FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Monday 25th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

1. In AM modulation, a signal is modulated on a carrier wave such that maximum and minimum amplitude are found to be 6V and 2V respectively. The modulation index is

(A) 100% (B) 80%

(C) 60% (D) 50%

Official Ans. by NTA (D)

Sol. modulation index = $\frac{V_{max} - V_{min.}}{V_{max} + V_{min.}} \times 100\%$

$= \frac{6-2}{6+2} \times 100\% = 50\%$

2. The electric current in a circular coil of 2 turns produces a magnetic induction B_1 at its centre. The coil is unwound and is rewound into a circular coil of 5 turns and the same current produces a magnetic induction B_2 at its centre.

The ratio of $\frac{B_2}{B_1}$ is :

(A) $\frac{5}{2}$ (B) $\frac{25}{4}$

(C) $\frac{5}{4}$ (D) $\frac{25}{2}$

Official Ans. by NTA (B)

Sol. $B = \frac{N\mu_0 i}{2R}$

$B_1 = \frac{N_1\mu_0 i}{2R_1}$

For $N_2 = 5$

Radius of coil = $R_2 = \frac{N_1 \times R_1}{N_2}$

$B_2 = \frac{N_2\mu_0 i}{R_2}$

$\frac{B_2}{B_1} = \frac{N_2}{N_1} \frac{R_1}{R_2} = \frac{N_2}{N_1} \times \frac{N_2}{N_1}$; $\frac{B_2}{B_1} = \frac{25}{4}$

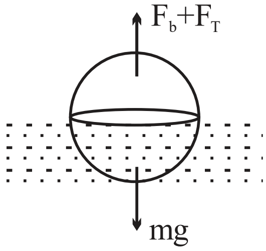
3. A drop of liquid of density ρ is floating half immersed in a liquid of density σ and surface tension 7.5×10^{-4} Ncm^{-1}. The radius of drop in cm will be : (Take : g = 10 m/s^2)

(A) $\frac{15}{\sqrt{2\rho-\sigma}}$ (B) $\frac{15}{\sqrt{\rho-\sigma}}$

(C) $\frac{3}{2\sqrt{\rho-\sigma}}$ (D) $\frac{3}{20\sqrt{2\rho-\sigma}}$

Official Ans. by NTA (A)

Sol.



Boyant force + surace tension = mg

$\sigma\frac{V}{2}g + 2\pi RT = \rho Vg$

$2\pi RT = \frac{(2\rho-\sigma)}{2}\frac{4}{3}\pi R^3 g;\ \left[V = \frac{4}{3}\pi R^3\right]$

$R^3 = \frac{3T}{(2\rho-\sigma)g} \Rightarrow R = \sqrt{\frac{3\times 7.5\times 10^{-2}\,N-m^{-1}}{(2\rho-\sigma)\times 10}}$

$R = \frac{3}{20\sqrt{(2\rho-\sigma)}}$ m = $\frac{15}{\sqrt{2\rho-\sigma}}$ cm

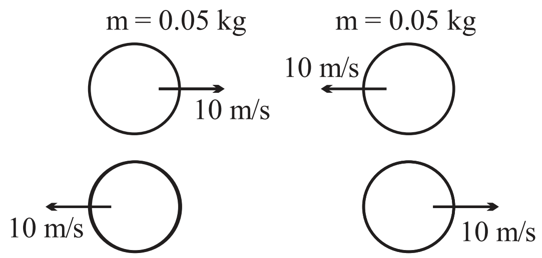
4. Two billiard balls of mass 0.05 kg each moving in opposite directions with 10 ms^{-1} collide and rebound with the same speed. If the time duration of contact is $t = 0.005$ s, then what is the force exerted on the ball due to each other?

(A) 100 N (B) 200 N

(C) 300 N (D) 400 N

Official Ans. by NTA (B)

Sol.



Change in momentum of any one ball

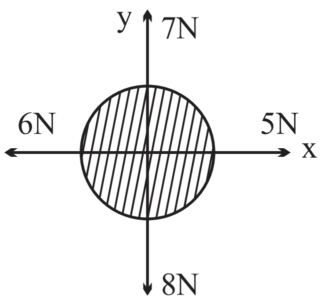
$$|\Delta\vec{P}| = 2\times0.05\times10$$

$$|\Delta\vec{P}| = 1$$

$$|\vec{F}_{av}| = \frac{|\Delta\vec{P}|}{\Delta t}$$

$F_{av.} = 200$ N

5. For a free body diagram shown in the figure, the four forces are applied in the 'x' and 'y' directions. What additional force must be applied and at what angle with positive x-axis so that the net acceleration of body is zero?



(A) $\sqrt{2}$ N, 45° (B) $\sqrt{2}$ N, 135°

(C) $\frac{2}{\sqrt{3}}$ N, 30° (D) 2 N, 45°

Official Ans. by NTA (A)

Sol. Let addition force required is = $\vec{F}$

$$\vec{F} + 5\hat{i} - 6\hat{i} + 7\hat{j} - 8\hat{j} = 0$$

$$\vec{F} = \hat{i} + \hat{j}, |\vec{F}| = \sqrt{2}$$

Angle with x-axis: $\tan\theta = \frac{\text{y component}}{\text{x component}} = \frac{1}{1}$

$\theta = 45°$

6. Capacitance of an isolated conducting sphere of radius R_1 becomes n times when it is enclosed by a concentric conducting sphere of radius R_2 connected to earth. The ratio of their radii $\left(\frac{R_2}{R_1}\right)$ is:

(A) $\frac{n}{n-1}$ (B) $\frac{2n}{2n+1}$

(C) $\frac{n+1}{n}$ (D) $\frac{2n+1}{n}$

Official Ans. by NTA (A)

Sol. Capacitance of isolated Conducting sphere

$= 4\pi\varepsilon_0 R_1$

By enclosing inside another sphere of radius

R_2, new capacitance = $\frac{4\pi\varepsilon_0 R_1 R_2}{(R_2 - R_1)}$

Given: $\frac{4\pi\varepsilon_0 R_1 R_2}{(R_2-R_1)} = n\times4\pi\varepsilon_0 R_1$

$$\Rightarrow \frac{R_2}{(R_2-R_1)} = n \Rightarrow \frac{\frac{R_2}{R_1}}{\left(\frac{R_2}{R_1}-1\right)} = n$$

$$\Rightarrow \frac{R_2}{R_1} = n\frac{R_2}{R_1} - n \Rightarrow \frac{R_2}{R_1} = \frac{n}{(n-1)}$$

7. The ratio of wavelengths of proton and deuteron accelerated by potential V_p and V_d is $1:\sqrt{2}$. Then, the ratio of V_p to V_d will be

(A) 1 : 1 (B) $\sqrt{2}:1$

(C) 2 : 1 (D) 4 : 1

Official Ans. by NTA (D)

Sol. Kinetic energy gained by a charged particle accelerated by a potential V is qV

KE = qV

$$\Rightarrow \frac{p^2}{2m} = qV \Rightarrow p = \sqrt{2mqV}$$

$$p = \frac{h}{\lambda}, \text{ thus } \lambda = \frac{h}{\sqrt{2mqV}}$$

$$\text{now } \frac{\lambda_p}{\lambda_d} = \sqrt{\frac{m_d V_d}{m_p V_p}}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \sqrt{\frac{2V_d}{V_p}} \Rightarrow \frac{V_p}{V_d} = 4$$

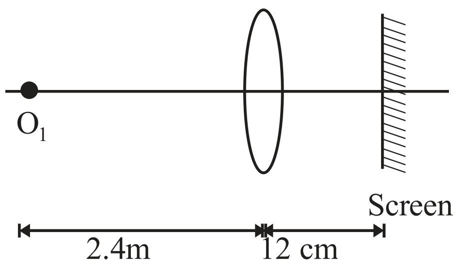
8. For an object placed at a distance 2.4 m from a lens, a sharp focused image is observed on a screen placed at a distance 12 cm from the lens. A glass plate of refractive index 1.5 and thickness 1 cm is introduced between lens and screen such that the glass plate plane faces parallel to the screen. By what distance should the object be shifted so that a sharp focused image is observed again on the screen?

(A) 0.8 m (B) 3.2 m

(C) 1.2 m (D) 5.6 m

Official Ans. by NTA (B)

Sol.



Applying lens formula

$$\frac{1}{0.12} + \frac{1}{2.4} = \frac{1}{f} \Rightarrow \frac{1}{f} = \frac{210}{24}$$

Upon putting the glass slab, shift of image is

$$\Delta x = t\left(1 - \frac{1}{\mu}\right) = \frac{1}{3}\text{cm}$$

$$\text{Now } v = 12 - \frac{1}{3} = \frac{35}{3}\text{cm}$$

Again apply lens formula

$$\frac{1}{0.12} + \frac{1}{u} = \frac{1}{f} = \frac{210}{24}$$

Solving u = – 5.6 m

Thus shift of object is

5.6 – 2.4 = 3.2 m

9. Light wave traveling in air along x-direction is given by $E_y = 540 \sin \pi \times 10^4(x - ct)\ Vm^{-1}$. Then, the peak value of magnetic field of wave will be (Given $c = 3 \times 10^8\ ms^{-1}$)

(A) 18×10^{-7} T (B) 54×10^{-7} T

(C) 54×10^{-8} T (D) 18×10^{-8} T

Official Ans. by NTA (A)

Sol. $E_y = 540 \sin \pi \times 10^4(x - ct)\ Vm^{-1}$

$E_0 = 540\ Vm^{-1}$

$$B_0 = \frac{E_0}{C} = \frac{540}{3\times10^8} = 18\times10^{-7}\,T$$

10. When you walk through a metal detector carrying a metal object in your pocket, it raises an alarm. This phenomenon works on

(A) Electromagnetic induction

(B) Resonance in ac circuits

(C) Mutual induction in ac circuits

(D) interference of electromagnetic waves

Official Ans. by NTA (B)

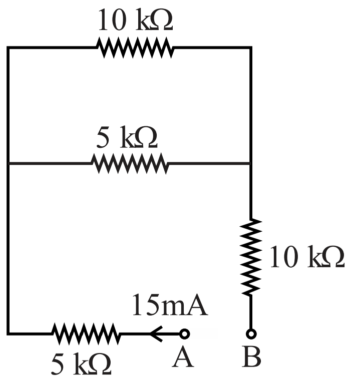
Sol. Metal detector works on the principle of transmitting an electromagnetic signal and analyses a return signal from the target. So it works on the principle of resonance in AC circuit.

