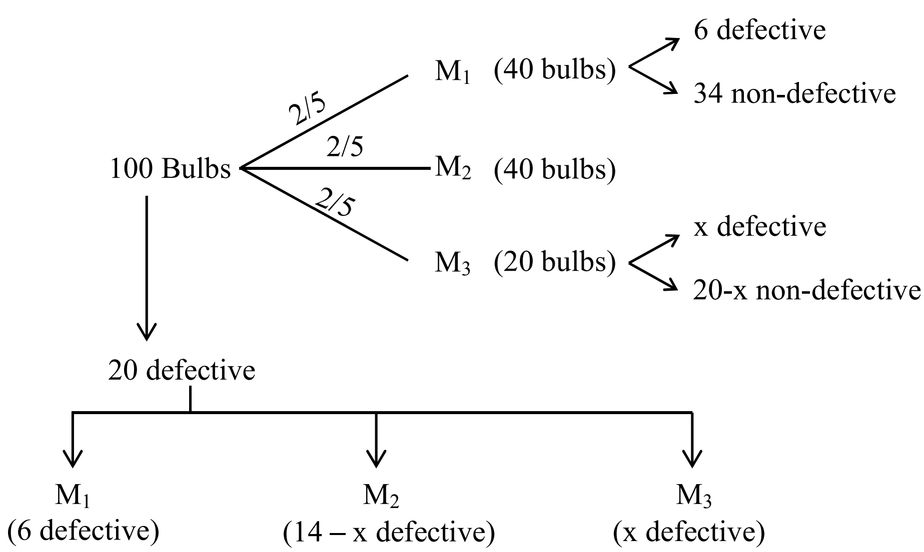
Hence r = 6

11. A factory has a total of three manufacturing units, M_1, M_2, and M_3, which produce bulbs independent of each other. The units M_1, M_2, and M_3 produce bulbs in the proportions of 2: 2: 1, respectively. It is known that 20% of the bulbs produced in the factory are defective. It is also known that, of all the bulbs produced by M_1, 15% are defective. Suppose that, if a randomly chosen bulb produced in the factory is found to be defective, the probability that it was produced by M_2 is $\frac{2}{5}$.

If a bulb is chosen randomly from the bulbs produced by M_3, then the probability that it is defective is___________.

Ans. **(0.30)**

Sol.



Now given probability

$$P\left(\frac{\text{Produced by } M_2}{\text{defective}}\right)=\frac{\frac{40}{100}\times\frac{14-x}{40}}{\frac{20}{100}}=\frac{2}{5}$$

$$\Rightarrow \frac{14-x}{40}=\frac{1}{5}$$

$$\Rightarrow 14-x=8$$

$$\Rightarrow x=6$$

So, the required probability

$$=\frac{6}{20}=0.3$$

12. Consider the vectors

$\vec{x}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{y}=2\hat{i}+3\hat{j}+\hat{k}$, and $\vec{z}=3\hat{i}+\hat{j}+2\hat{k}$.

For two distinct positive real numbers α and β, define

$\vec{X}=\alpha\vec{x}+\beta\vec{y}-\vec{z}$, $\vec{Y}=\alpha\vec{y}+\beta\vec{z}-\vec{x}$, and $\vec{Z}=\alpha\vec{z}+\beta\vec{x}-\vec{y}$.

If the vectors $\vec{X},\vec{Y},$ and $\vec{Z}$ lie in a plane, the value of $\alpha+\beta-3$ is____________.

