JEE Advance-2025 Paper With Solutions

CatalyseR

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10 TIMES JEE INDORE TOPPERS in 13 Yrs

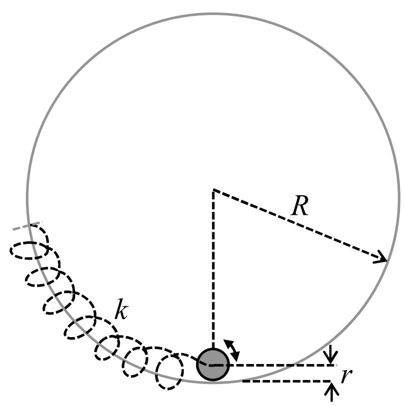
PHYSICS

SECTION-1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme:**

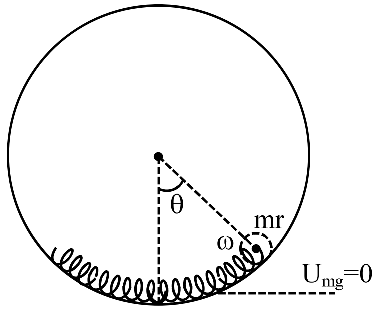
Full Marks	: +3	If **ONLY** the correct option is chosen;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: −1	In all other cases.

1. The center of a disk of radius r and mass m is attached to a spring of spring constant k, inside a ring of radius $R > r$ as shown in the figure. The other end of the spring is attached on the periphery of the ring. Both the ring and the disk are in the same vertical plane. The disk can only roll along the inside periphery of the ring, without slipping. The spring can only be stretched or compressed along the periphery of the ring, following the Hooke's law. In equilibrium, the disk is at the bottom of the ring. Assuming small displacement of the disc, the time period of oscillation of center of mass of the disk is written as $T = \frac{2\pi}{\omega}$. The correct expression for ω is (g is the acceleration due to gravity):



(A) $\sqrt{\frac{2}{3}\left(\frac{g}{R-r}+\frac{k}{m}\right)}$ (B) $\sqrt{\frac{2g}{3(R-r)}+\frac{k}{m}}$ (C) $\sqrt{\frac{1}{6}\left(\frac{g}{R-r}+\frac{k}{m}\right)}$ (D) $\sqrt{\frac{1}{4}\left(\frac{g}{R-r}+\frac{k}{m}\right)}$

Ans. (A)

Sol. 

$$E=\frac{1}{2}k(R-r)^2\theta^2+mg(R-r)(1-\cos\theta)+\frac{1}{2}mv^2+\frac{1}{2}\frac{mr^2}{2}\omega^2$$

Differentiating wrt t,

$$0=\frac{1}{2}k(R-r)^2\cdot 2\theta\frac{d\theta}{dt}+mg(R-r)\cdot\frac{d}{dt}\left(2\frac{\theta^2}{4}\right)+\frac{1}{2}m.2v\frac{dv}{dt}+\frac{mr^2}{4}\cdot 2\omega\frac{d\omega}{dt}$$

$$\Rightarrow 0=k(R-r)^2\theta\frac{d\theta}{dt}+mg(R-r)\theta\frac{d\theta}{dt}+mv\frac{dv}{dt}+\frac{mr^2}{2}\omega\frac{d\omega}{dt}$$

Also, $\frac{d\theta}{dt}=\frac{V}{(R-r)} \Rightarrow \frac{d^2\theta}{dt^2}=\frac{1}{(R-r)}\frac{dv}{dt}=\frac{1}{R-r}a$

$$\therefore k(R-r)^2\cdot\theta\frac{V}{R-r}+mg(R-r)\theta\frac{V}{R-r}=-mv\alpha r-\frac{mr^2}{2}\frac{v}{r}\alpha$$

$$\Rightarrow k(R-r)+mg\theta=-\frac{3}{2}mr\alpha$$

$$\Rightarrow -[k(R-r)+mg]\theta=\frac{3}{2}m(R-r)\frac{d^2\theta}{dt^2}$$

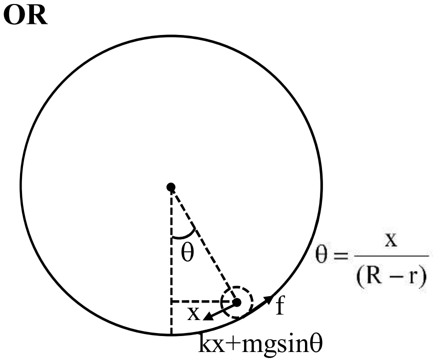
$$\Rightarrow -\frac{2}{3}\left[\frac{k}{m}+\frac{g}{R-r}\right]=\frac{d^2\theta}{dt^2}$$

Compering with standard equation of SHM

$$\omega=\sqrt{\frac{2}{3}\left[\frac{k}{m}+\frac{g}{R-r}\right]}$$

Hence answer is option(A)

OR



$kx + mg\sin\theta - f = ma$

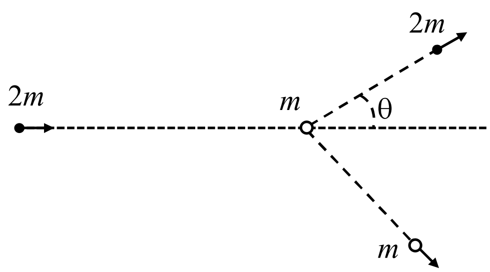
$$\Rightarrow kx + mg\frac{x}{(R-r)} - f = ma$$

$$fr = \frac{mr^2}{2}\cdot\alpha \Rightarrow f = \frac{ma}{2}$$

$$\therefore \left(\alpha + \frac{mg}{R-r}\right)x = \frac{3ma}{2}$$

$$\therefore \omega = \sqrt{\frac{2}{3}\left[\frac{k}{m} + \frac{g}{R-r}\right]}$$

2. In a scattering experiment, a particle of mass $2m$ collides with another particle of mass m, which is initially at rest. Assuming the collision to be perfectly elastic, the maximum angular deviation θ of the heavier particle, as shown in the figure, in radians is:



(A) π (B) $\tan^{-1}\left(\frac{1}{2}\right)$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

Ans. **(D)**

Sol. $2mv_1 = 2mv_{1f}\cos\theta + 2mv_{2f}\cos\phi$ ….(i)

$2m_{1f}\sin\theta = mv_{2f}\sin\phi$ ….(ii)

$$\frac{1}{2}(2m)v_1^2 + \frac{1}{2}m(0)^2 = \frac{1}{2}(2m)v_{1f}^2 + \frac{1}{2}mv_{2f}^2$$

$2v_1^2 = 2v_{1f}^2 + v_{2f}^2$ ….(iii)

From (i), (ii), (iii),

$$3v_{1f}^2 - 4v_1v_{1f}\cos\theta + v_1^2 = 0$$

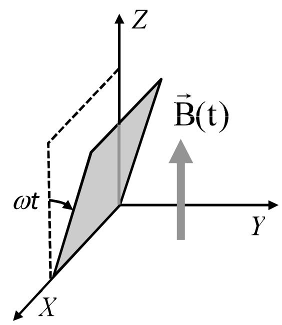
$$(-4v_1\cos\theta)^2 - 4(3)(v_1^2) \ge 0$$

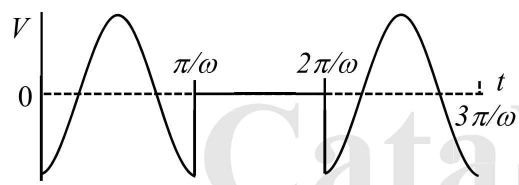
$$\cos^2\theta \ge \frac{3}{4}$$

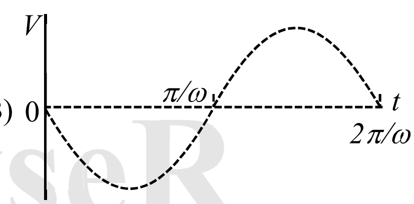
$$\cos^2\theta \ge \frac{\sqrt{3}}{2}$$

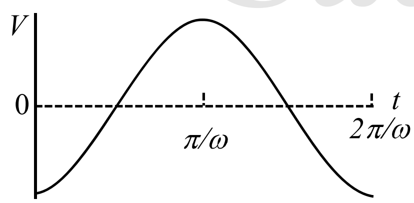
$$\therefore \theta = \frac{\pi}{6}$$

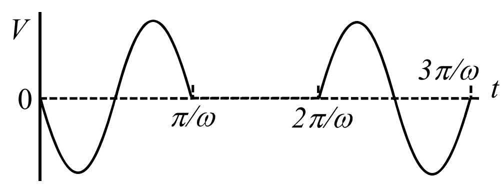
3. A conducting square loop initially lies in the XZ plane with its lower edge hinged along the X-axis. Only in the region $y \geq 0$, there is a time dependent magnetic field pointing along the Z-direction, $\vec{B}(t) = B_0(\cos \omega t)\hat{K}$, where B_0 is a constant. The magnetic field is zero everywhere else. At time $t = 0$, the loop starts rotating with constant angular speed ω about the X axis in the clockwise direction as viewed from the $+X$ axis (as shown in the figure). Ignoring self-inductance of the loop and gravity, which of the following plots correctly represents the induced e.m.f. (V) in the loop as a function of time:



(A) 

(B) 

(C) 

(D) 

Ans. **(A)**

Sol. $\phi = B_0 \cos \omega t A \sin \omega t = \dfrac{B_0 A \sin 2\omega t}{2}$

$$\varepsilon = -\frac{d\phi}{dt} = -B_0 A \cos 2\omega t = \left(0 \leq t \leq \frac{\pi}{\omega}\right)$$

$$\varepsilon = 0 \quad \left(\frac{\pi}{\omega} \leq t \leq \frac{2\pi}{\omega}\right)$$

Ans. Option (A)

4. Figure 1 shows the configuration of main scale and Vernier scale before measurement. Fig. 2 shows the configuration corresponding to the measurement of diameter D of a tube. The measured value of D is:

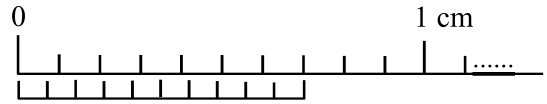


Fig.1

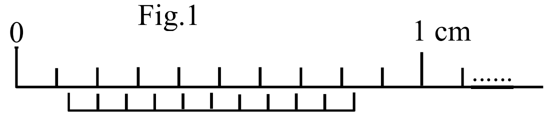


Fig.2

(A) 0.12 cm (B) 0.11 cm (C) 0.13 cm (D) 0.14 cm

Ans. **(C)**

Sol. 10 MSD = 1cm; 1 MSD = 0.1 cm

7 MSD = 10 VSD

1 VSD = 0.07 cm

Reading = 2 MSD – VSD

= 0.2 cm – 0.07 cm = 0.13 cm

Ans. Option (C)

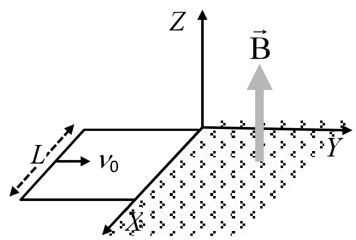
SECTION-2 : (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2 marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 marks;
choosing ONLY (B) will get +1 marks;
choosing ONLY (D) will get +1 marks;
choosing no option (i.e. the question is unanswered) will get 0 marks; and
choosing any other combination of options will get –2 marks.

5. A conducting square loop of side L, mass M and resistance R is moving in the XY plane with its edges parallel to the X and Y axes. The region $y \geq 0$ has a uniform magnetic field, $\vec{B} = B_0 \hat{k}$. The magnetic field is zero everywhere else. At time $t = 0$, the loop starts to enter the magnetic field with an initial velocity $v_0 \hat{j}$ m/s, as shown in the figure. Considering the quantity $K = \frac{B_0^2 L^2}{RM}$ in appropriate units, ignoring self-inductance of the loop and gravity, which of the following statements is/are correct:



(A) If $v_0 = 1.5KL$, the loop will stop before it enters completely inside the region of magnetic field.

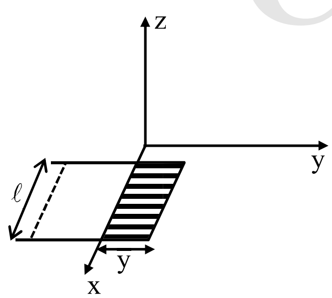
(B) When the complete loop is inside the region of magnetic field, the net force acting on the loop is zero.