11. An electron with energy 0.1 keV moves at right angle to the earth's magnetic field of 1×10^{-4} Wbm^{-2}. The frequency of revolution of the electron will be
(Take mass of electron = 9.0×10^{-31} kg)
(A) 1.6×10^5 Hz (B) 5.6×10^5 Hz
(C) 2.8×10^6 Hz (D) 1.8×10^6 Hz
Official Ans. by NTA (C)

Sol. $f = \frac{1}{T} = \frac{eB}{2\pi m}$

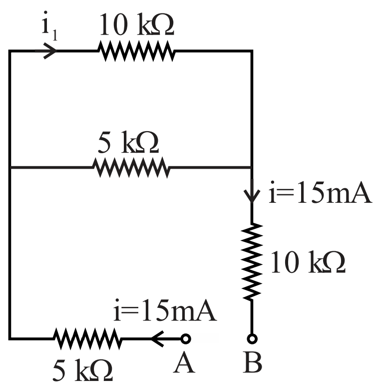
$$= \frac{1.6\times10^{-19}\times10^{-4}}{2\pi\times9\times10^{-31}} = 2.8\times10^6 \text{Hz}$$

12. A current of 15 mA flows in the circuit as shown in figure. The value of potential difference between the points A and B will be



(A) 50V (B) 75V
(C) 150V (D) 275V
Official Ans. by NTA (D)

Sol.



$i_1 = \frac{5}{10+5}\times15\text{mA} = 5\text{mA}$

$V_A - 5i - 10i_1 - 10i = V_B$

$V_A - V_B = 75 + 50 + 150 = 275$ V

13. The length of a seconds pendulum at a height h = 2R from earth surface will be:
(Given: R=Radius of earth and acceleration due to gravity at the surface of earth $g = \pi^2 \text{m/s}^{-2}$)

(A) $\frac{2}{9}$m (B) $\frac{4}{9}$m

(C) $\frac{8}{9}$m (D) $\frac{1}{9}$m

Official Ans. by NTA (D)

Sol. $T = 2\pi\sqrt{\frac{L}{g}}$, $g' = \frac{GM}{9R^2} = \frac{g}{9} = \frac{\pi^2}{9}$

$$2 = 2\pi\sqrt{\frac{L}{\pi^2}\times9}$$

$$\Rightarrow 1 = \pi\sqrt{L}\times\frac{3}{\pi} \Rightarrow L = \frac{1}{9}\text{m}$$

14. Sound travels in a mixture of two moles of helium and n moles of hydrogen. If rms speed of gas molecules in the mixture is $\sqrt{2}$ times the speed of sound, then the value of n will be
(A) 1 (B) 2
(C) 3 (D) 4
Official Ans. by NTA (B)

Sol. $v_s = \sqrt{\frac{\gamma RT}{M}}$

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\frac{v_s}{v_{rms}} = \sqrt{\frac{\gamma}{3}} = \frac{1}{\sqrt{2}} \Rightarrow \frac{\gamma}{3} = \frac{1}{2} \Rightarrow \gamma = \frac{3}{2}$$

$$\gamma = 1 + \frac{2}{f_{mix.}}$$

$$f_{mix.} = \frac{2\times3+n\times5}{n+2} = \frac{6+n\times5}{(n+2)}$$

$$\gamma = 1 + \frac{2(n+2)}{6+n\times5} = \frac{6+5n+2n+4}{6+5n}$$

$$\gamma = \frac{7n+10}{6+5n} = \frac{3}{2}$$

$14n + 20 = 18 + 15n$

$n = 2$

15. Let η_1 is the efficiency of an engine at T_1 = 447°C and T_2 = 147°C while η_2 is the efficiency at T_1 = 947°C and T_2 = 47°C. The ratio $\frac{\eta_1}{\eta_2}$ will be :

(A) 0.41 (B) 0.56

(C) 0.73 (D) 0.70

Official Ans. by NTA (B)

Sol. Efficiency $\eta = 1-\frac{T_L}{T_H}$

$$\eta_1 = 1-\frac{147+273}{447+273} = 1-\frac{420}{720}$$

$$\eta_1 = \frac{300}{720}$$

$$\eta_2 = 1-\frac{47+273}{947+273} = 1-\frac{320}{1220}$$

$$\eta_2 = \frac{900}{1220}$$

$$\frac{\eta_1}{\eta_2} = \frac{300}{720}\times\frac{1220}{900} = \frac{122}{72\times3}$$

$$\frac{\eta_1}{\eta_2} = 0.56$$

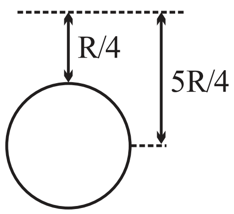
16. An object is taken to a height above the surface of earth at a distance $\frac{5}{4}R$ from the centre of the earth. Where radius of earth, R = 6400 km. The percentage decrease in the weight of the object will be

(A) 36% (B) 50%

(C) 64% (D) 25%

Official Ans. by NTA (A)

Sol.



$$g_{eff} = \frac{g}{\left(1+\frac{h}{R}\right)^2};\ g_{eff} = \frac{g}{\left(1+\frac{1}{4}\right)^2} = \frac{16g}{25}$$

$$\text{change} = \frac{g_{eff}-g}{g}\times100 = \frac{\frac{16}{25}-1}{1}\times100$$

$$= \frac{-9}{25}\times100 = -36\%$$

Hence % decrease in the weight = 36%

17. A bag of sand of mass 9.8 kg is suspended by a rope. A bullet of 200 g travelling with speed 10 ms^{-1} gets embedded in it, then loss of kinetic energy will be

(A) 4.9 J (B) 9.8 J

(C) 14.7 (D) 19.6 J

Official Ans. by NTA (B)

Sol. $P_i = P_f$ (no any external force)

0.2 × 10 = 10 × v

v = 0.2 m/sec

$$\text{Loss in K.E.} = \frac{1}{2}\times(0.2)\times10^2 - \frac{1}{2}\times10(0.2)^2$$

$$= \frac{1}{2}\times10\times(0.2)[10-0.2]$$

= 9.8 J

18. A ball is projected from the ground with a speed 15 ms^{-1} at an angle θ with horizontal so that its range and maximum height are equal, then'tan θ' will be equal to

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$

(C) 2 (D) 4

Official Ans. by NTA (D)

Sol. R = H

$$\frac{2v_x\times v_y}{g} = \frac{v_y^2}{2g}$$

$$v_x = \frac{v_y}{4};\ u\cos\theta = \frac{u\sin\theta}{4}$$

tan θ = 4

19. The maximum error in the measurement of resistance, current and time for which current flows in an electrical circuit are 1%, 2% and 3% respectively. The maximum percentage error in the detection of the dissipated heat will be:

(A) 2 (B) 4

(C) 6 (D) 8

Official Ans. by NTA (D)

Sol. $E_H = I^2R \times t$

$$\frac{\Delta E}{E}\times 100 = \frac{2\Delta I}{I}\times 100 + \frac{\Delta R}{R}\times 100 + \frac{\Delta T}{T}\times 100$$

$= 2 \times 2 + 1 + 3 = 8$

20. Hydrogen atom from excited state comes to the ground by emitting a photon of wavelength λ. The value of principal quantum number 'n' of the excited state will be :

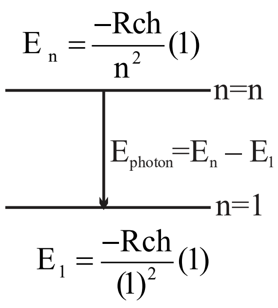
(R : Rydberg constant)

(A) $\sqrt{\frac{\lambda R}{\lambda - 1}}$ (B) $\sqrt{\frac{\lambda R}{\lambda R - 1}}$

(C) $\sqrt{\frac{\lambda}{\lambda R - 1}}$ (D) $\sqrt{\frac{\lambda R^2}{\lambda R - 1}}$

Official Ans. by NTA (B)

Sol.



$$\frac{-Rch}{(n)^2} + \frac{Rch}{1} = \frac{hc}{\lambda}$$

$$\frac{-R}{n^2} + R = \frac{1}{\lambda}$$

$$R - \frac{1}{\lambda} = \frac{R}{n^2}$$

$$\frac{\lambda R - 1}{\lambda} = \frac{R}{n^2}$$

$$n^2 = \frac{\lambda R}{\lambda R - 1} \Rightarrow n = \sqrt{\frac{\lambda R}{\lambda R - 1}}$$

SECTION-B

1. A particle is moving in a straight line such that its velocity is increasing at 5 ms^{-1} per meter. The acceleration of the particle is _____ ms^{-2} at a point where its velocity is 20 ms^{-1}.

Official Ans. by NTA (100)

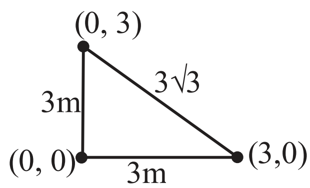
Sol. $\frac{dv}{ds} = 5$

$a = v\frac{dv}{ds} = 20 \times 5 = 100$ m/sec^2

2. Three identical spheres each of mass M are placed at the corners of a right angled triangle with mutually perpendicular sides equal to 3 m each. Taking point of intersection of mutually perpendicular sides as origin, the magnitude of position vector of centre of mass of the system will be $\sqrt{x}$ m. The value of x is

Official Ans. by NTA (2)

Sol.



$$\vec{r}_{com} = \frac{M(0\hat{i} + 0\hat{j}) + M(3\hat{i}) + M(3\hat{j})}{3M}$$

$$\vec{r}_{com} = \hat{i} + \hat{j}$$

$$|\vec{r}_{com}| = \sqrt{2} = \sqrt{x}$$

$x = 2$

3. A block of ice of mass 120 g at temperature 0°C is put in 300 gm of water at 25°C. The xg of ice melts as the temperature of the water reaches 0°C. The value of x is

[Use: Specific heat capacity of water = 4200 $Jkg^{-1}K^{-1}$, Latent heat of ice = 3.5×10^5 Jkg^{-1}]

Official Ans. by NTA (90)

Sol. Energy released by water

$= 0.3 \times 25 \times 4200 = 31500$ J

let m kg ice melts

$m \times 3.5 \times 10^5 = 31500$

$$m = \frac{31500 \times 10^{-5}}{3.5} = 9000 \times 10^{-5}$$

m = 0.09 kg = 90 gm

x = 90

4. $\frac{x}{x+4}$ is the ratio of energies of photons produced due to transition of an electron of hydrogen atom from its

(i) third permitted energy level to the second level and

(ii) the highest permitted energy level to the second permitted level.