Ans. **(–2.00)**

Sol. $\left[\vec{x}\,\vec{y}\,\vec{z}\right]=0$

$$\Rightarrow \begin{vmatrix}\alpha & \beta & -1\\ -1 & \alpha & \beta\\ \beta & -1 & \alpha\end{vmatrix}\underbrace{\begin{vmatrix}1 & 2 & 3\\ 2 & 3 & 1\\ 3 & 1 & 2\end{vmatrix}}_{\neq 0}=0$$

$$\Rightarrow\left(\alpha^3+\beta^3-1\right)-\left(-\alpha\beta-\alpha\beta-\alpha\beta\right)=0$$

$$\Rightarrow \alpha^3+\beta^3+3\alpha\beta=1$$

$$\Rightarrow \alpha^3+\beta^3+(-1)^3=3(\alpha)(\beta)(-1)$$

$$\Rightarrow \alpha+\beta-1=0$$

So, $\boxed{\alpha+\beta-3=-2}$

13. For a non-zero complex number z, let $\arg(z)$ denote the principal argument of z, with $-\pi < \arg(z) \le \pi$. Let ω be the cube root of unity for which $0 < \arg(\omega) < \pi$. Let

$$\alpha = \arg\left(\sum_{n=1}^{2025}(-\omega)^n\right).$$

Then the value of $\frac{3\alpha}{\pi}$ is ________________.

Ans. **(–2.00)**

Sol. $\alpha = \arg(-\omega + \omega^2 - \omega^3 + \ldots\ldots + (-\omega)^{2025})$

$$\alpha = \arg\left(\frac{-\omega\left((-\omega)^{2025} - 1\right)}{-\omega - 1}\right)$$

$$\alpha = \arg\left(\frac{-\omega}{-\omega - 1}(-2)\right)$$

$$\alpha = \arg\left(\frac{-2\omega}{\omega + 1}\right)$$

$$\alpha = \arg\left(\frac{-2\omega}{-\omega^2}\right)$$

$$\alpha = \arg\left(\frac{2}{\omega}\right)$$

$\alpha = \arg(2\omega^2)$

$$\alpha = \frac{-2\pi}{3}$$

$$\frac{3\alpha}{\pi} = -2$$

14. Let $\mathbb{R}$ denote the set of all real numbers. Let $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to (0, 4)$ be functions defined by

$$f(x) = \log_e\left(x^2 + 2x + 4\right), \text{ and } g(x) = \frac{4}{1 + e^{-2x}}.$$

Define the composite function $f \circ g^{-1}$ by $(f \circ g^{-1})(x) = (g^{-1}(x))$, where g^{-1} is the inverse of the function g.

Then the value of the derivative of the composite function $f \circ g^{-1}$ at $x = 2$ is ______________.

Ans. **(0.25)**

Sol. Let $h(x) = f(g^{-1}(x))$ and $g(0) = 2$

$h'(x) = f'(g^{-1}(x))\cdot(g^{-1}(x))'$

$h'(2) = f'(g^{-1}(2)\cdot(g^{-1})'(2)$

$= f'(0)\cdot(g^{-1})'(2)$

Now

$f(x) = \log_e (x^2 + 2x + 4)$

$f'(x) = \dfrac{2x+2}{x^2+2x+4}$

$f'(0) = \dfrac{1}{2}$

$g(x) = \dfrac{4}{1+e^{-2x}}$, $g(0) = 2$

$g^{-1}(g(x)) = x$

$\left((g^{-1})'(g(x))\right)g'(x) = 1$

$(g^{-1})'(2) = \dfrac{1}{g'(0)}$

$= \dfrac{1}{2}$

So $h'(2) = \dfrac{1}{4} = 0.25$

$g(x) = \dfrac{4}{1+e^{-2x}}$

$g'(x) = \dfrac{8e^{-2x}}{(1+e^{-2x})^2}$

$g'(0) = \dfrac{8}{4} = 2$

15. Let

$$\alpha = \frac{1}{\sin 60^o \sin 61^o} + \frac{1}{\sin 62^o \sin 63^o} + \ldots + \frac{1}{\sin 118^o \sin 119^o}.$$

Then the value of

$$\left(\frac{\operatorname{cosec} 1^o}{\alpha}\right)^2$$

is ____________.