(C) If $v_0 = \frac{KL}{10}$, the loop comes to rest at $t = \left(\frac{1}{K}\right) \ln\left(\frac{5}{2}\right)$.

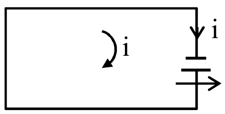
(D) If $v_0 = 3KL$, the complete loop enters inside the region of magnetic field at time $t = \left(\frac{1}{K}\right) \ln\left(\frac{3}{2}\right)$.

Ans. **(B,D)**

Sol.



$$\Rightarrow \frac{-d\phi}{dt} = \frac{d}{dt}(B_0 \times \ell \times y) = BV\ell$$



$$\vec{F} = B(\hat{i})(\ell)(-\hat{j})$$

$$ma = -B_0 \left[\frac{B_0 V \ell}{R}\right](\ell)$$

$$a = -\frac{B_0^2 \ell^2 V}{mR}$$

Also $K = \frac{B_0^2 \ell^2 V}{RM}$

So $[a = -kv]$

$$\frac{dv}{dt} = -kv$$

$$\int_{v_0}^{v} \frac{dv}{dt} = \int_0^t -kdt$$

$$\ell n \frac{v}{v_0} = -kt$$

$[v = v_0 e^{-kt}]$ ....(i)

$$\frac{dx}{dt} = v_0 e^{-kt} \qquad (x \le \ell)$$

$$\int_0^x dx = \int_0^t v_0 e^{-kt} dt$$

$$= \frac{v_0}{k}(1 - e^{-kt})$$

When $x = \ell$

$$\ell = \frac{v_0}{k}(1 - e^{-kt})$$

Option (D) $(v_0 = 3k\ell)$

$$\ell = \frac{3k\ell}{k}(1 - e^{-kt})$$

$$\frac{1}{3} = 1 - e^{-kt}$$

$$f\,\frac{2}{3} = 2e^{-kt}$$

$$-kt = 8n\left(\frac{2}{3}\right)$$

$$t = \frac{1}{k}\ell n\left(\frac{2}{3}\right)$$

Complete loop will enter at $t = \frac{1}{k}\ell n\left(\frac{2}{3}\right)$

Option (B)

$\frac{d\phi}{dt} = 0$, $\underline{e} = 0$, $i = 0$, $F = 0$

Ans. B,D)

6. Length, breadth and thickness of a strip having a uniform cross section are measured to be 10.5 cm, 0.05 mm, and 6.0 μm, respectively. Which of the following option(s) give(s) the volume of the strip in cm^3 with correct significant figures:

(A) 3.2×10^{-5} (B) 32.0×10^{-6} (C) 3.0×10^{-5} (D) 3×10^{-5}

Ans. (D)

Sol. L = 10.5 cm → 3 significant digits

b = 0.05 cm → 1 significant digit

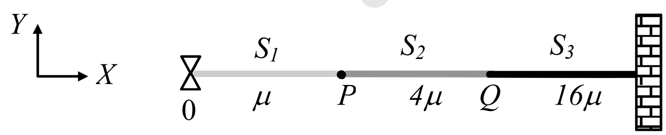
t = 6.0 μm → 2 significant digits

Volume, V = Lbt must have only 1 significant digit

$\Rightarrow V = 10.5 \times 0.05 \times 10^{-1} \times 6.0 \times 10^{-4}\ cm^3$

$= 3 \times 10^{-5}$ cc

7. Consider a system of three connected strings, S_1, S_2 and S_3 with uniform linear mass densities μ kg/m, 4μ kg/m and 16μ kg/m, respectively, as shown in the figure. S_1 and S_2 are connected at the point P, whereas S_2 and S_3 are connected at the point Q, and the other end of S_3 is connected to a wall. A wave generator O is connected to the free end of S_1. The wave from the generator is represented by $y = y_0 \cos(\omega t - kx)$ cm, where y_0, ω and k are constants of appropriate dimensions. Which of the following statements is/are correct:



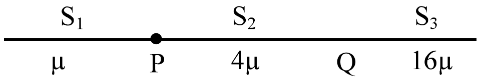
(A) When the wave reflects from P for the first time, the reflected wave is represented by $y = \alpha_1 y_0 \cos(\omega t + kx + \pi)$ cm, where α_1 is a positive constant.

(B) When the wave transmits through P for the first time, the transmitted wave is represented by $y = \alpha_2 y_0 \cos(\omega t - kx)$ cm, where α_2 is a positive constant.

(C) When the wave reflects from Q for the first time, the reflected wave is represented by $y = \alpha_3 y_0 \cos(\omega t - kx + \pi)$ cm, where α_3 is a positive constant.

(D) When the wave transmits through Q for the first time, the transmitted wave is represented by $y = \alpha_4 y_0 \cos(\omega t - 4kx)$ cm, where α_4 is a positive constant.

Ans. (A,D)

Sol. (A) 

$y_1 = y_0 \cos(\omega t - kx)$

when wave going from Rarer to Denser,

$y_r = A_r \cos(\omega t + kx + \pi)$

$y_r = a_1 y_0 \cos(\omega t + kx + \pi)$

option (A) correct

(B) For transmitted from point P

$y_t = A_t \cos[\omega t - k_1 x]$

$$\frac{k_1}{k} = \sqrt{\frac{\mu_1}{\mu}} = \frac{k_1}{k} = \sqrt{\frac{4\mu}{\mu}}$$

$k_1 = 2k$

$y_t = a_2 y_0 \cos[\omega t - 2kx]$

option (B) incorrect

(C) when reflected from Q

$y_i = a_2 y_0 \cos[\omega t - 2kx]$

$y_r = a_3 y_0 \cos[\omega t + 2kx + \pi]$

option (C) incorrect

(D) when transmitted from Q

$y_t = a_4 y_0 \cos[\omega t = k_2 x]$

$$\frac{k_2}{2k} = \sqrt{\frac{16\mu}{4\mu}} \Rightarrow k_2 = 4k$$

$y_t = a_4 y_0 \cos[\omega t - 4kx]$

option (D) correct

SECTION-3 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : +4 If **ONLY** the correct numerical value is entered in the designated place;
  Zero Marks : 0 In all other cases.

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8. A person sitting inside an elevator performs a weighing experiment with an object of mass 50 kg. Suppose that the variation of the height y (in m) of the elevator, from the ground, with time t (in s) is given by $y = 8\left[1+\sin\left(\frac{2\pi t}{T}\right)\right]$, where $T = 40\pi$ s. Taking acceleration due to gravity, $g = 10\ \text{m/s}^2$, the maximum variation of the object's weight (in N) as observed in the experiment is _______.

Ans. **(2.00)**

Sol. $y = 8 + 8\sin\frac{2\pi t}{T}$

With respect to elevator, variation in weight will be

$\Delta W = m(\Delta a)_{max}$

$\Delta W = m \times 2\omega^2 A$

Here elevator is performing SHM

$$\Delta W = 2m \times \left(\frac{2\pi}{T}\right)^2 \times A\ \ N$$

$$\Delta W = 2 \times 50 \times \left(\frac{2\pi}{40\pi}\right)^2 \times 8\ \ N$$

$$\Delta W = 2 \times 50 \times \frac{1}{400} \times 8\ \ N$$

$$\Delta W = \frac{800}{400} N = 2N$$

Ans. is (B)

9. A cube of unit volume contains 35×10^7 photons of frequency 10^{15} Hz. If the energy of all the photons is viewed as the average energy being contained in the electromagnetic waves within the same volume, then the amplitude of the magnetic field is $\alpha \times 10^{-9}$ T. Taking permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ Tm/A, Planck's constant $h = 6 \times 10^{-34}$ Js and $\pi = \frac{22}{7}$, the value of α is____

Ans. **(22.98)**

Sol. Total energy in cube $= 35 \times 10^7 \times hf$

$= 35 \times 10^7 \times 6 \times 10^{-34} \times 10^{15}$

$= 2.1 \times 10^{-10}$ J

Total energy of EM waves = $\dfrac{B_0^2}{2\mu_0} \times$ volume

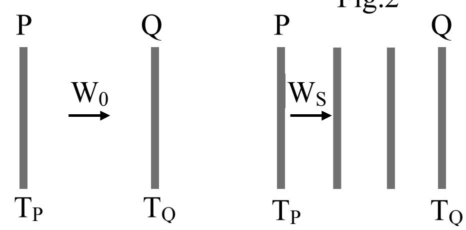
$$B_0^2 = \frac{2.1\times10^{-10}\times8\pi\times10^{-7}}{1^3}$$

$$\Rightarrow B_0 = 22.98 \times 10^{-9}\ \text{T}$$

Ans. 22.98

10. Two identical plates P and Q, radiating as perfect black bodies, are kept in vacuum at constant absolute temperatures T_P and T_Q, respectively, with $T_Q < T_P$, as shown in Fig. 1. The radiated power transferred per unit area from P to Q is W_0. Subsequently, two more plates, identical to P and Q, are introduced between P and Q, as shown in Fig. 2. Assume that heat transfer takes place only between adjacent plates. If the power transferred per unit area in the direction from P to Q (Fig. 2) in the steady state is W_S, then the ratio $\dfrac{W_0}{W_S}$ is ___

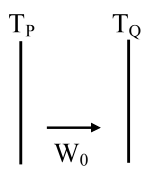
Fig.1 Fig.2



Ans. **(3.00)**

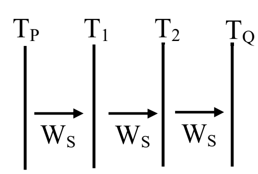
Sol. Initially :

$$W_0 = \sigma\left(T_P^4 - T_Q^4\right)$$



Finally :

Putting heat currents equal in steady state :



$$\sigma\left(T_P^4 - T_1^4\right) = \sigma\left(T_1^4 - T_2^4\right)$$

$$\sigma\left(T_1^4 - T_2^4\right) = \sigma\left(T_2^4 - T_Q^4\right)$$

Adding :

$$T_P^4 - T_1^4 = T_2^4 - T_Q^4$$

$$\Rightarrow T_1^4 + T_2^4 = T_P^4 + T_Q^4$$

and $$\Rightarrow T_1^4 - T_2^4 = T_P^4 - T_1^4$$

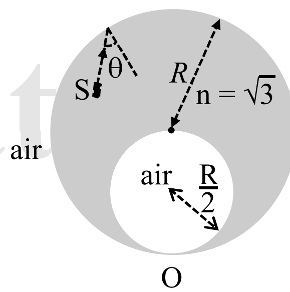
Adding : $T_1^4 = \dfrac{2T_P^4 + T_Q^4}{3}$

So $W_S = \sigma\left(T_P^4 - T_1^4\right)$

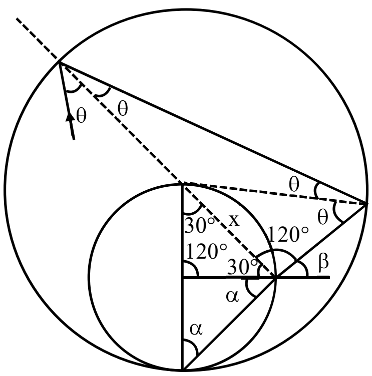
$$= \sigma\left(T_P^4 - \left(\frac{2T_P^4 + T_Q^4}{3}\right)\right) = \sigma\left(\frac{T_P^4 - T_Q^4}{3}\right)$$

hence $\dfrac{W_S}{W_0} = 3$

11. A solid glass sphere of refractive index $n = \sqrt{3}$ and radius R contains a spherical air cavity of radius $\dfrac{R}{2}$, as shown in the figure. A very thin glass layer is present at the point O so that the air cavity (refractive index $n = 1$) remains inside the glass sphere. An unpolarized, unidirectional and monochromatic light source S emits a light ray from a point inside the glass sphere towards the periphery of the glass sphere. If the light is reflected from the point O and is fully polarized, then the angle of incidence at the inner surface of the glass sphere is θ. The value of sin θ is ____



Ans. **(0.75)**



Sol.