The value of x will be

Official Ans. by NTA (5)

Sol. $$\frac{13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right)}{13.6\left(\frac{1}{2^2} - 0\right)} = \frac{x}{x+4}; \quad \frac{\frac{1}{4} - \frac{1}{9}}{\frac{1}{4}} = \frac{x}{x+4}$$

$$\frac{5}{9} = \frac{x}{x+4}$$

5x + 20 = 9x

4x = 20

x = 5

5. In a potentiometer arrangement, a cell of emf 1.20 V gives a balance point at 36 cm length of wire. This cell is now replaced by another cell of emf 1.80 V. The difference in balancing length of potentiometer wire in above conditions will be _________ cm.

Official Ans. by NTA (18)

Sol. 1.2 = (Potential Gradient) × 36

1.8 = (Potential Gradient) × x

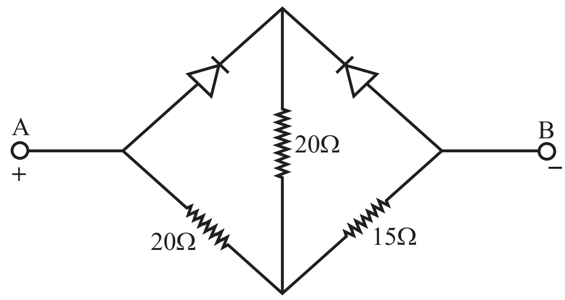
On dividing, we get

$$\frac{2}{3} = \frac{36}{x}$$

x = 18 × 3 = 54 cm

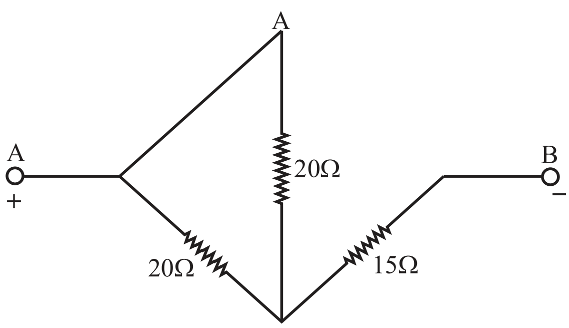
Hence difference = 54 – 36 = 18 cm

6. Two ideal diodes are connected in the network as shown in figure. The equivalent resistance between A and B is _______Ω.

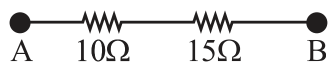


Official Ans. by NTA (25)

Sol.



The forward biased diode will conduct while the reverse biased will not



∴ Equivalent resistance = 10 + 15 = 25Ω

7. Two waves executing simple harmonic motion travelling in the same direction with same amplitude and frequency are superimposed. The resultant amplitude is equal to the $\sqrt{3}$ times of amplitude of individual motions. The phase difference between the two motions is _____ (degree)

Official Ans. by NTA (60)

Sol. $A_{resultant} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$\Rightarrow \sqrt{3}A = \sqrt{A^2 + A^2 + 2A^2\cos\phi}$

$\Rightarrow 3A^2 = 2A^2 + 2A^2\cos\phi$

$\Rightarrow \cos\phi = \frac{1}{2}$

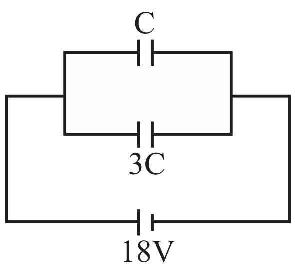
$\therefore \phi = 60°$

$\therefore$ Phase difference = 60 degree

8. Two parallel plate capacitors of capacity C and 3C are connected in parallel combination and charged to a potential difference 18V. The battery is then disconnected and the space between the plates of the capacitor of capacity C is completely filled with a material of dielectric constant 9. The final potential difference across the combination of capacitors will be _______ V

Official Ans. by NTA (6)

Sol.



Initial charge on C = 18 CV

initial charge on 3C = 54 CV

Let final common potential difference = V'

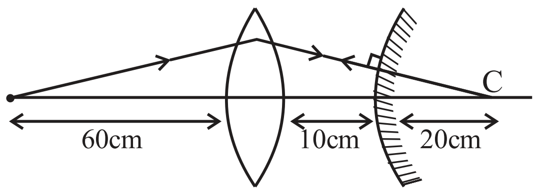
9CV' + 3CV' = 18CV + 54CV

$\Rightarrow$ 12CV' = 72 CV $\Rightarrow$ V' = 6 V

9. A convex lens of focal length 20 cm is placed in front of convex mirror with principal axis coinciding each other. The distance between the lens and mirror is 10 cm. A point object is placed on principal axis at a distance of 60 cm from the convex lens. The image formed by combination coincides the object itself. The focal length of the convex mirror is _____ cm.

Official Ans. by NTA (10)

Sol.



For lens

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \frac{1}{v} - \frac{1}{(-60)} = \frac{1}{20} \Rightarrow \frac{1}{v} + \frac{1}{60} = \frac{1}{20}$

v = 30 cm

For final image to be formed on the object itself, after refraction from lens the ray should meet the mirror perpendicularly and the image by lens should be on the centre of curvature of mirror

R = 30 – 10 = 20 cm

Focal length of mirror = R/2 = 10 cm

10. Magnetic flux (in weber) in a closed circuit of resistance 20 Ω varies with time t(s) as $\phi = 8t^2 - 9t + 5$. The magnitude of the induced current at t = 0.25 s will be _______ mA

Official Ans. by NTA (250)

Sol. $\phi = 8t^2 - 9t + 5$

$\text{emf} = -\frac{d\phi}{dt} = -(16t - 9)$

At t = 0.25 s

Emf = –[(16 × 0.25)–9] = 5V

$\text{Current} = \frac{\text{Emf}}{\text{Resistance}} = \frac{5V}{20\Omega}$

$= \frac{1}{4}A = \frac{1000}{4}mA = 250mA$

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(Held On Monday 25th July, 2022) **TIME : 3 : 00 PM to 06 : 00 PM**

CHEMISTRY

TEST PAPER WITH SOLUTION

SECTION-A

1. Match **List I** with **List II** :

List-I (molecule)	List-II (hybridization; shape)
A. XeO_3	I. sp^3d ; linear
B. XeF_2	II. sp^3; pyramidal
C. $XeOF_4$	III. sp^3d^3; distorted octahedral
D. XeF_6	IV. sp^3d^2 ;square pyramidal

Choose the correct answer from the options given below:

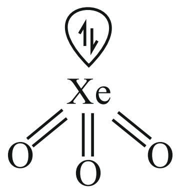
(A) A-II, B-I, C-IV, D-III

(B) A-II, B-IV, C-III, D-I

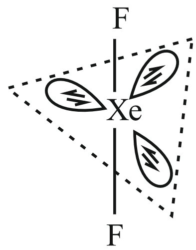
(C) A-IV, B-II, C-III, D-I

(D) A-IV, B-II, C-I, D-III

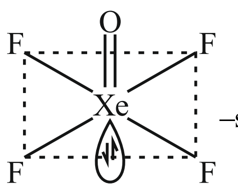
Official Ans. by NTA (A)

Sol. (A) 

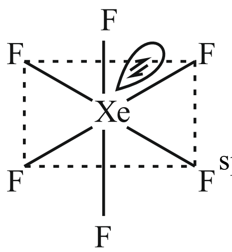
– sp^3 – pyramidal

(B) 

– sp^3d – linear

(C) 

–sp^3d^2–square pyramidal

(D) 

sp^3d^3–distorted octahedral

2. Two solutions A and B are prepared by dissolving 1 g of non-volatile solutes X and Y. respectively in 1 kg of water. The ratio of depression in freezing points for A and B is found to be 1 : 4. The ratio of molar masses of X and Y is :

(A) 1 : 4

(B) 1 : 0.25

(C) 1 : 0.20

(D) 1 : 5

Official Ans. by NTA (B)

Sol. $$\frac{\Delta T_{fx}}{\Delta T_{fy}} = \frac{k_f \cdot m_x}{k_f \cdot m_y} = \frac{\dfrac{1/M_x}{1}}{\dfrac{1/M_y}{1}}$$

$$\Rightarrow \frac{1}{4} = \frac{M_y}{M_x}$$

$$\Rightarrow M_x : M_y = 1 : 0.25$$

3. Ka_1, Ka_2 and Ka_3 are the respective ionization constants for the following reactions (a),(b), and (c).

(a) $H_2C_2O_4 \rightleftharpoons H^+ + HC_2O_4^-$

(b) $HC_2O_4^- \rightleftharpoons H^+ + HC_2O_4^{2-}$

(c) $H_2C_2O_4 \rightleftharpoons 2H^+ + C_2O_4^{2-}$

The relationship between K_{a_1}, K_{a_2} and K_{a_3} is given as

(A) $K_{a_3} = K_{a_1} + K_{a_2}$ (B) $K_{a_3} = K_{a_1} - K_{a_2}$

(C) $K_{a_3} = K_{a_1} / K_{a_2}$ (D) $K_{a_3} = K_{a_1} \times K_{a_2}$

Official Ans. by NTA (D)

Sol. $H_2C_2O_4 \rightleftharpoons H^+ + HC_2O_4^-$ K_{a_1}

$H_2C_2O_4^- \rightleftharpoons H^+ + C_2O_4^{2-}$ K_{a_2}

$H_2C_2O_4 \rightleftharpoons 2H^+ + C_2O_4^{2-}$ $K_{a_3} = K_{a_1} \times K_{a_2}$

4. The molar conductivity of a conductivity cell filled with 10 moles of 20 mL NaCl solution is Λ_{m1} and that of 20 moles another identical cell heaving 80 mL NaCl solution is Λ_{m2}, The conductivities exhibited by these two cells are same.

The relationship between Λ_{m2} and Λ_{m1} is

(A) $\Lambda_{m2} = 2\Lambda_{m1}$ (B) $\Lambda_{m2} = \Lambda_{m1}/2$

(C) $\Lambda_{m2} = \Lambda_{m1}$ (D) $\Lambda_{m2} = 4\Lambda_{m1}$

Official Ans. by NTA (A)

Sol. $\Lambda_m = \kappa \times \frac{1000}{M}$

$\Rightarrow \Lambda_m \propto \frac{1}{M}$

$$\frac{\Lambda_{m_1}}{\Lambda_{m_2}} = \frac{M_2}{M_1} = \frac{\frac{20}{80}}{\frac{10}{20}} = \frac{1}{4} \times \frac{2}{1} = \frac{1}{2}$$

$\Rightarrow \Lambda_{m_2} = 2\Lambda_{m_1}$

5. For micelle formation, which of the following statements are correct?

(A) Micelle formation is an exothermic process.

(B) Micelle formation is an endothermic process.

(C) The entropy change is positive.

(D) The entropy change is negative.