Ans. **(3.00)**

Sol. $\alpha = \sum_{r=30}^{59} \dfrac{1}{\sin(2r)^o \sin(2r+1)^o}$

$\dfrac{\alpha}{\operatorname{cosec} 1^o} = \sum_{r=30}^{59} \dfrac{\sin 1^o}{\sin(2r)^o \sin(2r+1)^o}$

$= \sum_{r=30}^{59} \left(\cot(2r)^o - \cot(2r+1)^o\right)$

$= \cot 60^o - \cot 61^o$

$+ \cot 62^o - \cot 63^o$

$\vdots$

$+ \cot(118^o) - \cot(119^o)$

$\dfrac{\alpha}{\operatorname{cosec} 1^o} = \cot 60^o = \dfrac{1}{\sqrt{3}}$

$\left(\dfrac{\operatorname{cosec} 1^o}{\alpha}\right)^2 = 3$

16. If

$$\alpha = \int_{\frac{1}{2}}^{2} \frac{\tan^{-1} x}{2x^2 - 3x + 2} dx,$$

then the value of $\sqrt{7}\tan\left(\frac{2\alpha\sqrt{7}}{\pi}\right)$ is ______________.

(Here, the inverse trigonometric function $\tan^{-1} x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.)

Ans. **(21.00)**

Sol. $\alpha = \int_{\frac{1}{2}}^{2} \frac{\tan^{-1} x}{2x^2 - 3x + 2} dx \quad ...(i)$

Let $x = \frac{1}{t}$

$dx = -\frac{1}{t^2} dt$

$$\alpha = \int_{2}^{\frac{1}{2}} \frac{\tan^{-1}\left(\frac{1}{t}\right)}{\frac{2}{t^2} - \frac{3}{t} + 2} \left(\frac{-1}{t^2}\right) dt$$

$$\alpha = \int_{\frac{1}{2}}^{2} \frac{\cot^{-1} t}{2t^2 - 3t + 2} dt \quad ...(ii)$$

Now by (i) + (ii)

$$2\alpha = \int_{\frac{1}{2}}^{2} \frac{\frac{\pi}{2}}{2x^2 - 3x + 2} dx$$

$$\alpha = \frac{\pi}{8} \int_{\frac{1}{2}}^{2} \frac{dx}{x^2 - \frac{3x}{2} + 1}$$

$$\alpha = \frac{\pi}{8} \int_{\frac{1}{2}}^{2} \frac{dx}{\left(x - \frac{3}{4}\right)^2 + \frac{7}{16}}$$

$$\alpha = \frac{\pi}{8 \times \frac{\sqrt{7}}{4}} \left[\tan^{-1}\left(\frac{x - \frac{3}{4}}{\frac{\sqrt{7}}{4}}\right)\right]_{\frac{1}{2}}^{2}$$

$$\alpha = \frac{\pi}{2\sqrt{7}}\left[\tan^{-1}\frac{4x-3}{\sqrt{7}}\right]_{\frac{1}{2}}^{2}$$

$$\alpha = \frac{\pi}{2\sqrt{7}}\left[\tan^{-1}\frac{5}{\sqrt{7}} - \tan^{-1}\left(-\frac{1}{\sqrt{7}}\right)\right]$$

$$\alpha = \frac{\pi}{2\sqrt{7}}\tan^{-1}\frac{\left(\frac{5}{\sqrt{7}} + \frac{1}{\sqrt{7}}\right)}{1-\frac{5}{7}}$$

$$\alpha = \frac{\pi}{2\sqrt{7}}\tan^{-1}(3\sqrt{7})$$

Now $\sqrt{7}\tan\left(\frac{2\sqrt{7}\alpha}{\pi}\right)$

$$\sqrt{7}\times\tan\left(\tan^{-1}(3\sqrt{7})\right)$$

$$\sqrt{7}\times 3\sqrt{7}$$

$= 21$