$\tan\alpha = \sqrt{3}$

$\alpha = 60°$

$$\sqrt{3}\sin\beta = 1\times\sin\alpha \Rightarrow \beta = 30°$$

$$\frac{R}{2\sin 30°} = \frac{x}{\sin 120°}$$

$$\frac{R}{\sin 120°} = \frac{R\sqrt{3}}{2\times\sin\theta} \Rightarrow \sin\theta = \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}$$

$$\sin\theta = \frac{3}{4}$$

12. A single slit diffraction experiment is performed to determine the slit width using the equation, $\frac{bd}{D} = m\lambda$, where b is the slit width, D the shortest distance between the slit and the screen, d the distance between the m^{th} diffraction maximum and the central maximum, and λ is the wavelength. D and d are measured with scales of least count of 1 cm and 1 mm, respectively. The values of λ and m are known precisely to be 600 nm and 3, respectively. The absolute error (in μm) in the value of b estimated using the diffraction maximum that occurs for $m = 3$ with $d = 5$ mm and $D = 1$ m is ___

Ans. **(75.60 OR 94.50)**

Sol. **Solution-1**

If we can consider

$$\frac{\Delta b}{b} = \frac{\Delta m}{m} + \frac{\Delta\lambda}{\lambda} + \frac{\Delta D}{D} + \frac{\Delta d}{d}$$

$$\frac{\Delta b}{b} = 0 + 0 + \frac{1cm}{1m} + \frac{1mm}{5mm} = 0.21$$

$$b = \frac{m\lambda D}{d} = \frac{3\times 600\times 10^{-3}\times 1}{5\times 10^{-3}}\mu m = 360\ \mu m$$

$\Rightarrow \Delta b = 360 \times 0.21\ \mu m = 75.6\ \mu m$

However, error in d is too large (20%) for the solution-1 to be correct. Hence, we propose solution-2

Solution-2

$$b = \frac{m\lambda D}{d} = 360\ \mu m$$

$$b_{max} = \frac{3\times 600\times 10^{-3}\times 1.01}{4\times 10^{-3}}\mu m = 454.5\ \mu m$$

$$b_{min} = \frac{3\times 600\times 10^{-3}\times 0.99}{6\times 10^{-3}}\mu m = 297\ \mu m$$

Maximum value of b gives error, $\Delta b_1 = 94.5\ \mu m$

Minimum value of b gives error, $\Delta b_2 = 63\ \mu m$

$\therefore$ We always report the largest error, hence correct answer should be 94.5μm

13. Consider an electron in the $n = 3$ orbit of a hydrogen-like atom with atomic number Z. At absolute temperature T, a neutron having thermal energy $k_B T$ has the same de Broglie wavelength as that of this electron. If this temperature is given by $T = \frac{Z^2 h^2}{\alpha \pi^2 a_0^2 m_N k_B}$, (where h is the Planck's constant, k_B is the Boltzmann constant, m_N is the mass of the neutron and a_0 is the first Bohr radius of hydrogen atom) then the value of α is ___

Ans. **(72.00)**

Sol. $$\frac{mv^2}{r} = \frac{KZe^2}{r^2}$$

$$mv^2 r = \frac{1}{4\pi \in_0} Ze^2 \qquad \ldots (1)$$

$$mvr = \frac{nh}{2\pi} \qquad \ldots (2)$$

(1)/(2) gives

$$v = \frac{\frac{Ze^2}{4\pi \in_0}}{\frac{nh}{2\pi}} = \frac{Ze^2}{2 \in_0 nh}$$

$$\frac{h}{mv} = \frac{h}{\sqrt{2m_N \cdot K_B T}}$$

$$T = \frac{m^2 Z^2 e^4}{8 \in_0^2 n^2 h^2 m_N K_B}$$

$$n = 3 \Rightarrow T = \frac{m^2 Z^2 e^4}{72 \in_0^2 h^2 m_N K_B}$$

$$\frac{(1)}{(2)^2} \Rightarrow \frac{1}{mr} = \frac{\frac{Ze^2}{4\pi \in_0}}{\frac{n^2 h^2}{4\pi^2}}$$

$$r = \frac{n^2 h^2 \in_0}{\pi Ze^2 \cdot m} \Rightarrow a_0 = \frac{h^2 \in_0}{\pi e^2 m}$$

$$a_0^2 = \frac{h^4 \in_0^2}{\pi^2 e^4 m^2}$$

$$Ta_0^2 = \frac{m^2 Z^2 e^4}{72 \in_0 h^2 m_N k_B} \cdot \frac{h^4 \in_0^2}{\pi^2 e^4 m^2}$$

$$T = \frac{h^2 Z^2}{72\pi^2 a_0^2 m_N k_B} \Rightarrow \alpha = 72$$

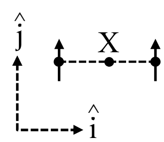
SECTION-4 : (Maximum Marks : 12)

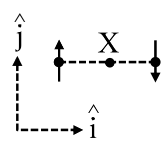
- This section contains **THREE (03)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists : **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : +4 **ONLY** if the option corresponding to the correct combination is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : –1 In all other cases.

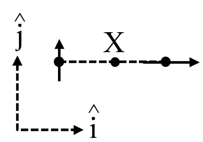
---

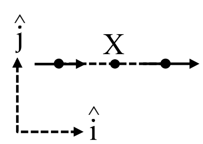
14. List-I shows four configurations, each consisting of a pair of ideal electric dipoles. Each dipole has a dipole moment of magnitude p, oriented as marked by arrows in the figures. In all the configurations the dipoles are fixed such that they are at a distance $2r$ apart along the x direction. The midpoint of the line joining the two dipoles is X. The possible resultant electric fields $\vec{E}$ at X are given in List-II. Choose the option that describes the correct match between the entries in **List-I** to those in **List-II**.

List-I

(P) 

(Q) 

(R) 

(S) 

List-II

(1) $\vec{E} = 0$

(2) $\vec{E} = -\dfrac{p}{2\pi \epsilon_0 r^3}\hat{j}$

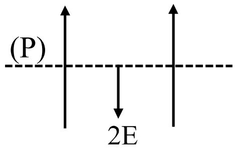
(3) $\vec{E} = -\dfrac{p}{4\pi \epsilon_0 r^3}\left(\hat{i} - \hat{j}\right)$

(4) $\vec{E} = \dfrac{p}{4\pi \epsilon_0 r^3}\left(2\hat{i} - \hat{j}\right)$

(5) $\vec{E} = \dfrac{p}{\pi \epsilon_0 r^3}\hat{i}$

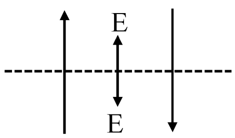
(A) P→3, Q→1, R→2, S→4
(B) P→4, Q→5, R→3, S→1
(C) P→2, Q→1, R→4, S→5
(D) P→2, Q→1, R→3, S→5

Ans. **(C)**

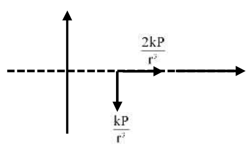
Sol. (P) 

$$E_{net} = \frac{-2kP}{r^3}\hat{j}$$

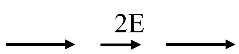
$$E_{net} = \frac{-P\hat{j}}{2\pi\epsilon_0 r^3}$$

(Q) 

$$E_{net} = 0$$

(R) 

$$\frac{2P\hat{i}}{4\pi\epsilon_0 r^3} - \frac{P\hat{j}}{4\pi\epsilon_0 r^3}$$

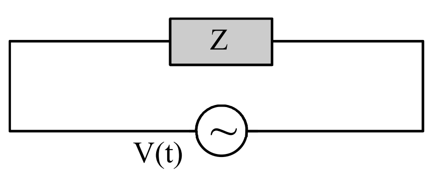
(S) 

$$E_{net} = \frac{4kP\hat{i}}{r^3}$$

$P \rightarrow 2, Q \rightarrow 1, R \rightarrow 4, S \rightarrow 5$

Ans. (C)

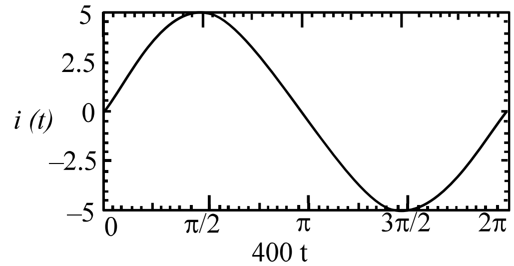
15. A circuit with an electrical load having impedance Z is connected with an AC source as shown in the diagram. The source voltage varies in time as $V(t) = 300\sin(400t)$ V, where t is time in s. List-I shows various options for the load. The possible currents $i(t)$ in the circuit as a function of time are given in List-II.

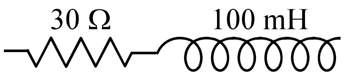


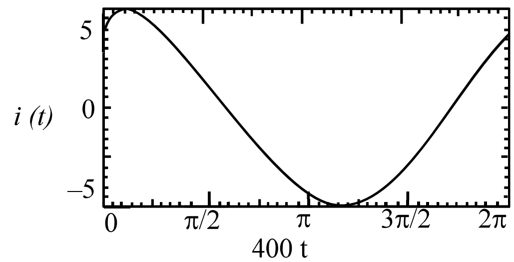
Choose the option that describes the correct match between the entries in **List-I** to those in **List-II.**

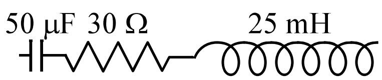
List-I | **List-II**

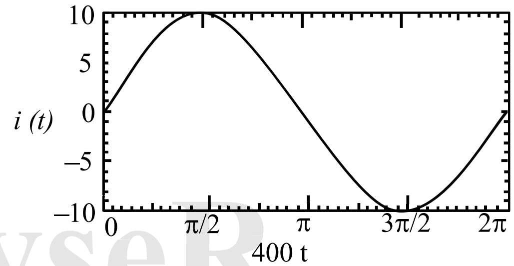
(P)

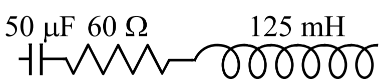

(1)


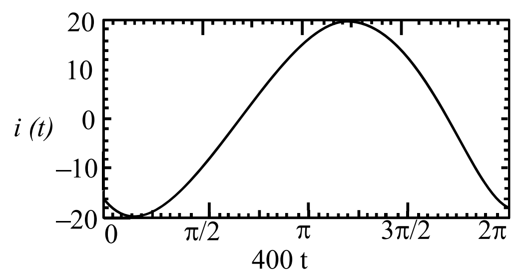
(Q)


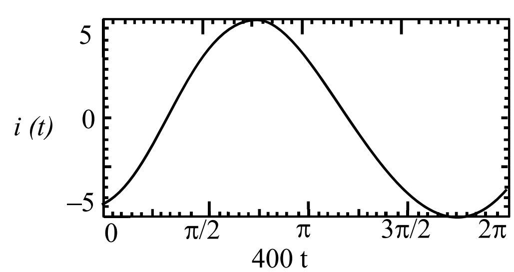
(2)


(R)


(3)


(S)


(4)


(5)


(A) P→3, Q→5, R→2, S→1

(B) P→1, Q→5, R→2, S→3

(C) P→3, Q→4, R→2, S→1

(D) P→1, Q→4, R→2, S→5

Ans. **(A)**

Sol. For P

$i = \dfrac{V}{R} = 10 \sin 400t \Rightarrow (3)$

For Q

$X_L = \omega L = 400 \times 100 \times 10^{-3} = 40\Omega$

$\therefore Z = 50\Omega$

$\therefore\ i = \dfrac{300}{50}\sin\left(400t - 53°\right)$ [current will lag by $\tan^{-1}\dfrac{X_L}{R}$] $\Rightarrow (5)$

For R

$X_C = \dfrac{10^6}{400 \times 50}\Omega = 50\Omega$ and $X_L = 400 \times 25 \times 10^{-3} = 10\Omega$

$\therefore Z = 50\Omega$

$\therefore\ i = \dfrac{300}{50}\sin\left(400t + 53°\right)$ [Current will lead by $\tan^{-1}\dfrac{X_C - X_L}{R}$] $\Rightarrow (2)$

For S

$X_C = 50\Omega$ and $X_L = 400 \times 125 \times 10^{-3} = 50\Omega$

$R = 60\Omega$

$\therefore i = \dfrac{300}{60}\sin\left(400t\right)$ $X_L = X_C \Rightarrow$ Resonance $\Rightarrow (1)$

16. List-I shows various functional dependencies of energy (E) on the atomic number (Z). Energies associated with certain phenomena are given in List-II.

Choose the option that describes the correct match between the entries in **List-I** to those in **List-II**.