(A) A and D only (B) A and C only

(C) B and C only (D) B and D only

Official Ans. by NTA (A)

Sol. For micelle formation, $\Delta S > 0$ (hydrophobic effect) This is possible because, the decrease in entropy due to clustering is offset by increase in entropy due to desolvation of the surfactant, Also $\Delta H > 0$

6. The first ionization enthalpies of Be, B, N and O follow the order

(A) O < N < B < Be (B) Be < B < N < O

(C) B < Be < N < O (D) B < Be < O < N

Official Ans. by NTA (D)

Sol. 1^{st} **I.E.** $\underset{(2p^3)}{N} > \underset{(2p^4)}{O} > \underset{(2s^2)}{Be} > \underset{(2p^1)}{B}$

7. Given below are two statements.

Statement I : Pig iron is obtained by heating cast iron with scrap iron.

Statement II: Pig iron has a relatively lower carbon content than that of cast iron. In the light of the above statements, choose the correct answer from the options given below.

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are not correct.

(C) Statement I is correct but Statement II is not correct

(D) Statement I is not correct but Statement II is correct.

Official Ans. by NTA (B)

Sol. Statement –I is incorrect because cast iron is obtained by heating pig iron with scrap iron Statement–II is also incorrect because pig iron has more carbon content (~4%) than cast iron (~3%)

8. High purity (>99.95%) dihydrogen is obtained by

(A) reaction of zinc with aqueous alkali.

(B) electrolysis of acidified water using platinum electrodes.

(C) electrolysis of warm aqueous barium hydroxide solution between nickel electrodes.

(D) reaction of zinc with dilute acid.

Official Ans. by NTA (C)

Sol. High purity (>99.95%) dihydrogen is obtained by electrolysis of warm aqueous $Ba(OH)_2$ solution between Ni-electrodes

9. The correct order of density is

(A) Be > Mg > Ca > Sr

(B) Sr > Ca > Mg > Be

(C) Sr > Be > Mg > Ca

(D) Be > Sr > Mg > Ca

Official Ans. by NTA (C)

Sol. In II'A' group density decreases down the group till Ca and after that it increases.

Correct order of density is

Sr > Be > Mg > Ca

10. The total number of acidic oxides from the following list is : NO, N_2O, B_2O_3, N_2O_5, CO, SO_3, P_4O_{10}

(A) 3 (B) 4

(C) 5 (D) 6

Official Ans. by NTA (B)

Sol. Neutral Oxides — N_2O, NO, CO

Acidic Oxides — B_2O_3, N_2O_5, SO_3, P_4O_{10}

11. The correct order of energy of absorption for the following metal complexes is

A: $[Ni(en)_3]^{2+}$, B: $[Ni(NH_3)_6]^{2+}$, C: $[Ni(H_2O)_6]^{2+}$

(A) C < B < A

(B) B < C < A

(C) C < A < B

(D) A < C < B

Official Ans. by NTA (A)

Sol. Stronger the ligand, larger the splitting & higher the energy of absorption.

$$\underset{(A)}{\left[Ni(en)_3\right]^{+2}} > \underset{(B)}{\left[Ni(NH_3)_6\right]^{+2}} > \underset{(C)}{\left[Ni(H_2O)_6\right]^{+2}}$$

12. Match List I with List II.

List–I		List–II	
A.	Sulphate	I.	Pesticide
B.	Fluoride	II.	Bending of bones
C.	Nicotine	III.	Laxative effect
D.	Sodium arsinite	IV.	Herbicide

Choose the correct answer from the options given below:

(A) A-II, B-III. C-IV, D-I

(B) A-IV, B-III, C-II, D-I

(C) A-III, B-II, C-I, D-IV

(D) A-III, B-II, C-IV, D-I

Official Ans. by NTA (C)

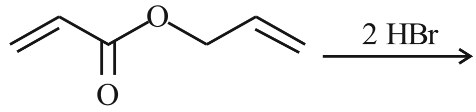
Sol. A-Sulphate – III (Laxative effect)

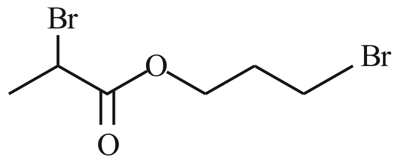
B-Fluoride – II (Bending of bones)

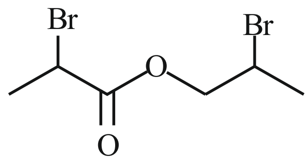
C-Nictoine – I (pesticides)

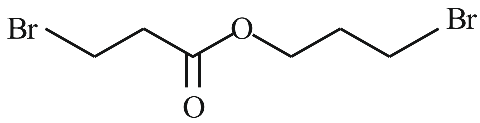
D-Sodium Arsinite – IV (herbicide)

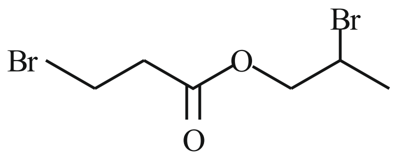
13. Major product of the following reaction is



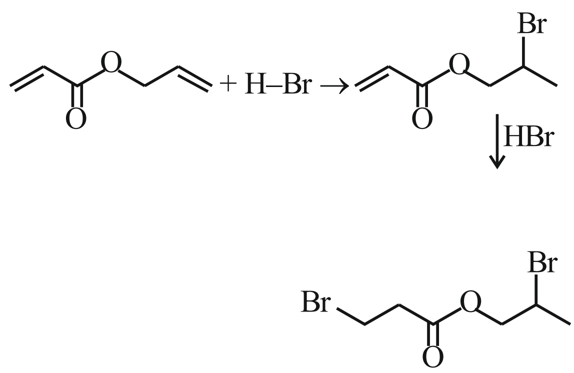
(A)


(B)


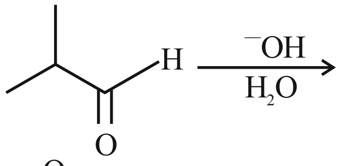
(C)


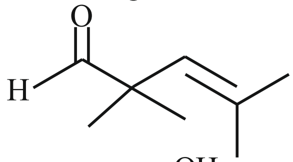
(D)


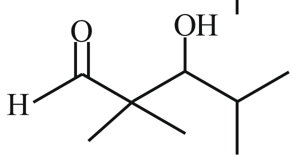
Official Ans. by NTA (D)

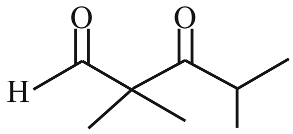
Sol.


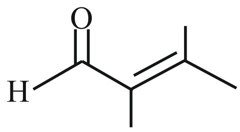
14. What is the major product of the following reaction?



(A) 

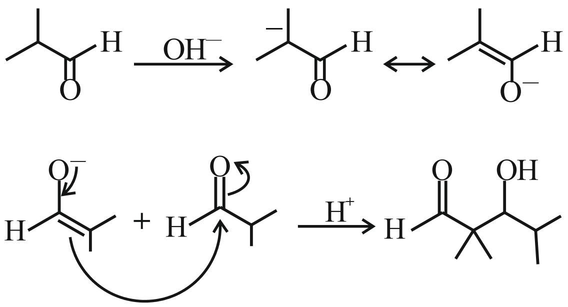
(B) 

(C) 

(D) 

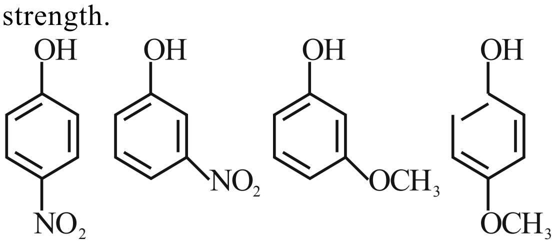
Official Ans. by NTA (B)

Sol.



Aldol formation takes place.

15. Arrange the following in decreasing acidic strength.



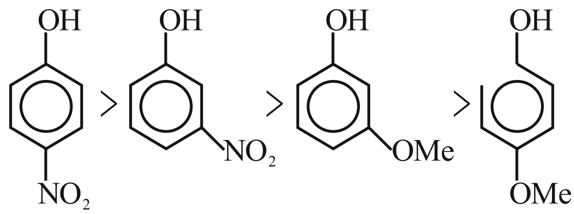
(A) (B) (C) (D)

(A) A > B > C > D (B) B > A > C > D

(C) D > C > A > B (D) D > C > B > A

Official Ans. by NTA (A)

Sol. The correct order of acid strength is



16. $CH_3-CH_2-CN \xrightarrow[\text{Ether}]{CH_3MgBr} A \xrightarrow{H_3O^+} B \xrightarrow[\text{HCl}]{\text{Zn–Hg}} C$

The correct structure of C is

(A) $CH_3—CH_2—CH_2—CH_3$

(B) $CH_3-CH_2-\overset{\overset{\large O}{||}}{C}-CH_3$

(C) $CH_3-CH_2-\overset{\overset{\large OH}{|}}{CH}-CH_3$

(D) $CH_3—CH_2—CH = CH_2$

Official Ans. by NTA (A)

Sol. $CH_3CH_2-C\equiv N \xrightarrow{CH_3MgBr} CH_3CH_2-\overset{\overset{\large NMgBr}{||}}{C}-CH_3$

$\downarrow H_3O^+$

$CH_3CH_2CH_2CH_3 \xleftarrow[\text{HCl}]{\text{Zn/Hg}} CH_3CH_2-\overset{\overset{\large O}{||}}{C}-CH_3$

(Clemmensen Reduction)

17. **Match List I with List II :**

List-I Polymer	**List-II used for items**
A. Nylon 6,6	I. Buckets
B. Low density polythene	II. Non-stick utensils
C. High density polythene	III. Bristles of brushes
D. Teflon	IV. Toys

Choose the correct answer from the options given below:

(A) A–III, B-I, C-IV, D-II

(B) A–III, B-IV, C-I, D-II

(C) A–II, B-I, C-IV, D-III

(D) A–II, B-IV, C-I, D-III

Official Ans. by NTA (B)

Sol. **LDPE → Toys**

HDPE → Buckets **(As per NCERT)**

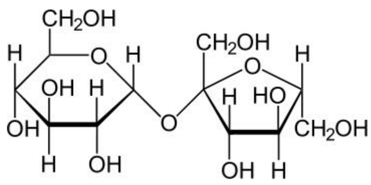
18. Glycosidic linkage between C_1 of α-glucose and C_2 of β-fructose is found in

(A) maltose (B) sucrose

(C) lactose (D) amylose

Official Ans. by NTA (B)

Sol. Theoretical



19. Some drugs bind to a site other than, the active site of an enzyme. This site is known as

(A) non-active site (B) allosteric site

(C) competitive site (D) therapeutic site

Official Ans. by NTA (B)

Sol. Theoretical

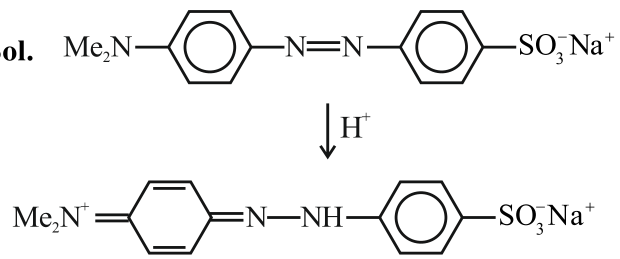
20. In base vs. Acid titration, at the end point methyl orange is present as

(A) quinonoid form (B) heterocyclic form

(C) phenolic form (D) benzenoid form

Official Ans. by NTA (A)

Sol.