List-I	**List-II**
(P) $E \propto Z^2$	(1) energy of characteristic x-rays
(Q) $E \propto (Z-1)^2$	(2) electrostatic part of the nuclear binding energy for stable nuclei with mass numbers in the range 30 to 170
(R) $E \propto Z(Z-1)$	(3) energy of continuous x-rays
(S) E is practically independent of Z	(4) average nuclear binding energy per nucleon for stable nuclei with mass number in the range 30 to 170
	(5) energy of radiation due to electronic transitions from hydrogen-like atoms

(A) P→4, Q→3, R→1, S→2
(B) P→5, Q→2, R→1, S→4
(C) P→5, Q→1, R→2, S→4
(D) P→3, Q→2, R→1, S→5

Ans. **(C)**

Sol. (P) Energy of H-like atom is

$$E=-13.6\frac{Z^2}{n^2}\ \text{So}$$

$E \propto Z^2$

$P \rightarrow (5)$

(Q) Energy of characteristic X-ray by moseley's correction

$$E=-13.6(Z-1)^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]\ \text{So}$$

$E \propto (Z-1)^2$

$Q \rightarrow (1)$

(R) Electrostatics binding energy is proportional to $Z(Z-1)$

$R \rightarrow (2)$

(S) For stable nuclei with mass no. in range 30 to 170. Binding energy per nucleon is constant & graph is straight line

$S \rightarrow (4)$

Ans. (C) is correct

PAPER-1

SECTION-1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks : +3 If **ONLY** the correct option is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : −1 In all other cases.

---

1. The heating of NH_4NO_2 at 60–70ºC and NH_4NO_3 at 200–250ºC is associated with the formation of nitrogen containing compounds **X** and **Y**, respectively. **X** and **Y**, respectively, are

(A) N_2 and N_2O (B) NH_3 and NO_2 (C) NO and N_2O (D) N_2 and NH_3

Ans. **(A)**

Sol. $NH_4NO_2 \xrightarrow[60-70°C]{\Delta} N_2 + 2H_2O$

$NH_4NO_3 \xrightarrow[200-250°C]{\Delta} N_2O + 2H_2O$

2. The correct order of the wavelength maxima of the absorption band in the ultraviolet-visible region for the given complexes is

(A) $[Co(CN)_6]^{3-} < [Co(NH_3)_6]^{3+} < [Co(NH_3)_5(H_2O)]^{3+} < [Co(NH_3)_5(Cl)]^{2+}$

(B) $[Co(NH_3)_5(Cl)]^{2+} < [Co(NH_3)_5(H_2O)]^{3+} < [Co(NH_3)_6]^{3+} < [Co(CN)_6]^{3-}$

(C) $[Co(CN)_6]^{3-} < [Co(NH_3)_5(Cl)]^{2+} < [Co(NH_3)_5(H_2O)]^{3+} < [Co(NH_3)_6]^{3+}$

(D) $[Co(NH_3)_6]^{3+} < [Co(CN)_6]^{3-} < [Co(NH_3)_5(Cl)]^{2+} < [Co(NH_3)_5(H_2O)]^{3+}$

Ans. **(A)**

Sol. $\Delta_o \propto \dfrac{1}{\lambda}$

$\therefore$ The absorb wave length order is

$[Co(CN)_6]^{3-} < [Co(NH_3)_6]^{3+} < [Co(NH_3)_5H_2O]^{3+} < [Co(NH_3)_5Cl]^{2+}$

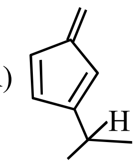
3. One of the products formed from the reaction of permanganate ion with iodide ion in neutral aqueous medium is

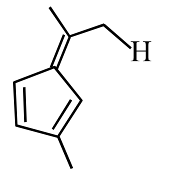
(A) I_2 (B) IO_3^- (C) IO_4^- (D) IO_2^-

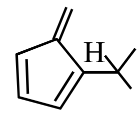
Ans. (B)

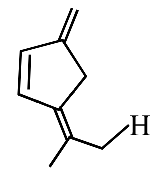
Sol. $I^- + 2MnO_4^- + H_2O \xrightarrow{\text{Neutral solution}} 2MnO_2 + IO_3^- + 2OH^-$

4. Consider the depicted hydrogen **(H)** in the hydrocarbons given below. The most acidic hydrogen **(H)** is

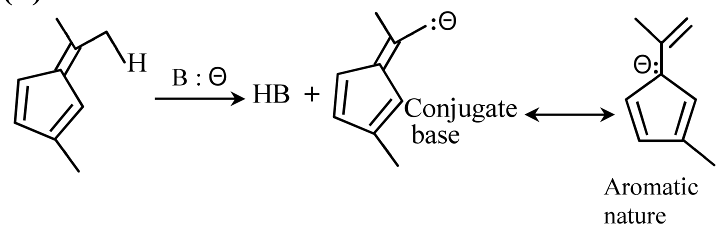
(A) 

(B) 

(C) 

(D) 

Ans. (B)

Sol. 

B is most acidic : It forms most stable conjugate base.

SECTION-2 : (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks	: +4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	: +3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	: +2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	: +1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: −2	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 marks;
  choosing ONLY (B) will get +1 marks;
  choosing ONLY (D) will get +1 marks;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –2 marks.

5. Regarding the molecular orbital (MO) energy levels for homonuclear diatomic molecules, the **INCORRECT** statement(s) is(are)

(A) Bond order of Ne_2 is zero.

(B) The highest occupied molecular orbital (HOMO) of F_2 is σ-type.

(C) Bond energy of O_2^+ is smaller than the bond energy of O_2.

(D) Bond length of Li_2 is larger than the bond length of B_2

Ans. **(B,C)**

Sol. (i) $Ne_2 \Rightarrow (\sigma 1s^2)(\sigma^* 1s^2)(\sigma 2s^2)(\sigma^* 2s^2)(\sigma 2p_z^2)(\pi 2p_x^2 = \pi 2p_y^2)(\pi^* 2p_x^2 = \pi^* 2p_y^2)(\sigma^* 2p_z^2)$

$$\text{B.O.} = \frac{6-6}{2} = 0$$

(ii) $F_2 \Rightarrow (\sigma 1s^2)(\sigma^* 1s^2)(\sigma 2s^2)(\sigma^* 2s^2)(\sigma 2p_z^2)(\pi 2p_x^2 = \pi 2p_y^2)(\underbrace{\pi^* 2p_x^2 = \pi^* 2p_y^2}_{\text{HOMO}})$

(iii) $O_2^{\oplus} \Rightarrow (\sigma 1s^2)(\sigma^* 1s^2)(\sigma 2s^2)(\sigma^* 2s^2)(\sigma 2p_z^2)(\pi 2p_x^2 = \pi 2p_y^2)(\pi^* 2p_x^1 = \pi^* 2p_y)$

$$\text{B.O.} = \frac{6-1}{2} = 2.5$$

$O_2 \Rightarrow (\sigma 1s^2)(\sigma^* 1s^2)(\sigma 2s^2)(\sigma^* 2s^2)(\sigma 2p_z^2)(\pi 2p_x^2 = \pi 2p_y^2)(\pi^* 2p_x^1 = \pi^* 2p_y^1)$

$\text{B.O.} \frac{6-2}{2} = 2$ (Bond order increases, Bond strength increases)

(iv) Size of atom increases, Bond length increases

Size of Li > B

So, Bond length of Li_2 > B_2

6. The pair(s) of diamagnetic ions is(are)

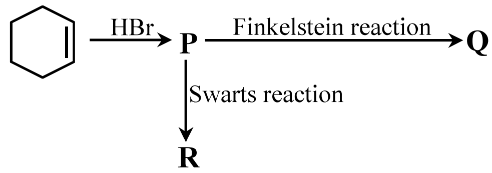
(A) La^{3+}, Ce^{4+} (B) Yb^{2+}, Lu^{3+} (C) La^{2+}, Ce^{3+} (D) Yb^{3+}, Lu^{2+}

Ans. **(A,B)**

Sol.

$La^{+3} \rightarrow [_{54}Xe]\ 4f^0$	diamagnetic
$Yb^{+2} \rightarrow [_{54}Xe]\ 4f^{14}$	diamagnetic
$Lu^{+3} \rightarrow [_{54}Xe]\ 4f^{14}$	diamagnetic
$La^{+2} \rightarrow [_{54}Xe]\ 5d^1$	paramagnetic
$Ce^{+4} \rightarrow [_{54}Xe]\ 4f^0$	diamagnetic
$Ce^{+3} \rightarrow [_{54}Xe]\ 4f^1$	paramagnetic
$Yb^{+3} \rightarrow [_{54}Xe]\ 4f^{13}$	paramagnetic
$Lu^{+2} \rightarrow [_{54}Xe]\ 4f^{14}\ 5d^1$	paramagnetic

7. For the reaction sequence given below, the correct statement(s) is(are)



(In the options, X is any atom other than carbon and hydrogen, and it is different in **P**, **Q** and **R**)

(A) C–X bond length in **P**, **Q** and **R** follows the order **Q** > **R** > **P**.

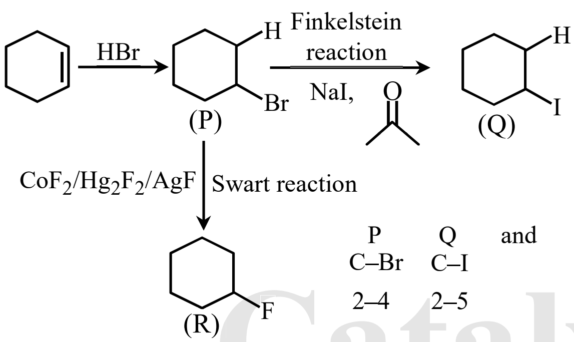
(B) C–X bond enthalpy in **P**, **Q** and **R** follows the order **R** > **P** > **Q**.

(C) Relative reactivity toward S_N2 reaction in **P**, **Q** and **R** follows the order **P** > **R** > **Q**.

(D) pK_a value of the conjugate acids of the leaving groups in **P**, **Q** and **R** follows the order **R** > **Q** > **P**.

Ans. **(B)**

Sol.



P	Q	and	R
C–Br	C–I		C–F
2–4	2–5		2–2

Period no.↑ Size↑ B.L.↑ B.O.↓ B.E.↓

(A) Wrong: bond length order	Q > P > R
(B) **Correct**: Bond enthalpy	R > P > Q
(C) **Wrong:** S_N2 reactivity	Q > P > R
(D) **Wrong:** pKa (HI < HBr < HF)	R > P > Q (pKa)

SECTION-3 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**

  Full Marks : +4 If **ONLY** the correct numerical value is entered in the designated place;

  Zero Marks : 0 In all other cases.

8. In an electrochemicalcell, dichromate ions in aqueous acidic medium are reduced to Cr^{3+}. The current (in amperes) that flows through the cell for 48.25 minutes to produce 1 mole of Cr^{3+} is _______.

Use: 1 Faraday = 96500 C mol^{-1}

Ans. **(100.00)**

Sol. For reduction of dichromate, balanced reaction is :

$$Cr_2O_7^{-2}(aq) + 6e^- + 14H^+(aq) \rightarrow 2Cr^{3+}(aq) + 7H_2O(l)$$

3 mol 1 mol

Number of Farads required = 3 mol

Let current = I amperes

$$\Rightarrow \frac{I \times 48.25 \times 60}{96500} = 3$$

I = 100 A

9. At 25°C, the concentration of H^+ ions in 1.00×10^{-3} M aqueous solution of a weak monobasic acid having acid dissociation constant $(K_a) = 4.00 \times 10^{-11}$ is $\boldsymbol{X} \times 10^{-7}$ M. The value of $\boldsymbol{X}$ is _______.

Use: Ionic product of water $(K_w) = 1.00 \times 10^{-14}$ at 25°C

Ans. **(2.23 or 2.24)**

Sol. Because concentration of H+ from weak acid is less we need to consider self ionization of H_2O also.

$$HX(aq) \rightleftharpoons H^+ + X^-(aq)$$

10^{-3}–x x + y x

$$H_2O(l) \rightleftharpoons H^+(aq) + OH^-(aq)$$

x + y y

Approximation : $(10^{-3} - x) \simeq 10^{-3}$

$$\Rightarrow \frac{x(x+y)}{10^{-3}} = K_a = 4 \times 10^{-11} \quad \text{.........(1)}$$

$$\Rightarrow y(x+y) = Kw = 10^{-14} \quad \text{.........(2)}$$

Add (1) + (2)

$\Rightarrow (x + y)^2 = 5 \times 10^{-14}$

$\Rightarrow x + y = [H^+] = \sqrt{5} \times 10^{-7}$

$\Rightarrow x = \sqrt{5} = 2.236$

Answer 2.23 or 2.24.