(QUINONOID FORM)

SECTION-B

1. 56.0 L of nitrogen gas is mixed with excess of hydrogen gas and it is found that 20 L of ammonia gas is produced. The volume of unused nitrogen gas is found to be____ L.

Official Ans. by NTA (46)

Sol.

N_2	+	$3H_2$	$\rightarrow$	$2NH_3$
56L		excess		O
–10L		–30L		+20L
46L				20L

2. A sealed flask with a capacity of 2 dm^3 contains 11 g of propane gas. The flask is so weak that it will burst if the pressure becomes 2 MPa. The minimum temperature at which the flask will burst is _______ °C. [Nearest integer]

(Given: R = 8.3 J K^{-1} mol^{-1}. Atomic masses of C and H are 12u and 1u respectively.) (Assume that propane behaves as an ideal gas.)

Official Ans. by NTA (1655)

Sol. Moles of $C_3H_8 = \frac{11}{44} = 0.25\,\text{moles}$

PV = nRT

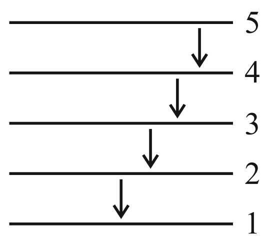
$\Rightarrow\ 2 \times 10^6 \times 2 \times 10^{-3} = 0.25 \times 8.3 \times T$

$\Rightarrow\ T = 1927.710\ K = 1654.56°C$

3. When the excited electron of a H atom from n = 5 drops to the ground state, the maximum number of emission lines observed are _____

Official Ans. by NTA (10)

Sol. Since only a single H atom is present, maximum number of spectral lines = 4



4. While performing a thermodynamics experiment, a student made the following observations,

$HCl + NaOH \rightarrow NaCl + H_2O\ \Delta H = -57.3\ kJ\ mol^{-1}$

$CH_3COOH + NaOH \rightarrow CH_3COONa + H_2O$

$\Delta H = -55.3\ kJ\ mol^{-1}$.

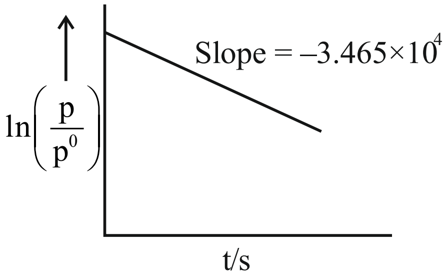
The enthalpy of ionization of CH_3COOH as calculated by the student is _______ kJ mol^{-1}. (nearest integer)

Official Ans. by NTA (2)

Sol. $\Delta H_{ionisation}\,\text{of}\,CH_3COOH = |-57.3 - (-55.3)|$

$= 2\ KJ/mol$

5. For the decomposition of azomethane.
$CH_3N_2CH_3(g) \rightarrow CH_3CH_3(g)+N_2(g)$ a first order reaction, the variation in partial pressure with time at 600 K is given as



The half life of the reaction is _____ $\times 10^{-5}$s. [Nearest integer]

Official Ans. by NTA (2)

Sol. For first order reaction

$$k = \frac{1}{t}\ln\left(\frac{P_0}{P}\right)$$

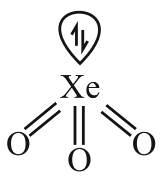
$$\ln\left(\frac{P_0}{P}\right) = kt$$

$$t_{1/2} = \frac{\ln 2}{k} = \frac{0.693}{3.465\times 10^4} = 2\times 10^{-5}$$

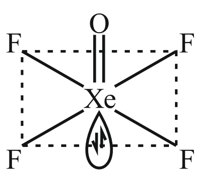
6. The sum of number of lone pairs of electrons present on the central atoms of XeO_3, $XeOF_4$ and XeF_6 is _____________

Official Ans. by NTA (3)

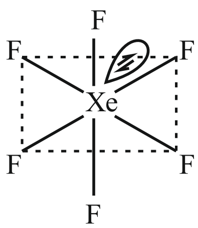
Sol.



- one lone pair on central atom



- one lone pair on central atom



- one lone pair on central atom

7. The spin-only magnetic moment value of M^{3+} ion (in gaseous state) from the pairs Cr^{3+}/Cr^{2+}, Mn^{3+}/Mn^{2}, Fe^{3+}/Fe^{2+} and Co^{3+}/Co^{2+} that has negative standard electrode potential, is B.M.

[Nearest integer]

Official Ans. by NTA (4)

Sol. $E^0_{Cr^{+3}}\Big|_{Cr^{+2}} = -0.41V$

$$[Cr^{+3}] = 4s^0 3d^3$$

$$\mu = \sqrt{n(n+2)}\text{ B.M}$$

$$= \sqrt{15}\text{ B.M} \sim 4 \text{ B.M}$$

8. A sample of 4.5 mg of an unknown monohydric alcohol, R–OH was added to methylmagnesium iodide. A gas is evolved and is collected and its volume measured to be 3.1 mL. The molecular weight of the unknown alcohol is ____ g/mol. [Nearest integer]

Official Ans. by NTA (33)

Sol. $ROH + CH_3MgI \rightarrow ROMgI + CH_4(g)$

moles of CH_4 = moles of ROH

$$\Rightarrow \frac{V}{22400} = \frac{m}{M.M} \text{ (Assuming NTP Condition)}$$

$$\Rightarrow \frac{3.1}{22400} = \frac{4.5\times 10^{-3}}{M.M}$$

$$\Rightarrow \quad MM = 32.51$$

Nearest Integer = 33

9. The separation of two coloured substances was done by paper chromatography. The distances travelled by solvent front, substance A and substance B from the base line are 3.25 cm. 2.08 cm and 1.05 cm. respectively. The ratio of R_f values of A to B is ______

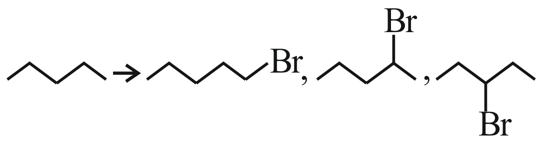
Official Ans. by NTA (2)

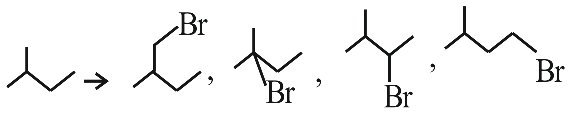
Sol. $\frac{R_{F_A}}{R_{F_B}} = \frac{\frac{2.08}{3.25}}{\frac{1.05}{3.25}} = \frac{2.08}{1.05} \simeq 2$

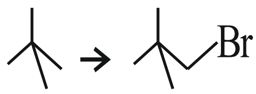
10. The total number of monobromo derivatives formed by the alkanes with molecular formula C_5H_{12} is (excluding stereo isomers)_____

Official Ans. by NTA (8)

Sol. The Alkanes and their monobromodervative are

1. 

2. 

3. 

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Monday 25th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

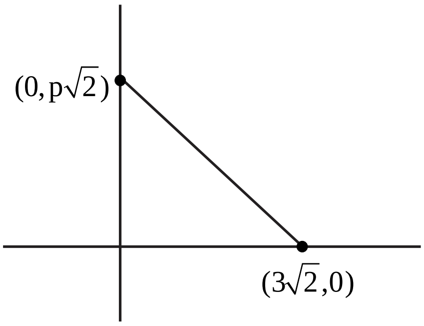
1. For $z \in \mathbb{C}$ if the minimum value of $\left(|z-3\sqrt{2}|+|z-p\sqrt{2}i|\right)$ is $5\sqrt{2}$, then a value of p is ____________

(A) 3 (B) $\frac{7}{2}$

(C) 4 (D) $\frac{9}{2}$

Official Ans. by NTA (C)

Sol.



$\sqrt{(3\sqrt{2})^2+(p\sqrt{2})^2}=5\sqrt{2}$

18 + 2p^2 = 50

p^2 = 16

$p = \pm 4$

As per option (C) is correct

2. The number of real values λ, such that the system of linear equations

2x – 3y + 5z = 9

x + 3y – z = –18

3x – y + (λ^2 – |λ|)z = 16

has no solution, is :-

(A) 0 (B) 1

(C) 2 (D) 4

Official Ans. by NTA (C)

Sol. 2x – 3y + 5z = 9

x + 3y – z = –18

3x – y + (λ^2 – |λ|)z = 16

$$D=\begin{vmatrix}2 & -3 & 5\\ 1 & 3 & -1\\ 3 & -1 & \lambda^2-|\lambda|\end{vmatrix}=0$$

$\Rightarrow 3\lambda^2-3|\lambda|-11=0$

Clearly one negative and one positive root since $|\lambda|$ is there so negative not possible and two values of λ corresponding to positive value

$$D_3=\begin{vmatrix}2 & -3 & 9\\ 1 & 3 & -18\\ 3 & -1 & 16\end{vmatrix}\neq 0$$ so no solution.

3. The number of bijective functions f : {1, 3, 5, 7,........99} $\rightarrow$ {2, 4, 6, 8,......., 100}, such that $f(3) \geq f(9) \geq f(15) \geq f(21) \geq ..... \geq f(99)$, is _____

(A) $^{50}P_{17}$ (B) $^{50}P_{33}$

(C) 33! × 17! (D) $\frac{50!}{2}$

Official Ans. by NTA (B)

Sol. f : {1, 3, 5, 7, ...99} $\rightarrow$ {2, 4, 6, 8,....,100}

$f(3) \geq f(9) \geq f(15) \geq ....f(99)$ ..... (1)

3, 9, 15,.....99 $\Rightarrow$ 17 numbers

for condition one we have $^{50}C_{17}$ × 1 way rest 33 elements 33 !

= $^{50}C_{17}$ × 33!

= $^{50}C_{33}$ × 33!

= $^{50}P_{33}$.