10. Molar volume (V_m) of a van der Waals gas can be calculated by expressing the van der Waals equation as a cubic equation with V_m as the variable. The ratio (in mol dm^{-3}) of the coefficient of V_m^2 to the coefficient of V_m for a gas having van der Waals constants a = 6.0 dm^6 atm mol^{-2} and b = 0.060 dm^3 mol^{-1} at 300 K and 300 atm is _______.

Use: Universal gas constant (R) = 0.082 dm^3 atm mol^{-1} K^{-1}

Ans. **(–7.10)**

Sol. $\left(P+\dfrac{a}{V_m^2}\right)(V_m - b) = RT$

$$PV_m - bP + \frac{a}{V_m} - \frac{ab}{V_m^2} - RT = 0$$

$$\Rightarrow \quad PV_m^2 - (bP + RT)\ V_m^2 + aV_m - ab = 0$$

Coefficient of $V_m^2 = -(bP + RT)$

Coefficient of $V_m = a$

$$\text{Ratio} = -\frac{(bP+RT)}{a} = -\left[\frac{0.06\times 300+24.6}{6}\right] = -7.1.$$

11. Considering ideal gas behavior, the expansion work done (in kJ) when 144 g of water is electrolyzed completely under constant pressure at 300 K is _______.

Use: Universal gas constant (R) = 8.3 J K^{-1} mol^{-1}; Atomic mass (in amu): H = 1, O = 16

Ans. **(29.88)**

Sol. $H_2O(l) \rightarrow H_2(g) + \frac{1}{2}O_2(g)$

144 g – –

8 mol 8 mol 4 mol

W = – PΔV = –(Δn)RT

Change in gaseous moles = 12

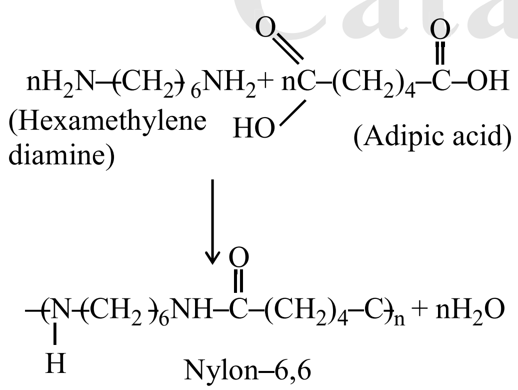
$\Rightarrow \quad W = -\frac{12 \times 8.3 \times 300}{1000} \text{kJ} = -29.88 \text{ kJ.}$

12. The monomer (**X**) involved in the synthesis of Nylon 6,6 gives positive carbylamine test. If 10 moles of **X** are analyzed using Dumas method, the amount (in grams) of nitrogen gas evolved is _______.

Use: Atomic mass of N (in amu) = 14

Ans. **(280.00)**

Sol.



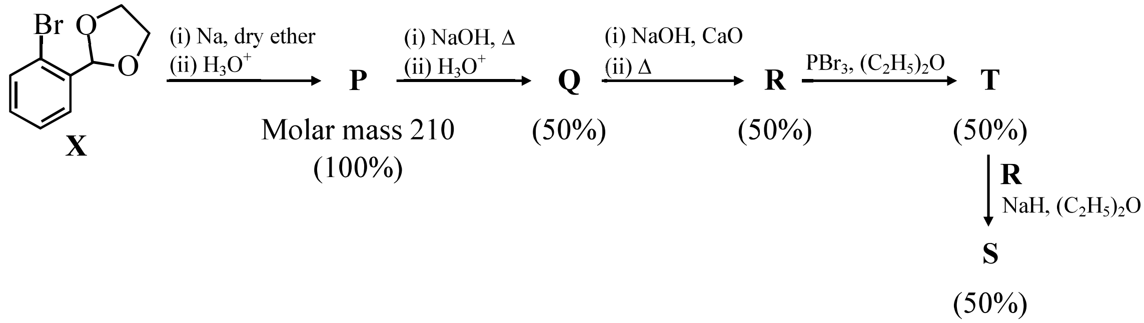
Hexamethylene diamine series give (+) carbylamine test.

$C_x H_y N_z + (2x + 1/2)CuO \rightarrow xCO_2 + \frac{y}{2} H_2O + \frac{z}{2} N_2$ (Dumas method)

$C_6H_{16}N_2$ then $\frac{z}{2} N_2 = \frac{2}{2} N_2 = 1$ mole

Given 10 mole in this reaction so $w_{N_2} = 10 \times 28 = 280\text{gm}$

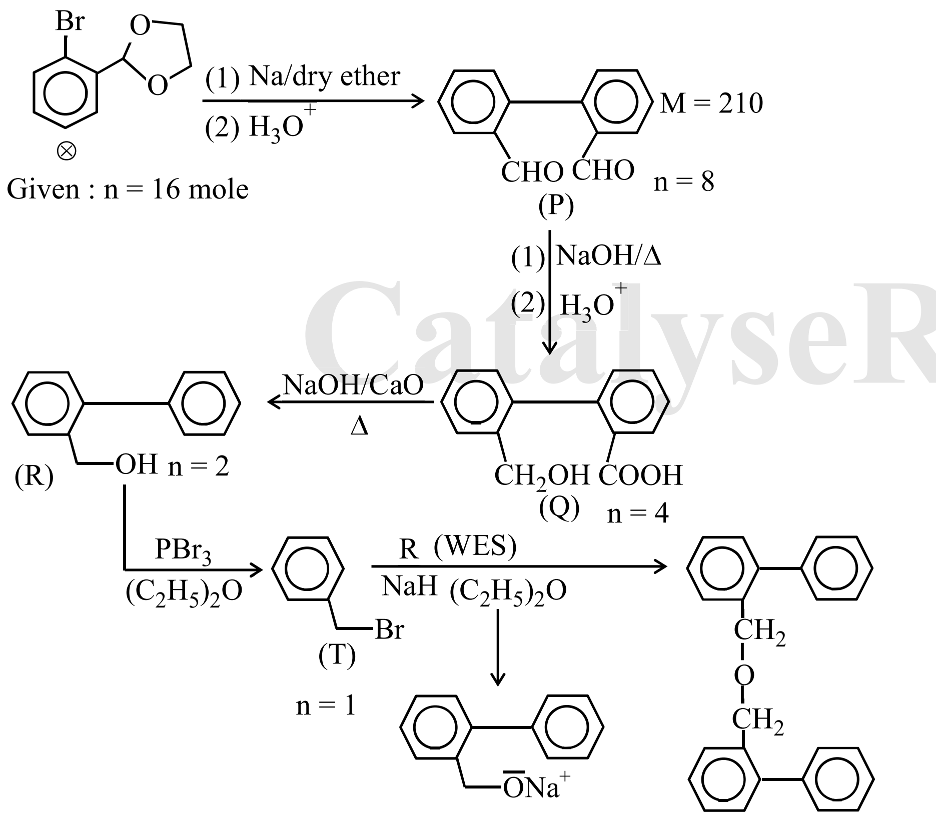
13. The reaction sequence given below is carried out with 16 moles of **X**. The yield of the major product in each step is given below the product in parentheses. The amount (in grams) of **S** produced is_______.



Use: Atomic mass (in amu): H = 1, C = 12, O = 16, Br = 80

Ans. **(175.00)**

Sol.



$M = (C_6H_4)_2(C_6H_5)_2C_2H_4O$

$= 350$

Weight of product of S = 350 × 1/2 = 175 gm Ans.

* WES : Williamson ether synthesis

SECTION-4 : (Maximum Marks : 12)

- This section contains **THREE (03)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists : **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks : +4 **ONLY** if the option corresponding to the correct combination is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : –1 In all other cases.

---

14. The correct match of the group reagents in **List-I** for precipitating the metal ion given in **List-II** from solutions, is

List-I		List-II	
(P)	Passing H_2S in the presence of NH_4OH	(1)	Cu^{2+}
(Q)	$(NH_4)_2CO_3$ in the presence of NH_4OH	(2)	Al^{3+}
(R)	NH_4OH in the presence of NH_4Cl	(3)	Mn^{2+}
(S)	Passing H_2S in the presence of dilute HCl	(4)	Ba^{2+}
		(5)	Mg^{2+}

(A) $P \to 3$; $Q \to 4$; $R \to 2$; $S \to 1$

(B) $P \to 4$; $Q \to 2$; $R \to 3$; $S \to 1$

(C) $P \to 3$; $Q \to 4$; $R \to 1$; $S \to 5$

(D) $P \to 5$; $Q \to 3$; $R \to 2$; $S \to 4$

Ans. **(A)**

Sol. $Mn^{+2} \xrightarrow{H_2S+NH_4OH} \underset{\text{Pink/buff ppt.}}{MnS\downarrow}$

$Ba^{+2} \xrightarrow{(NH_4)_2CO_3+NH_4OH} \underset{\text{White ppt.}}{BaCO_3\downarrow}$

$Al^{+3} \xrightarrow{NH_4Cl+NH_4OH} \underset{\text{White ppt.}}{Al(OH)_3\downarrow}$

$Cu^{+2} \xrightarrow{H_2S+HCl(dil.)} \underset{\text{Black ppt.}}{CuS\downarrow}$

15. The major products obtained from the reactions in **List-II** are the reactants for the named reactions mentioned in **List-I**. Match each entry in **List-I** with the appropriate entry in **List-II** and choose the correct options.

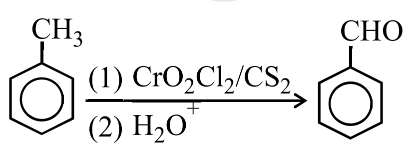
List-I	**List-II**
(P) Stephen reaction	(1) Toluene $\xrightarrow[\text{(ii) } H_3O^+]{\text{(i) } CrO_2Cl_2/CS_2}$
(Q) Sandmeyer reaction	(2) Benzoic acid $\xrightarrow[\text{(iii) } P_4O_{10},\Delta]{\text{(i) } PCl_5,\ \text{(ii) } NH_3}$
(R) Hoffmann bromamide degradation reaction	(3) Nitrobenzene $\xrightarrow[(273-278\text{ K}),\ H_2O]{\text{(i) Fe, HCl; (ii) HCl, } NaNO_2}$
(S) Cannizzaro reaction	(4) Toluene $\xrightarrow[\text{(iii) } SO_2Cl_2,\ \text{(iv) } NH_3]{\text{(i) } Cl_2/h\nu,\ H_2O,\ \text{(ii) Tollen's reagent}}$
	(5) Aniline $\xrightarrow[\text{(iii) aq. NaOH}]{\text{(i) } (CH_3CO)_2O, \text{ Pyridine; (ii) } HNO_3, H_2SO_4, 288K}$

(A) P → 2 ; Q → 4 ; R → 1 ; S → 3 (B) P → 2 ; Q → 3 ; R → 4 ; S → 1

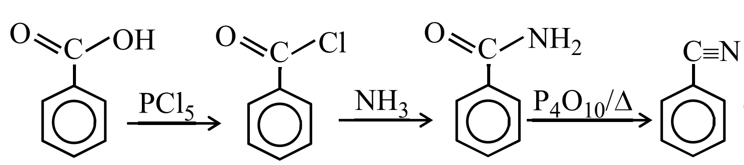
(C) P → 5 ; Q → 3 ; R → 4 ; S → 2 (D) P → 5 ; Q → 4 ; R → 2 ; S → 1

Ans. (B)

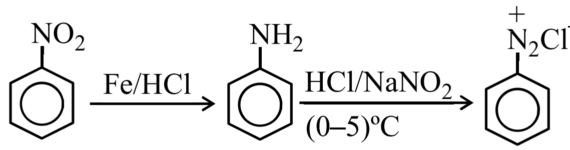
Sol.

(1) 

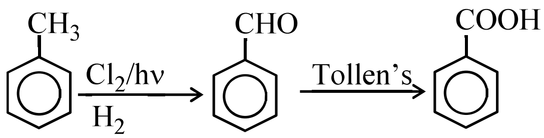
(S) Cannizaro

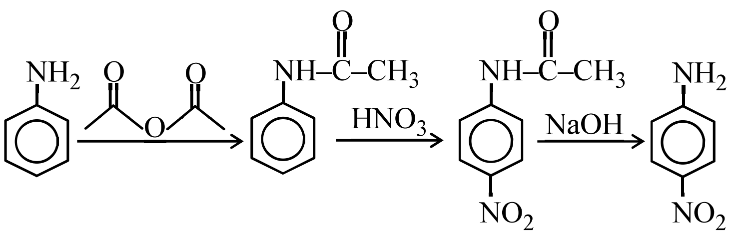
(2) 

(P) Stephen

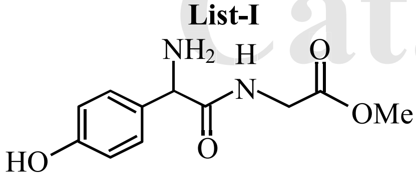
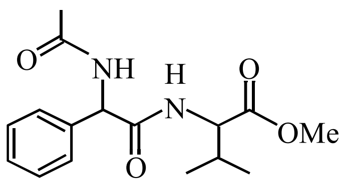
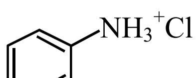
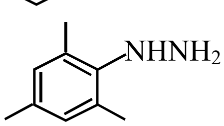
(3) 

(Q) Sandymeyer

(4) 

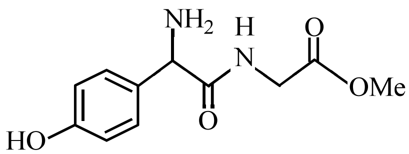
(5) 

16. Match the compounds in **List-I** with the appropriate observations in **List-II** and choose the correct option.

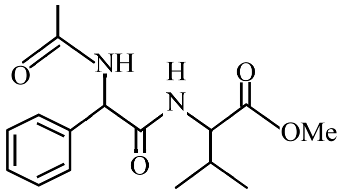
List-I		List-II
(P)		(1) Reaction with phenyl diazonium salt gives yellow dye.
(Q)		(2) Reaction with ninhydrin gives purple color and it also reacts with $FeCl_3$ to give violet color.
(R)		(3) Reaction with glucose will give corresponding hydrazone.
(S)		(4) Lassiagne extract of the compound treated with dilute HCl followed by addition of aqueous $FeCl_3$ gives blood red color.
		(5) After complete hydrolysis, it will give ninhydrin test and it **DOES NOT** give positive phthalein dye test.

(A) P $\rightarrow$ 1 ; Q $\rightarrow$ 5 ; R $\rightarrow$ 4 ; S $\rightarrow$ 2
(B) P $\rightarrow$ 2; Q $\rightarrow$ 5 ; R $\rightarrow$ 1 ; S $\rightarrow$ 3
(C) P $\rightarrow$ 5 ; Q $\rightarrow$ 2 ; R $\rightarrow$ 1 ; S $\rightarrow$ 4
(D) P $\rightarrow$ 2 ; Q $\rightarrow$ 1 ; R $\rightarrow$ 5 ; S $\rightarrow$ 3

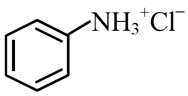
Ans. **(B)**

Sol. **(P)** 

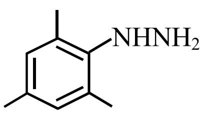
Gives $FeCl_3$ test because of presence of phenolic group and ninhydrin test.

(Q) 

Amino acid obtained gives (+) Ninhydrin test but not phthalein dye test.

(R) 

Reaction with phenyl diazonium salt gives yellow dye.

(S) 

Compound will form hydrazone derivative with aldehydic group of glucose

PAPER-1

SECTION-1 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme:**

Full Marks	: +3	If **ONLY** the correct option is chosen;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: −1	In all other cases.

---

1. Let $\mathbb{R}$ denote the set of all real numbers. Let $a_i, b_i \in \mathbb{R}$ for $i \in \{1, 2, 3\}$.

Define the functions $f: \mathbb{R} \to \mathbb{R}$, $g: \mathbb{R} \to \mathbb{R}$, and $h: \mathbb{R} \to \mathbb{R}$ by

$f(x) = a_1 + 10x + a_2x^2 + a_3x^3 + x^4$,

$g(x) = b_1 + 3x + b_2x^2 + b_3x^3 + x^4$,

$h(x) = f(x+1) - g(x+2)$.

If $f(x) \neq g(x)$ for every $x \in \mathbb{R}$, then the coefficient of x^3 in $h(x)$ is

(A) 8 (B) 2 (C) –4 (D) –6

Ans. **(C)**

Sol. $h(x) = f(x + 1) - g(x + 2)$

$= a_1 + 10(x + 1) + a_2(x + 1)^2 + a_3(x + 1)^3 + (x + 1)^4 - b_1 - 3(x + 2) - b_2(x + 2)^2 - b_3(x + 2)^3 - (x + 2)^4$

Coeff. of x^3 in h(x)

$= a_3 - b_3 - 4$

$f(x) - g(x) \neq 0 \quad \forall x \in R$

$\Rightarrow a_1 + 10x + a_2x^2 + a_3x^3 + x^4 - b_1 - 3x - b_2x^2 - b_3x^3 - x^4 \neq 0$

$\Rightarrow x^3(a_3 - b_3) + x^2(a_2 - b_2) + 7x + (a_1 - b_1) \neq 0$

Cubic Eq. will become zero at atleast are value of x

So it will be quadratic

$\Rightarrow a_3 - b_3 = 0$

2. Three students S_1, S_2 and S_3 are given a problem to solve. Consider the following events :

U : At least one of S_1, S_2 and S_3 can solve the problem,

V : S_1 can solve the problem, given that neither S_2 nor S_3 can solve the problem,

W : S_2 can solve the problem and S_3 cannot solve the problem,

T : S_3 can solve the problem.

For any event E, let $P(E)$ denote the probability of E.

If $P(U) = \frac{1}{2}$, $P(V) = \frac{1}{10}$ and $P(W) = \frac{1}{12}$,

then $P(T)$ is equal to

(A) $\frac{13}{36}$ (B) $\frac{1}{3}$ (C) $\frac{19}{60}$ (D) $\frac{1}{4}$

Ans. **(A)**

Sol. $P(U) = 1 - P\left(S_1' \cap S_2' \cap S_3'\right) = \frac{1}{2}$

$\Rightarrow P\left(S_1' \cap S_2' \cap S_3'\right) = \frac{1}{2}$; $P(S_1') \cdot P(S_2') \cdot P(S_3') = \frac{1}{2}$

$\Rightarrow (1 - P(S_1))(1 - P(S_2))(1 - P(S_3)) = \frac{1}{2}$ ...(1)

$P(V) = \frac{P(S_1 \cap S_2' \cap S_3')}{P(S_2' \cap S_3')} = \frac{1}{10}$

$\Rightarrow P\left(S_1\right).P(S_2')P(S_3') = \frac{1}{10}P(S_2').P(S_3')$

$\Rightarrow P\left(S_1\right) = \frac{1}{10}$

$P(W) = P\left(S_2 \cap S_3'\right) = \frac{1}{12}$

$P(S_2) \cdot P(S_3') = \frac{1}{12}$

$P(S_2)(1 - P(S_3)) = \frac{1}{12}$ ...(2)

Eq. (1)

$\left(1 - \frac{1}{10}\right)\left(1 - P(S_2)\right)\left(1 - P(S_3)\right) = \frac{1}{2}$

$$\left(1-P(S_2)\right)\left(1-P(S_3)\right)=\frac{5}{9} \qquad \ldots(3)$$

$$\frac{\text{Eq.(2)}}{\text{Eq.(3)}} \Rightarrow \frac{P(S_2)}{1-P(S_2)}=\frac{1}{12}\times\frac{9}{5}$$

$$P(S_2)=\frac{3}{23}$$

Put in Eq. (2)

$$\frac{3}{23}\left(1-P(S_3)\right)=\frac{1}{12}$$

$$1-P(S_3)=\frac{23}{36}$$

$$P(S_3)=\frac{13}{36}$$

$$P(T)=\frac{13}{36}$$

3. Let $\mathbb{R}$ denote the set of all real numbers. Define the function $f: \mathbb{R} \to \mathbb{R}$ by

$$f(x)=\begin{cases} 2-2x^2-x^2\sin\frac{1}{x} & \text{if } x \neq 0 \\ 2 & \text{if } x = 0 \end{cases}$$

Then which one of the following statements is TRUE ?

(A) The function f is **NOT** differentiable at $x = 0$

(B) There is a positive real number δ, such that f is a decreasing function on the interval (0, δ)

(C) For any positive real number δ, the function f is **NOT** an increasing function on the interval (–δ, 0)

(D) $x = 0$ is a point of local minima of f

Ans. **(C)**

Sol. (A) RHD at x = 0 : $\lim\limits_{h\to 0}\frac{\left(2-2h^2-h^2\sin\frac{1}{h}\right)-2}{h}=0$

Similarly LHD at x = 0 is also equal to 0.

$\therefore$ Differentiable at x = 0

(B) $f'(x)=-4x-2x\sin\frac{1}{x}-x^2\left(\cos\frac{1}{x}\right)\left(\frac{-1}{x^2}\right)$

$$f'(x) = -\left(4x + 2x\sin\frac{1}{x}\right) + \cos\frac{1}{x}$$

$$f'(x) = -\left(2x\left(4 - \sin\frac{1}{x}\right)\right) + \cos\frac{1}{x}$$

for $x \in (0, \delta)$

$\Rightarrow$ We can't say f(x) is decreasing on $(0, \delta)$ as $\cos\frac{1}{x}$ oscillates.

(C) for $x \in (-\delta, 0)$, for any $\delta > 0$

$\Rightarrow$ f(x) is not increasing on $(-\delta, 0)$ as $\cos\frac{1}{x}$ oscillates from -1 to 1.

(D) $f(0) = 2$

$f(0 + h) < 2$

$f(0 - h) < 2$

$\therefore$ $x = 0$ is local maxima

4. Consider the matrix

$$P = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

Let the transpose of a matrix X be denoted by X^T. Then the number of 3×3 invertible matrices Q with integer entries, such that

$$Q^{-1} = Q^T \text{ and } PQ = QP,$$

is

(A) 32 (B) 8 (C) 16 (D) 24

Ans. **(C)**

Sol. $Q^{-1} = Q^T \Rightarrow QQ^T = I$

Q is orthogonal matrix

$$\text{Let } Q = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

$$PQ = QP \Rightarrow \begin{bmatrix} 2a_1 & 2b_1 & 2c_1 \\ 2a_2 & 2b_2 & 2c_2 \\ 3a_3 & 3b_3 & 3c_3 \end{bmatrix} = \begin{bmatrix} 2a_1 & 2b_1 & 3c_1 \\ 2a_2 & 2b_2 & 3c_2 \\ 2a_3 & 2b_3 & 3c_3 \end{bmatrix}$$

$c_1 = 0, c_2 = 0, a_3 = 0, b_3 = 0$

$$Q = \begin{bmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ 0 & 0 & c_3 \end{bmatrix}$$

$a_1a_2 + b_1b_2 = 0$

$a_1^2 + b_1^2 = 1, \quad a_2^2 + b_2^2 = 1, \quad c_3^2 = 1$

a_1	b_1	a_2	b_2	c_3
1	0	0	1, –1	+1, –1
–1	0	0	1, –1	1, –1
0	1	1, –1	0	1, –1
0	–1	1, –1	0	1, –1

Total 16 matrices

SECTION-2 : (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme:**

  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : −2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 marks;
  choosing ONLY (B) will get +1 marks;
  choosing ONLY (D) will get +1 marks;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –2 marks.

5. Let L_1 be the line of intersection of the planes given by the equations

$$2x + 3y + z = 4 \text{ and } x + 2y + z = 5.$$

Let L_2 be the line passing through the point $P(2, -1, 3)$ and parallel to L_1. Let M denote the plane given by the equation

$$2x + y - 2z = 6$$

Suppose that the line L_2 meets the plane M at the point Q. Let R be the foot of the perpendicular drawn from P to the plane M.

Then which of the following statements is (are) TRUE ?