4. The remainder when $(11)^{1011}$ + $(1011)^{11}$ is divided by 9 is

(A) 1 (B) 4

(C) 6 (D) 8

Official Ans. by NTA (D)

Sol. $\dfrac{(9+2)^{1011}}{9}+\dfrac{(1008+3)^{11}}{9}$

$=\dfrac{2^{1011}}{9}+\dfrac{3^{11}}{9}$

$=\dfrac{(8)^{337}}{9}+\dfrac{3^{11}}{9}$

$=\dfrac{(9-1)^{337}}{9}+0$

$= (-1)^{337} + 9$

$= 8$

5. The sum $\sum\limits_{n=1}^{21}\dfrac{3}{(4n-1)(4n+3)}$ is equal to

(A) $\dfrac{7}{87}$ (B) $\dfrac{7}{29}$

(C) $\dfrac{14}{87}$ (D) $\dfrac{21}{29}$

Official Ans. by NTA (B)

Sol. $\sum\limits_{n=1}^{21}\dfrac{3}{(4n-1)(4n+3)}$

$=\dfrac{3}{4}\sum\limits_{n=1}^{21}\dfrac{(4n+3)-(4n-1)}{(4n-1)(4n+3)}$

$=\dfrac{3}{4}\sum\limits_{n=1}^{21}\dfrac{1}{4n-1}-\dfrac{1}{4n+3}$

$=\dfrac{3}{4}\left(\dfrac{1}{3}-\dfrac{1}{7}+\dfrac{1}{7}-\dfrac{1}{11}+\dfrac{1}{11}-....+\dfrac{1}{83}-\dfrac{1}{87}\right)$

$=\dfrac{3}{4}\left(\dfrac{1}{3}-\dfrac{1}{87}\right)=\dfrac{7}{29}$

6. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{8\sqrt{2}-(\cos x+\sin x)^7}{\sqrt{2}-\sqrt{2}\sin 2x}$ is equal to

(A) 14 (B) 7

(C) $14\sqrt{2}$ (D) $7\sqrt{2}$

Official Ans. by NTA (A)

Sol. $\sin x + \cos x = t$

$1 + \sin 2x = t^2$

$\sin 2x = t^2 - 1$

$\lim\limits_{t\to\sqrt{2}}\dfrac{8\sqrt{2}-t^7}{\sqrt{2}-\sqrt{2}(t^2-1)}$

$\lim\limits_{t\to\sqrt{2}}\dfrac{8\sqrt{2}-t^7}{2\sqrt{2}-\sqrt{2}t^2}$ (L-Hospital Rule)

$\lim\limits_{t\to\sqrt{2}}\dfrac{-7t^6}{-2\sqrt{2}t}=\lim\limits_{t\to\sqrt{2}}\dfrac{7}{2\sqrt{2}}\times t^5$

$=\dfrac{7}{2\sqrt{2}}\times(\sqrt{2})^5=14$

7. $\lim\limits_{n\to\infty}\dfrac{1}{2^n}\left(\dfrac{1}{\sqrt{1-\frac{1}{2^n}}}+\dfrac{1}{\sqrt{1-\frac{2}{2^n}}}+\dfrac{1}{\sqrt{1-\frac{3}{2^n}}}+......+\dfrac{1}{\sqrt{1-\frac{2^n-1}{2^n}}}\right)$

is equal to

(A) $\dfrac{1}{2}$ (B) 1

(C) 2 (D) –2

Official Ans. by NTA (C)

Sol. $\lim\limits_{n\to\infty}\dfrac{1}{2^n}\sum\limits_{r=1}^{2^n}\dfrac{1}{\sqrt{1-\frac{r}{2^n}}}$

$\therefore \dfrac{1}{2^n}\to dx \Leftarrow \dfrac{r}{2^n}=x \quad \left(\dfrac{r}{n'}=x,\dfrac{1}{x}=dx\right)$

$2^n = n'$

$\lim\limits_{n'\to\infty}\dfrac{1}{n'}\sum\limits_{r=1}^{n'-1}\dfrac{1}{\sqrt{1-\frac{r}{n'}}}=\int_0^1\dfrac{1}{\sqrt{1-x}}dx$

$=-\left.\dfrac{(1-x)^{1/2}}{1/2}\right]_0^1=-2[0-1]=2$

8. If A and B are two events such that $P(A)=\frac{1}{3}$, $P(B)=\frac{1}{5}$ and $P(A\cup B)=\frac{1}{2}$, then $P(A|B')+P(B|A')$ is equal to

(A) $\frac{3}{4}$ (B) $\frac{5}{8}$

(C) $\frac{5}{4}$ (D) $\frac{7}{8}$

Official Ans. by NTA (B)

Sol. $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$\frac{1}{2}=\frac{1}{3}+\frac{1}{5}-P(A\cap B)$$

$$P(A\cap B)=\frac{1}{30}$$

$$P\left(\frac{A}{\bar{B}}\right)+P\left(\frac{B}{\bar{A}}\right)=\frac{P(A\cap\bar{B})}{P(\bar{B})}+\frac{P(B\cap\bar{A})}{P(\bar{A})}$$

$$=\frac{P(A)-P(A\cap B')}{1-P(B)}+\frac{P(B)-P(A\cap B)}{1-P(A)}$$

$$=\frac{\frac{1}{3}-\frac{1}{30}}{\frac{4}{5}}+\frac{\frac{1}{5}-\frac{1}{30}}{\frac{2}{3}}=\frac{5}{8}$$

9. Let [t] denote the greatest integer less than or equal to t. Then the value of the integral $\int_{-3}^{101}([\sin(\pi x)]+e^{[\cos(2\pi x)]})dx$ is equal to

(A) $\frac{52(1-e)}{e}$ (B) $\frac{52}{e}$

(C) $\frac{52(2+e)}{e}$ (D) $\frac{104}{e}$

Official Ans. by NTA (B)

Sol. $\int_{-3}^{101}([\sin\pi x]+e^{[\cos 2\pi x]})dx$

$$52\int_0^2([\sin\pi x]+e^{[\cos 2\pi x]})dt$$

$$\frac{52}{\pi}\int_0^{2\pi}([\sin t]+e^{[\cos 2t]})dt$$

$$\frac{52}{\pi}\left(\int_0^{2\pi}([\sin t]dt+\int_0^{2\pi}e^{[\cos 2t]}dt\right)$$

$$I_1=\int_0^{2\pi}[\sin t]dt$$

Using King

$$I_1=\int_0^{2\pi}[-\sin t]dt$$

$$2I_1=\int_0^{2\pi}(-1)dt=-2\pi$$

$$I_1=-\pi$$

$$I_2=2\int_0^{\pi}e^{[\cos 2t]}dt$$

$$=2.2\int_0^{\pi/2}e^{[\cos 2t]}dt$$

$$=4\left(\int_0^{\pi/4}e^0.dt+\int_{\pi/4}^{\pi/2}e^{-1}dt\right)$$

$$4\left(\frac{\pi}{4}+e^{-1}\left(\frac{\pi}{4}\right)\right)=\pi(1+e^{-1})$$

$$I=\frac{52}{\pi}(-\pi+\pi+\pi e^{-1})=\frac{52}{e}$$

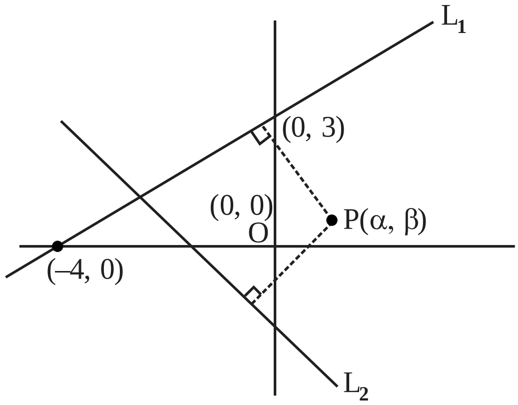
10. Let the point P (α, β) be at a unit distance from each of the two lines $L_1 : 3x - 4y + 12 = 0$, and $L_2 : 8x + 6y + 11 = 0$. If P lies below L_1 and above L_2, then $100(\alpha + \beta)$ is equal to

(A) –14 (B) 42

(C) –22 (D) 14

Official Ans. by NTA (D)

Sol.



By observing origin and P lies in same region.

$L_1(0, 0) > 0$; $L_1(\alpha, \beta) > 0 \Rightarrow 3\alpha - 4\beta + 12 > 0$

$$1 = \left|\frac{3\alpha - 4\beta + 12}{5}\right|$$

$3\alpha - 4\beta + 12 = 5$ ....(1)

Similarly for L_2

$L_2(0, 0) > 0$; $L_2(\alpha, \beta) > 0$

$$1 = \left|\frac{8\alpha + 6\beta + 11}{10}\right| \Rightarrow 8\alpha + 6\beta + 11 = 10 \quad ....(2)$$

Solving (1) and (2)

$$\alpha = -\frac{23}{25}; \beta = \frac{106}{100}$$

$$100(\alpha + \beta) = 100\left(\frac{-92}{100} + \frac{106}{100}\right) = 14$$

11. Let a smooth curve y = f(x) be such that the slope of the tangent at any point (x, y) on it is directly proportional to $\left(\frac{-y}{x}\right)$. If the curve passes through the point (1, 2) and (8, 1), then $\left|y\left(\frac{1}{8}\right)\right|$ is equal to

(A) $2\log_e 2$ (B) 4

(C) 1 (D) $4\log_e 2$

Official Ans. by NTA (B)

Sol. $\frac{dy}{dx} = -\frac{\alpha y}{x}$

$$\frac{dy}{y} = -\frac{\alpha}{x}dx$$

$$\Rightarrow \frac{dy}{y} + \frac{\alpha}{x}dx = 0$$

$$\Rightarrow \ell ny + \alpha \ell nx = \ell nc$$

$$\Rightarrow yx^{\alpha} = c$$

For (1, 2) $\Rightarrow 2.1^{\alpha} = c \Rightarrow c = 2$

For (8, 1) $\Rightarrow 1.8^{\alpha} = 2 \Rightarrow \alpha = \frac{1}{3}$

$\therefore$ curve is $y = 2x^{-1/3}$

At x = 1/8, y(1/8) = $2\left(\frac{1}{8}\right)^{-\frac{1}{3}} \Rightarrow y = 4$

12. If the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the line $\frac{x}{7} + \frac{y}{2\sqrt{6}} = 1$ on the x-axis and the line $\frac{x}{7} - \frac{y}{2\sqrt{6}} = 1$ on the y-axis, then the eccentricity of the ellipse is