(A) The length of the line segment PQ is $9\sqrt{3}$

(B) The length of the line segment QR is 15

(C) The area of ΔPQR is $\frac{3}{2}\sqrt{234}$

(D) The acute angle between the line segments PQ and PR is $\cos^{-1}\left(\frac{1}{2\sqrt{3}}\right)$

Ans. **(A,C)**

Sol. Let $L_1 : \vec{r} = \vec{a} + t\vec{b}$

$$\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 1 & 2 & 1 \end{vmatrix} = \hat{i}(1) - \hat{j}(1) + \hat{k}(1)$$

Dr's of $L_1 : <1, -1, 1>$

$L_1 : \dfrac{x+1}{1} = \dfrac{y-0}{-1} = \dfrac{z-6}{1}$

& $L_2 : \dfrac{x-2}{1} = \dfrac{y+1}{-1} = \dfrac{z-3}{1} = \lambda$

$M : 2x + y - 2z - 6 = 0$

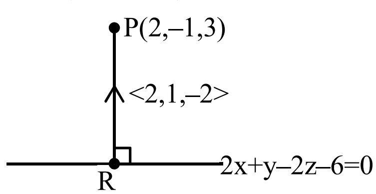
Let point on L_2 $(\lambda + 2, -\lambda - 1, \lambda + 3)$

$2(\lambda + 2) - \lambda - 1 - 2\lambda - 6 - 6 = 0$

$2\lambda + 4 - 3\lambda - 13 = 0$

$\lambda = -9$

$\therefore Q(-7, 8, -6)$



Line PR : $\frac{x-2}{2}=\frac{y+1}{1}=\frac{z-3}{-2}=\mu$

R(2μ + 2, μ – 1, –2μ + 3)

Put in plane

2(2μ + 2) + μ – 1 – 2(–2μ + 3) – 6 = 0

4μ + 4 + μ – 1 + 4μ – 6 – 6 = 0

9μ – 9 = 0 ⇒ μ = 1

R(4, 0, 1)

P(2, –1, 3) & Q(–7, 8, –6)

$PQ=\sqrt{81+81+81}=9\sqrt{3}$

Q(–7, 8, –6) & R(4, 0, 1)

$QR=\sqrt{121+64+49}=\sqrt{234}$

Area (ΔPQR)

$=\frac{1}{2}\left|\overrightarrow{QP}\times\overrightarrow{QR}\right|$

$=\frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 9 & -9 & 9\\ 11 & -8 & 7\end{vmatrix}$

$=\frac{3}{2}\sqrt{234}$

$\overrightarrow{PQ}=-9\hat{i}+9\hat{j}-9\hat{k}=-9\left(\hat{i}-\hat{j}+\hat{k}\right)$

$\overrightarrow{PR}=2\hat{i}+\hat{j}-2\hat{k}$

$\cos\theta=\left|\frac{\overrightarrow{PQ}\cdot\overrightarrow{PR}}{PQ\cdot PR}\right|$

$=\frac{9}{9\sqrt{3}\times 3}=\frac{1}{3\sqrt{3}}$

$\theta=\cos^{-1}\left(\frac{1}{3\sqrt{3}}\right)$

6. Let $\mathbb{N}$ denote the set of all natural numbers, and $\mathbb{Z}$ denote the set of all integers. Consider the functions $f : \mathbb{N} \to \mathbb{Z}$ and $g : \mathbb{Z} \to \mathbb{N}$ defined by

$$f(n) = \begin{cases} (n+1)/2 & \text{if } n \text{ is odd,} \\ (4-n)/2 & \text{if } n \text{ is even,} \end{cases}$$

and

$$g(n) = \begin{cases} 3+2n & \text{if } n \geq 0, \\ -2n & \text{if } n < 0. \end{cases}$$

Define $(g \circ f)(n) = g(f(n))$ for all $n \in \mathbb{N}$, and $(f \circ g)(n) = f(g(n))$ for all $n \in \mathbb{Z}$.

Then which of the following statements is (are) **TRUE** ?

(A) $g \circ f$ is **NOT** one-one and $g \circ f$ is **NOT** onto

(B) $f \circ$ g is **NOT** one-one but $f \circ g$ is onto

(C) g is one-one and g is onto

(D) f is **NOT** one-one but f is onto

Ans. **(A,D)**

Sol. $$\mathrm{f(n)} = \begin{cases} (\mathrm{n}+1)/2 & \text{if n is odd} \\ (4-\mathrm{n})/2 & \text{if n is even} \end{cases}$$

f(n) = {(1, 1), (2, 1), (3, 2), (4, 0), (5, 3), (6, –1), …}

$\therefore$ f(n) is many one and onto function

$$\mathrm{g(n)} = \begin{cases} 3+2\mathrm{n} & \text{if n} \geq 0 \\ -2\mathrm{n} & \text{if n} < 0 \end{cases}$$

g(n) = {(–3, 6), (–2, 4), (–1, 2), (0, 3), (1, 5), (2, 7), (3, 9), (4, 15), …}

$\therefore$ g(n) is one-one and into function

$f(g(n)) = 2 + n$, $n \in N$

fog is one-one and into

$$\mathrm{g(f(n))} = \begin{cases} 4+\mathrm{n} & \text{if n is odd natural number} \\ 7-\mathrm{n} & \text{if n} = 2, 4 \\ \mathrm{n}-4 & \text{if n is even natural number and n } \geq 6 \end{cases}$$

g(f(2)) = g(f(1)) = 5

$\therefore$ gof is many one and into

7. Let $\mathbb{R}$ denote the set of all real numbers. Let $z_1 = 1 + 2i$ and $z_2 = 3i$ be two complex numbers, where $i = \sqrt{-1}$. Let

$$S = \{(x,y) \in \mathbb{R} \times \mathbb{R} : |x + iy - z_1| = 2|x + iy - z_2|\}.$$

Then which of the following statements is (are) **TRUE** ?

(A) S is a circle with centre $\left(-\frac{1}{3}, \frac{10}{3}\right)$

(B) S is a circle with centre $\left(\frac{1}{3}, \frac{8}{3}\right)$

(C) S is a circle with radius $\frac{\sqrt{2}}{3}$

(D) S is a circle with radius $\frac{2\sqrt{2}}{3}$

Ans. **(A,D)**

Sol. $|x + iy - 1 - 2i| = 2\,|x + iy - 3i|$

$\Rightarrow \quad (x-1)^2 + (y-2)^2 = 4\,(x^2 + (y-3)^2)$

$\Rightarrow \quad 3x^2 + 3y^2 + 2x - 20y + 31 = 0$

$\Rightarrow \quad x^2 + y^2 + \frac{2x}{3} - \frac{20y}{3} + \frac{31}{3} = 0$

$\therefore$ S is a circle with centre $\left(-\frac{1}{3}, \frac{10}{3}\right)$ and radius $= \sqrt{\frac{1}{9} + \frac{100}{9} - \frac{31}{3}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$

SECTION-3 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct numerical value of the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : +4 If **ONLY** the correct numerical value is entered in the designated place;
  Zero Marks : 0 In all other cases.

---

8. Let the set of all relations R on the set $\{a, b, c, d, e, f\}$, such that R is reflexive and symmetric, and R contains exactly 10 elements, be denoted by S.

Then the number of elements in S is __________.

Ans. (105)

Sol.

$$\begin{vmatrix} (a,a) & (a,b) & (a,c) & (a,d) & (a,e) & (a,f) \\ (b,a) & (b,b) & \cdots & \cdots & \cdots & (b,f) \\ \vdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ \vdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ (f,a) & (f,b) & \cdots & \cdots & \cdots & (f,f) \end{vmatrix}$$

For relation to be reflexive all the diagonal elements must be taken and out of remaining 30 elements there are 15 pairs and we need 2 pairs such that R contains exactly 10 elements and is both reflexive and symmetric.

$\therefore$ number of ways = ${}^{15}C_2 = 105$

9. For any two points M and N in the XY-plane, let $\overrightarrow{MN}$ denote the vector from M to N, and $\vec{0}$ denote the zero vector. Let P, Q and R be three distinct points in the XY-plane. Let S be a point inside the triangle ΔPQR such that

$$\overrightarrow{SP} + 5\overrightarrow{SQ} + 6\overrightarrow{SR} = \vec{0}.$$

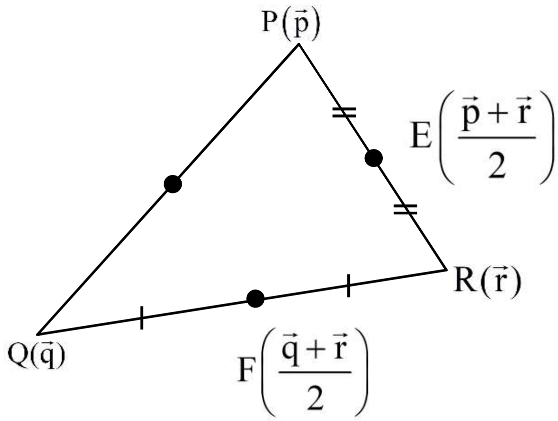
Let E and F be the mid-points of the sides PR and QR, respectively. Then the value of

$$\frac{\text{length of the line segment } EF}{\text{length of the line segment } ES}$$

is _______ .

Ans. (1.20)

Sol.



$\because\ \overrightarrow{SP}+5\overrightarrow{SQ}+6\overrightarrow{SP}=\vec{0}$

(Let P.V. of P be $\vec{p}$, Q be $\vec{q}$ and R be $\vec{r}$)

$\Rightarrow (\vec{p}-\vec{s})+5(\vec{q}-\vec{s})+6(\vec{r}-\vec{s})=\vec{0}$

$\Rightarrow \vec{S}=\dfrac{\vec{p}+5\vec{q}+6\vec{r}}{12}$

$\Rightarrow \overrightarrow{EF}=\left(\dfrac{\vec{q}-\vec{p}}{2}\right)$

$\Rightarrow \overrightarrow{ES}=\left(\dfrac{5\vec{q}-5\vec{p}}{12}\right)=\dfrac{5}{12}(\vec{q}-\vec{p})$

$\therefore\ \dfrac{|\overrightarrow{EF}|}{|\overrightarrow{ES}|}=\dfrac{6}{5}=1.2$

10. Let *S* be the set of all seven-digit numbers that can be formed using the digits 0, 1 and 2. For example, 2210222 is in *S*, but 0210222 is **NOT** in *S*.

Then the number of elements *x* in *S* such that at least one the digits 0 and 1 appears exactly twice in *x*, is equal to ______.

Ans. **(762)**

Sol. Let $A \rightarrow$ "0" appear exactly twice.

and $B \rightarrow$ "1" appear exactly twice.

$\therefore A \cap B \rightarrow$ "0" and "1" both appears exactly twice.

n(A) = $\_\_\ \underbrace{\_\_\ \_\_\ \_\_\ \_\_\ \_\_\ \_\_}$

$$= \underset{\text{placing zero}}{{}^6C_2 (1)}(2)^5 = \frac{6\times 5}{2}\times 2^5 = 480$$

for n(B)

C-I : 1 at first place

$$\underline{1}\quad \underbrace{\_\ \_\ \_\ \_\ \_\ \_}$$

Number of ways = $\underset{\text{placing 1}}{{}^6C_1 (1)}(2)^5 = 192$

C-II : 2 at first place

$$\underline{2}\quad \underbrace{\_\ \_\ \_\ \_\ \_\ \_}$$

Number of ways = $\underset{\text{placing1}}{{}^6C_2 (1)}(2)^4 = \frac{6\times 5}{2}\times 2^4 = 240$

n(B) = 240 + 192

for n($A \cap B$)

$$\_\quad \underbrace{\_\ \_\ \_\ \_\ \_\ \_}$$

for n($A \cap B$) = $\underset{\text{placing zero}}{{}^6C_2 (1)} \times \underset{\text{placing 1}}{{}^5C_2 (1)} \times \underset{\text{2 at rest places1}}{(1)} = \frac{6\times 5}{2}\times\frac{5\times 4}{2} = 150$

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$

= 480 + (192 + 240) – 150

= 762

11. Let α and β be the real numbers such that

$$\lim_{x\to 0}\frac{1}{x^3}\left(\frac{\alpha}{2}\int_0^x \frac{1}{1-t^2}dt + \beta x\cos x\right) = 2 .$$

Then the value of $\alpha + \beta$ is ______.

Ans. **(2.40)**

Sol. $$\lim_{x\to 0}\frac{\frac{\alpha}{2}\int_0^x \frac{1}{1-t^2}dt + \beta x\cos x}{x^3}$$

$$= \lim_{x\to 0} \frac{\frac{\alpha}{2}\left(\frac{1}{1-x^2}\right) + \beta\cos x - \beta x \sin x}{3x^2}$$

$$= \frac{\frac{\alpha}{2}\left(1-x^2\right)^{-1} + \beta\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!}....\right) - \beta x\left(x - \frac{x^3}{3!} + \frac{x^5}{5!}....\right)}{3x^2}$$

$$= \frac{\frac{\alpha}{2}\left(1 + x^2 + x^4...\right) + \beta\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!}....\right) - \beta\left(x^2 - \frac{x^4}{3!}...\right)}{3x^2}$$

$$= \frac{\left(\frac{\alpha}{2} + \beta\right) + x^2\left(\frac{\alpha}{2} - \frac{\beta}{2} - \beta\right) + x^4(\ )....}{3x^2} = 2(\text{Given})$$

$$\therefore \ \frac{\alpha}{2} + \beta = 0 \text{ and } \frac{\alpha - 3\beta}{6} = 2$$

$\Rightarrow \alpha = -2\beta$ and $\alpha = 12 + 3\beta$

$$\Rightarrow \beta = -\frac{12}{5} \text{ and } \alpha = \frac{24}{5}$$

$$\therefore \ \alpha + \beta = \frac{12}{5} = 2.40$$

12. Let $\mathbb{R}$ denote the set of all real numbers. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $f(x) > 0$ for all $x \in \mathbb{R}$, and $f(x + y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$.

Let the real numbers $a_1, a_2, \ldots, a_{50}$ be in an arithmetic progression. If $f(a_{31}) = 64 f(a_{25})$, and

$$\sum_{i=1}^{50} f(a_i) = 3\left(2^{25} + 1\right),$$

then the value of

$$\sum_{i=6}^{30} f(a_i)$$

is _________.

Ans. **(96)**

Sol. $\because f(x+y) = f(x).f(y)$

$\Rightarrow f(x) = k^x \qquad (f(x) > 0 \ \forall x \in R)$

$\because f(a_{31}) = 64 f(a_{25})$

$\Rightarrow k^{(a+30d)} = 64.k^{(a+24d)}$

$\Rightarrow k^{6d} = 64$

$\Rightarrow k^{d} = 2$

$$\sum_{i=1}^{50} f(a_i) = f(a_1) + f(a_2) + ... + f(a_{50})$$

$$= k^{a} + k^{a+d} + ... + k^{a+49d} = \frac{k^{a}\left(k^{50d}-1\right)}{k^{d}-1}$$

$$= k^{a}\left(2^{50}-1\right) = 3\left(2^{25}+1\right) \text{ (Given)}$$

$$\Rightarrow k^{a} = \frac{3}{2^{25}-1}$$

$$\therefore \sum_{i=6}^{30} f(a_i) = k^{a+5d} + k^{a+6d} + ... + k^{a+29d}$$

$$= k^{a+5d}\frac{\left(k^{25d}-1\right)}{k^{d}-1} = k^{a}.\left(k^{d}\right)^{5}\left(2^{25}-1\right)$$

$$= \frac{3}{2^{25}-1}.2^{5}\left(2^{25}-1\right) = 96$$

13. For all $x > 0$, let $y_1(x)$, $y_2(x)$, and $y_3(x)$ be the functions satisfying

$$\frac{dy_1}{dx} - (\sin x)^2 y_1 = 0, y_1(1) = 5,$$

$$\frac{dy_2}{dx} - (\cos x)^2 y_2 = 0, y_2(1) = \frac{1}{3},$$

$$\frac{dy_3}{dx} - \left(\frac{2-x^3}{x^3}\right) y_3 = 0, y_3(1) = \frac{3}{5e},$$

respectively. Then

$$\lim_{x \to 0^+} \frac{y_1(x) y_2(x) y_3(x) + 2x}{e^{3x} \sin x}$$

is equal to _______ .

Ans. **(2.00)**

Sol. $\frac{dy_1}{y_1}+\frac{dy_2}{y_2}+\frac{dy_3}{y_3}=\left(\sin^2 x+\cos^2 x+\frac{2-x^3}{x^3}\right)dx$

$\ell n(y_1y_2y_3)=\frac{-1}{x^2}+C$

$\ell n(y_1(x)y_2(x)y_3(x))=\frac{-1}{x^2}+C$

$\ell n\left(5.\frac{1}{3}.\frac{3}{5e}\right)=\frac{-1}{x^2}+C$

$\therefore$ C = 0

$y_1(x)y_2(x)y_3(x)=e^{\frac{-1}{x^2}}$

$\lim_{x\to 0}\frac{e^{\frac{-1}{x^2}}+2x}{e^{3x}\sin x}$

$\lim_{x\to 0}\frac{1}{e^{3x+\frac{1}{x^2}}\sin x}+\lim_{x\to 0}\frac{2x}{e^{3x}\sin x}$

$\lim_{x\to 0}\frac{1}{e^{3x}.\frac{\sin x}{x}.\frac{e^{\frac{1}{x^2}}}{\frac{1}{x}}}+2$

= 0 + 2 $\left\{\lim_{y\to\infty}\frac{e^{y^2}}{y}=\frac{e^{y^2}.2y}{1}=\infty\right\}$

= 2

SECTION-4 : (Maximum Marks : 12)

- This section contains **THREE (03)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists : **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated **according to the following marking scheme:**
  Full Marks : +4 **ONLY** if the option corresponding to the correct combination is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : –1 In all other cases.

14. Consider the following frequency distribution :

Value	4	5	8	9	6	12	11
Frequency	5	f_1	f_2	2	1	1	3

Suppose that the sum of the frequencies is 19 and the median of this frequency distribution is 6. For the given frequency distribution, let α denote the mean deviation about the mean, β denote the mean deviation about the median, and σ^2 denote the variance.

Match each entry in List-I to the correct entry in List-II and choose the correct option.

	List-I		List-II
(P)	$7f_1 + 9f_2$ is equal to	(1)	146
(Q)	19α is equal to	(2)	47
(R)	19β is equal to	(3)	48
(S)	$19\sigma^2$ is equal to	(4)	145
		(5)	55

(A) (P)→(5), (Q)→(3), (R)→(2), (S)→(4)

(B) (P)→(5), (Q)→(2), (R)→(3), (S)→(1)

(C) (P)→(5), (Q)→(3), (R)→(2), (S)→(1)

(D) (P)→(3), (Q)→(2), (R)→(5), (S)→(4)

Ans. **(C)**

Sol.

x_i	f_i	$\bar{x} = 7$ $d_i = \lvert x_i - \bar{x} \rvert$	$M = 6$ $e_i = \lvert x_i - M \rvert$	$\Sigma f_i d_i$	$\Sigma f_i e_i$	$\Sigma f_i d_i^2$
4	5	3	2	15	10	45
5	4	2	1	8	4	16
6	1	1	0	1	0	1
8	3	1	2	3	6	3
9	2	2	3	4	6	8
11	3	4	5	12	15	48
12	1	5	6	5	6	25
				48	47	146

$f_1 = 4$

$f_2 = 3$

$$\alpha = \frac{48}{19}, \beta = \frac{47}{19}, \sigma^2 = \frac{146}{19}$$

15. Let $\mathbb{R}$ denote the set of all real numbers. For a real number x, let $[x]$ denote the greatest integer less than or equal to x. Let n denote a natural number.

Match each entry in List-I to the correct entry in List-II and choose the correct option.

	List-I		List-II
(P)	The minimum value of n for which the function $f(x)=\left[\frac{10x^3-45x^2+60x+35}{n}\right]$ is continuous on the interval [1, 2], is	(1)	8
(Q)	The minimum value of n for which $g(x) = (2n^2 - 13n - 15)(x^3 + 3x)$, $x \in \mathbb{R}$, is an increasing function on $\mathbb{R}$, is	(2)	9
(R)	The smallest natural number n which is greater than 5, such that $x = 3$ is a point of local minima of $h(x) = (x^2 - 9)^n(x^2 + 2x + 3)$, is	(3)	5
(S)	Number of $x_0 \in \mathbb{R}$ such that $l(x)=\sum_{k=0}^{4}\left(\sin\lvert x-k\rvert+\cos\left\lvert x-k+\frac{1}{2}\right\rvert\right)$, $x \in \mathbb{R}$, is **NOT** differentiable at x_0, is	(4)	6
		(5)	10

(A) (P)→(1), (Q)→(3), (R)→(2), (S)→(5)

(B) (P)→(2), (Q)→(1), (R)→(4), (S)→(3)

(C) (P)→(5), (Q)→(1), (R)→(4), (S)→(3)

(D) (P)→(2), (Q)→(3), (R)→(1), (S)→(5)

Ans. **(B)**

Sol. $P(x) = 10x^3 - 45x^2 + 60x + 35$

$P'(x) = 30(x-1)(x-2)$

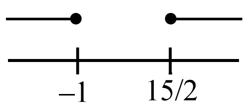
$P(x)$ decreases in $[1, 2]$

$\Rightarrow$ Range of $P(x) = [55, 60]$

$f(x) = \left[\frac{P(x)}{n}\right]$ min value of $n = 9$

(Q) For $g(x)$ to be increasing

$2n^2 - 13n - 15 \geq 0$



$n \in N$
$\Rightarrow n = 8$

(R) $h(x) = (x^2 - 9)^n (x^2 + 2x + 3)$

$h'(x) = (x^2 - 9)^n (2x + 2) + (x^2 + 2x + 3)\, n(x^2 - 9)^{n-1}.2x$

$= (x^2 - 9)^{n-1} [2(x^2 - 9)(x + 1) + 2nx(x^2 + 2x + 3)]$

$= (x + 3)^{n-1}(x - 3)^{n-1}.q(x)$

Derivative must change sign at $x = 3$

$\because \; n - 1 = \text{odd}$

$n = \text{even}$

$n = 6$

(S) $\cos\left|x - k + \frac{1}{2}\right|$ is differentiable everywhere

$\Rightarrow \sin|x - k|$ is **NOT** diff. at $k = 0,1,2,3,4$

16. Let $\vec{w} = \hat{i} + \hat{j} - 2\hat{k}$, and $\vec{u}$ and $\vec{v}$ be two vectors such that $\vec{u} \times \vec{v} = \vec{w}$ and $\vec{v} \times \vec{w} = \vec{u}$. Let α, β, γ and t be real numbers such that

$\vec{u} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$, $-t\alpha + \beta + \gamma = 0$, $\alpha - t\beta + \gamma = 0$, and $\alpha + \beta - t\gamma = 0$.

Match each entry in List-I to the correct entry in List-II and choose the correct option.

	List-I		**List-II**
(P)	$\|\vec{v}\|^2$ is equal to	(1)	0
(Q)	If $\alpha = \sqrt{3}$, then γ^2 is equal to	(2)	1
(R)	If $\alpha = \sqrt{3}$, then $(\beta + \gamma)^2$ is equal to	(3)	2
(S)	If $\alpha = \sqrt{2}$, then $t + 3$ is equal to	(4)	3
		(5)	5

(A) (P)→(2), (Q)→(1), (R)→(4), (S)→(5)

(B) (P)→(2), (Q)→(4), (R)→(3), (S)→(5)

(C) (P)→(2), (Q)→(1), (R)→(4), (S)→(3)

(D) (P)→(5), (Q)→(4), (R)→(1), (S)→(3)

Ans. **(A)**

Sol. $\vec{u} \times \vec{v} = \vec{w}$ $\quad \vec{u} \perp \vec{v} \quad \vec{u} \perp \vec{w}$

$\vec{v} \times \vec{w} = \vec{u}$ $\quad \vec{v} \perp \vec{w}$

$\vec{w} = \hat{i} + \hat{j} - 2\hat{k}$

$\vec{u} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$

$\vec{u}.\vec{w} = 0 = \alpha + \beta - 2\gamma$

$(\vec{v}.\vec{w}) \times \vec{v} = \vec{u} \times \vec{v}$

$\vec{w}|\vec{v}|^2 - 0 = \vec{w}$

$|\vec{v}| = 1(P)$

$$\begin{vmatrix} -t & 1 & 1 \\ 1 & -t & 1 \\ 1 & 1 & -t \end{vmatrix} = 0$$

$t = -1$ or 2

$t = -1$

$\alpha + \beta + \gamma = 0$

$t = 2$

$\alpha = \beta = \gamma$

Also $\vec{u}\times\vec{v}=\vec{w}$

$|\vec{u}||\vec{v}|=|\vec{w}|$

$\Rightarrow|\vec{u}|=|\vec{w}|=\sqrt{6}$

Case-1

$t = 2$

$\alpha = \beta = \gamma$

$\alpha^2 + \beta^2 + \gamma^2 = 6$

$\alpha=\sqrt{2},-\sqrt{2}$

(S) $t + 3 = 5$

Case-2

$t = -1$

$\alpha + \beta + \gamma = 0$

$\alpha + \beta - 2\gamma = 0$

$\gamma = 0, \alpha = -\beta$

(Q) $\alpha=\sqrt{3}$, $\gamma^2 = 0$, $\beta=-\sqrt{3}$

(R) $\alpha=\sqrt{3}$ $(\gamma + \beta)^2 = \beta^2 = 3$

(P)$\rightarrow$(2), (Q)$\rightarrow$(1), (R)$\rightarrow$(4), (S)$\rightarrow$(5)