(A) $\frac{5}{7}$ (B) $\frac{2\sqrt{6}}{7}$

(C) $\frac{3}{7}$ (D) $\frac{2\sqrt{5}}{7}$

Official Ans. by NTA (A)

Sol. Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through the point (7, 0) and $(0, -2\sqrt{6})$

Now $\frac{49}{a^2} + 0 = 1 \Rightarrow a^2 = 49$

and $0 + \frac{24}{b^2} = 1 \Rightarrow b^2 = 24$

Now a > b $\Rightarrow b^2 = a^2(1 - e^2)$

$$\Rightarrow 24 = 49(1 - e^2) \Rightarrow e^2 = \frac{25}{49}$$

$$\Rightarrow e = \frac{5}{7}$$

13. The tangents at the point A(1, 3) and B(1, –1) on the parabola $y^2 - 2x - 2y = 1$ meet at the point P. Then the area (in unit^2) of the triangle PAB is :-

(A) 4 (B) 6

(C) 7 (D) 8

Official Ans. by NTA (D)

Sol. Both point A(1, 3), B(1, –1) lies on the parabola $y^2 - 2y - 2x - 1 = 0$

Equation of tangent aty A(1, 3) is T = 0

$x - 2y + 5 = 0$

and equation of tangent at B(1, –1) is T = 0

$x + 2y + 1 = 0$

So point P is (–3, 1)

$$\Rightarrow A = \frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 1 & -1 & 1 \\ -3 & 1 & 1 \end{vmatrix} = 8$$

14. Let the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{7}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{\alpha}=\frac{1}{25}$ coincide. Then the length of the latus rectum of the hyperbola is:-

(A) $\frac{32}{9}$ (B) $\frac{18}{5}$

(C) $\frac{27}{4}$ (D) $\frac{27}{10}$

Official Ans. by NTA (D)

Sol. $\frac{x^2}{16}+\frac{y^2}{7}=1$

$$\Rightarrow 7 = 16(1-e^2) \Rightarrow e = \frac{3}{4}$$

Foci of ellipse is $(\pm ae, 0) \Rightarrow (\pm 3, 0)$

Now hyperbola be $\frac{x^2}{144}-\frac{y^2}{\alpha}=\frac{1}{25}$

$$\frac{x^2}{\frac{144}{25}}-\frac{y^2}{\frac{\alpha}{25}}=1$$

Now $a = \frac{12}{5},\ b^2 = \frac{\alpha}{25}$

Let eccentricity of hyperbola be e

ae = 3 (Given)

$$\Rightarrow \frac{12}{5}e = 3 \Rightarrow e = \frac{5}{4}$$

$b^2 = a^2(e^2 - 1)$

$$\frac{\alpha}{25} = \frac{144}{25}\left(\frac{25}{16}-1\right) \Rightarrow \alpha = 81$$

Hyperbola is $\frac{x^2}{\frac{144}{25}}-\frac{y^2}{\frac{81}{25}}=1$

Now length of LR = $\frac{2b^2}{a} = \frac{27}{10}$

15. A plane E is perpendicular to the two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$, and passes through the point P(1, –1, 1). If the distance of the plane E from the point Q(a, a, 2) is $3\sqrt{2}$, then $(PQ)^2$ is equal to

(A) 9 (B) 12

(C) 21 (D) 33

Official Ans. by NTA (C)

Sol. Let equation of plane be

$a(x - 1) + b(y + 1) + c(z - 1) = 0$ .....(1)

It is perpendicular to the given two planes

$2a - 2b + c = 0$

$a - b + 2c = 0$

$$\Rightarrow \frac{a}{3} = \frac{b}{3} = \frac{c}{0}$$

Equation of plane be x + y = 0

Now $\frac{|a+a|}{\sqrt{2}} = 3\sqrt{2} \Rightarrow |2a| = 6 \Rightarrow a = \pm 3$

P(3, 3, 2) or P(–3, –3, 2), Q(1, –1, 1)

$PQ^2 = (3 - 1)^2 + (3 + 1)^2 + (2 - 1)^2 = 21$

16. The shortest distance between the lines $\frac{x+7}{-6}=\frac{y-6}{7}=z$ and $\frac{7-x}{2}=y-2=z-6$ is

(A) $2\sqrt{29}$ (B) 1

(C) $\sqrt{\frac{37}{29}}$ (D) $\frac{\sqrt{29}}{2}$

Official Ans. by NTA (A)

Sol. $\frac{x+7}{-6}=\frac{y-6}{7}=\frac{z-0}{1}$:

point (–7, 6, 0) dr's –6, 7, 1

$\frac{x-7}{2}=\frac{y-2}{-1}=\frac{z-6}{-1}$:

point (7, 2, 6) dr's 2, –1, –1

Shortest distance

$$=\left|\frac{\begin{vmatrix}14 & -4 & 6\\ -6 & 7 & 1\\ 2 & -1 & -1\end{vmatrix}}{\sqrt{(-7+1)^2+(6-2)^2+(6-14)^2}}\right|=2\sqrt{29}$$

17. Let $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}$ be a vector such that $\vec{a}\times\vec{b}=2\hat{i}-\hat{k}$ and $\vec{a}.\vec{b}=3$. Then the projection of $\vec{b}$ on the vector $\vec{a}-\vec{b}$ is :-

(A) $\frac{2}{\sqrt{21}}$ (B) $2\sqrt{\frac{3}{7}}$

(C) $\frac{2}{3}\sqrt{\frac{7}{3}}$ (D) $\frac{2}{3}$

Official Ans. by NTA (A)

Sol. Projection of $\vec{b}$ on vector $\vec{a}-\vec{b}$ is

$$=\frac{\vec{b}.(\vec{a}-\vec{b})}{|\vec{a}-\vec{b}|}$$

$$=\frac{\vec{a}.\vec{b}-|b|^2}{\sqrt{a^2+b^2-2a\cdot b}}=\frac{3-b^2}{\sqrt{6+b^2-6}}=\frac{3-b^2}{b}$$

$$\left|\vec{a}\times\vec{b}\right|^2=5$$

$$a^2b^2-(a\cdot b)^2=5$$

$$6b^2=14\ \Rightarrow b^2=\frac{7}{3}$$

$$\therefore\ \frac{3-b^2}{b}=\frac{3-\frac{7}{3}}{\sqrt{\frac{7}{3}}}=2\times\sqrt{21}$$

18. If the mean deviation about median for the number 3, 5, 7, 2k, 12, 16, 21, 24 arranged in the ascending order, is 6 then the median is

(A) 11.5 (B) 10.5

(C) 12 (D) 11

Official Ans. by NTA (D)

Sol. 3, 5, 7, 2k, 12, 16, 21, 24

Median = $\frac{2k+12}{2}=k+6$

M.D. = $\frac{\Sigma|x_i-M|}{8}=6$

= (k + 3) + (k + 1) + (k – 1) + (6 – k) + (6 – k) + (10 – k) + (15 – k) + (18 – k) = 48

= 58 – 2k = 48

k = 5

Median = k + 6 = 11

19. $2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$

is equal to

(A) $\frac{3}{16}$ (B) $\frac{1}{16}$

(C) $\frac{1}{32}$ (D) $\frac{9}{32}$

Official Ans. by NTA (B)

Sol. $2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$

$$2\cos\left(\frac{\pi}{2}-\frac{\pi}{22}\right)\cos\left(\frac{\pi}{2}-\frac{3\pi}{22}\right)\cos\left(\frac{\pi}{2}-\frac{5\pi}{22}\right)\cos\left(\frac{\pi}{2}-\frac{7\pi}{15}\right)$$

$$\cos\left(\frac{\pi}{2}-\frac{9\pi}{22}\right)$$

$$2\cos\left(\frac{10\pi}{22}\right)\cos\left(\frac{8\pi}{22}\right)\cos\left(\frac{6\pi}{22}\right)\cos\left(\frac{4\pi}{22}\right)\cos\left(\frac{2\pi}{22}\right)$$

$$2\cos\left(\frac{\pi}{11}\right)\cos\left(\frac{2\pi}{11}\right)\cos\left(\frac{3\pi}{11}\right)\cos\left(\frac{4\pi}{11}\right)\cos\left(\frac{5\pi}{11}\right)$$

$$2\cos\left(\frac{\pi}{11}\right)\cos\left(\frac{2\pi}{11}\right)\cos\left(\frac{4\pi}{11}\right)\cos\left(\pi-\frac{3\pi}{11}\right)\cos\left(\pi+\frac{5\pi}{11}\right)$$

$$2\cos\left(\frac{\pi}{11}\right)\cos\left(\frac{2\pi}{11}\right)\cos\left(\frac{4\pi}{11}\right)\cos\left(\frac{8\pi}{11}\right)\cos\left(\frac{16\pi}{11}\right)$$

$$\frac{2.\sin\left(2^5\times\frac{\pi}{11}\right)}{2^5\sin\frac{\pi}{11}}$$

$$\frac{2.\sin\left(\frac{32\pi}{11}\right)}{32\sin\frac{\pi}{11}}=\frac{1}{16}$$

20. Consider the following statements :

P : Ramu is intelligent

Q : Ramu is rich

R : Ramu is not honest

The negation of the statement "Ramu is intelligent and honest if and only if Ramu is not rich" can be expressed as :

(A) $((P\wedge(\sim R))\wedge Q)\wedge((\sim Q)\wedge((\sim P)\vee R))$

(B) $((P\wedge R)\wedge Q)\vee((\sim Q)\wedge((\sim P)\vee(\sim R)))$

(C) $((P\wedge R)\wedge Q)\wedge((\sim Q)\wedge((\sim P)\vee(\sim R)))$

(D) $((P\wedge(\sim R))\wedge Q)\vee((\sim Q)\wedge((\sim P)\vee R))$

Official Ans. by NTA (D)

Sol. Negation of $(P\wedge\sim R)\leftrightarrow(\sim Q)$

$\Rightarrow$ $((P\wedge\sim R)\wedge Q)\vee(\sim Q\wedge\sim(P\wedge\sim R))$

$\Rightarrow((P\wedge\sim R)\wedge Q)\vee(\sim Q\wedge(\sim P\vee R))$

Answer D is correct

SECTION-B

1. Let A : {1, 2, 3, 4, 5, 6, 7}. Define B = {$T\subseteq A$: either $1\notin T$ or $2\in T$} and C = $T\subseteq A$: T the sum of all the elements of T is a prime number}. Then the number of elements in the set $B\cup C$ is _______________

Official Ans. by NTA (107)

Sol. A : {1, 2, 3, 4, 5, 6, 7}

Number of elements in set B

$=n(1\notin T)+n(2\in T)-n[(1\notin T)\cap(2\in T)]$

$=2^6+2^6-2^5=96$

Number of elements in set C

= {{2}, {3}, {5}, {7}, {1, 2}, {1, 4}, {1, 6}, {2, 3}, {2, 5}, {3, 4}, {4, 7}, {5, 6}, {6, 7}, {1, 2, 4}, {1, 3, 7}, {1, 4, 6}, {1, 5, 7}, {2, 3, 6}, {2, 4, 5}, {2, 4, 7}, {2, 5, 6}, {3, 4, 6}, {4, 6, 7}, {1, 2, 4, 6}, {2, 4, 6, 7}, {2, 4, 6, 5}, {3, 5, 7, 4}, {1, 3, 5, 4}, {1, 5, 7, 4}, {1, 2, 3, 5}, {1, 2, 3, 7}, {1, 3, 6, 7}, {1, 5, 6, 7}, {2, 3, 5, 7}, {1, 5, 7, 2, 4}, {3, 5, 7, 2, 6}, {1, 3, 7, 2, 4}, {1,4, 5, 6, 7}, {1, 3, 4, 5, 6}, {1, 2, 3, 6, 7}, {1, 2, 3, 5, 6}, {1, 2, 3, 4, 6, 7}

Number of elementrrs in C = 42

$\Rightarrow n(B\cup C)=n(B)+n(C)-n(B\cap C)$

= 96 + 42 – 31 = 107

2. Let f(x) be a quadratic polynomial with leading coefficient 1 such that f(0) = p, $p\neq 0$ and $f(1)=\frac{1}{3}$. If the equation f(x) = 0 and fofofof(x) = 0 have a common real root, then f(–3) is equal to......................

Official Ans. by NTA (25)

Sol. Let $f(x) = x^2 + bx + p$

$$f(1)=\frac{1}{3}\Rightarrow 1+b+p=\frac{1}{3} \qquad ...(1)$$

Assume common root be α

$f(\alpha)=0$ & $f(f(f(f(\alpha)))) = 0$

$\Rightarrow f(f(f(0)))=0$

$$\Rightarrow f\left(f\left(p\right)\right)=0$$

$$\Rightarrow f\left(p^2+bp+p\right)=0$$

$$\Rightarrow f\left(p\left(p+b+1\right)\right)=0$$

$$\Rightarrow f\left(\frac{p}{3}\right)=0$$

$$\Rightarrow \frac{p^2}{9}+b\cdot\frac{p}{3}+p=0$$

$$\Rightarrow \frac{p}{9}+\frac{b}{3}+1=0$$

p + 3b + 9 = 0 ...(2)

From (1) & (2) $\Rightarrow p=\frac{7}{2}$

Now, $f\left(-3\right)=9-3b+p$

$=9-\left(-p-9\right)+p$

= 18 + 2p = $18+2\times\frac{7}{2}=25$

3. Let $A=\begin{bmatrix}1 & a & a\\0 & 1 & b\\0 & 0 & 1\end{bmatrix}, a,b\in\mathbb{R}$. If for some $n\in N$,

$A^n=\begin{bmatrix}1 & 48 & 2160\\0 & 1 & 96\\0 & 0 & 1\end{bmatrix}$ then n + a + b is equal to

____

Official Ans. by NTA (24)

Sol. $A^2=\begin{bmatrix}1 & a & a\\0 & 1 & b\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & a & a\\0 & 1 & b\\0 & 0 & 1\end{bmatrix}$

$$=\begin{bmatrix}1 & 2a & 2a+ab\\0 & 1 & 2b\\0 & 0 & 1\end{bmatrix}$$

$$A^2A=\begin{bmatrix}1 & 2a & 2a+ab\\0 & 1 & 2b\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & a & a\\0 & 1 & b\\0 & 0 & 1\end{bmatrix}$$

$$A^3=\begin{bmatrix}1 & 3a & 3a+3ab\\0 & 1 & 3b\\0 & 0 & 1\end{bmatrix}$$

$$A^4=\begin{bmatrix}1 & 4a & 4a+6ab\\0 & 1 & 4b\\0 & 1 & 1\end{bmatrix}$$

$$A^n=\begin{bmatrix}1 & na & \frac{(n^2-n)}{2}ab+na\\0 & 1 & nb\\0 & 0 & 1\end{bmatrix}$$

na = 48, nb = 96

$na+\frac{nab}{2}(n-1)=2160$

48 + 24b(n – 1) = 2160

48 + 24 × 96 – 24b = 2160

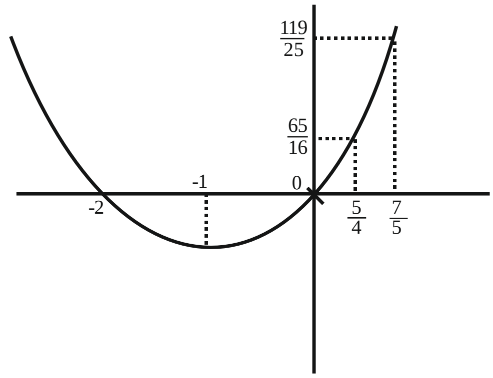
b = 8 and a = 4, n = 12

n + a + b = 24

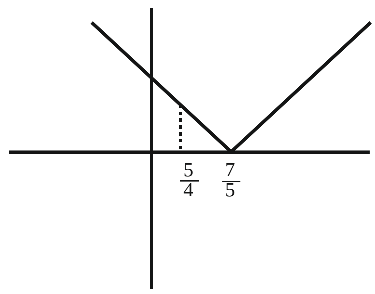
4. The sum of the maximum and minimum values of the function f(x) = |5x – 7| + [x^2 + 2x] is the interval $\left[\frac{5}{4},2\right]$, where [t] is the greatest integer $\le t$ is ______________

Official Ans. by NTA (15)

Sol. Graph of x^2 + 2x



Graph of |5x – 7|



$f(x)|_{min} = 4 + 0 = 4$, at $x = \frac{7}{5}$

$f(x)|_{max} = 8 + 3 = 11$, at $x = 2$

$\therefore$ Required sum = 15

5. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} = \frac{4y^3 + 2yx^2}{3xy^2 + x^3}, y(1) = 1$. If for some $n \in N, y(2) \in [n-1, n)$, then n is equal to ________

Official Ans. by NTA (3)

Sol. $\frac{dy}{dx} = \frac{4y^3 + 2yx^2}{3xy^2 + x^3}, y(1) = 1$

$$\frac{dy}{dx} = \frac{4(y/x)^3 + 2(y/x)}{3(y/x)^2 + 1}$$

y = xp

$$x\frac{dp}{dx} + p = \frac{4p^3 + 2p}{3p^2 + 1}$$

$$x\frac{dp}{dx} = \frac{p^3 + p}{3p^2 + 1}$$

$$\int \frac{3p^2 + 1}{p^3 + p} dp = \int \frac{dx}{x}$$

$\ln(p^3 + p) = \ln x + \ln C$

$p^3 + p = xC$

$$\left(\frac{y}{x}\right)^3 + \left(\frac{y}{x}\right) = xC$$

$y^3 + x^2y = x^4C$

x = 1, y = 1

$1 + 1 = C \Rightarrow C = 2$

$y^3 + x^2y = 2x^4$

Put x = 2

$y^3 + 4y - 32 = 0$

Having root between 2 and 3

$y(2) \in [2, 3)$

6. Let f be a twice differentiable function on R. If $f'(0) = 4$ and

$$f(x) + \int_0^x (x-t)f'(t)dt = (e^{2x} + e^{-2x})\cos 2x + \frac{2}{a}x ,$$

then $(2a + 1)^5 a^2$ is equal to ________

Official Ans. by NTA (8)

Sol. f'(0) = 4

$$f(x) + \int_0^x (x-t)f'(t)dt = (e^{2x} + e^{-2x})\cos 2x + \frac{2}{a}x$$

Put x = 0 : f(0) = 2

$$f'(x) + x(f'(x)) + \int_0^x f'(t)dt - xf'(x)$$

$$= (e^{2x} + e^{-2x})(-2\sin 2x) + \cos 2x(2e^{2x} - 2e^{-2x}) + \frac{2}{a}$$

$$\Rightarrow f'(x) + f(x) - 2 = (e^{2x} + e^{-2x})(-2\sin 2x)$$

$$+ \cos 2x(2e^{2x} - 2e^{-2x}) + \frac{2}{a}$$

Put x = 0

4 + 2 – 2 = 0 + (2 – 2) + 2/a

$$\Rightarrow a = \frac{1}{2}$$

$$(2a+1)^5 a^2 = 2^5 . \frac{1}{2^2} = 8$$

7. Let $a_n = \int_{-1}^{n}\left(1 + \frac{x}{2} + \frac{x^2}{2} + \frac{x^3}{3} + ... + \frac{x^{n-1}}{n}\right)dx$ for $n \in N$. Then the sum of all the elements of the set $\{n \in N : a_n \in (2, 30)\}$ is ______

Official Ans. by NTA (5)

Sol. $\int_{-1}^{n}\left(1 + \frac{x}{2} + \frac{x^2}{3} + ... + \frac{x^{n-1}}{n}\right)dx$

$$\left[x + \frac{x^2}{2} + \frac{x^3}{3^2} + ... + \frac{x^n}{n^2}\right]_{-1}^{n}$$

$$\left(n+\frac{n^2}{2^2}+\frac{n^3}{3^2}+\ldots+\frac{n^n}{n^2}\right)$$

$$-\left(-1+\frac{1}{2^2}-\frac{1}{3^2}+\frac{1}{4^2}+\ldots+\frac{(-1)^n}{n^2}\right)$$

$$a_n=(n+1)+\frac{1}{2^2}(n^2-1)+\frac{1}{3^2}(n^3+1)$$

$$+\ldots+\frac{1}{n^2}\left(n^n-(-1)^n\right)$$

if $n = 1 \Rightarrow a_n = 2 \not\in (2,30)$

if $n = 2 \Rightarrow a_n=(2+1)+\frac{1}{2^2}(2^2-1)=3+\frac{3}{4}<30$

if $n = 3 \Rightarrow a_n=(3+1)+\frac{1}{4}(8)+\frac{1}{9}(28)=11+\frac{28}{9}<30$

If $n = 4 \Rightarrow a_n=(4+1)+\frac{1}{4}(16-1)+\frac{1}{9}(64+1)+\frac{1}{16}$

$$=5+\frac{15}{4}+\frac{65}{9}+\frac{255}{16}>30$$

Test {2, 3} sum of elements 5

8. If the circles $x^2 + y^2 + 6x + 8y + 16 = 0$ and $x^2+y^2+2(3-\sqrt{3})x+x+2(4-\sqrt{6})y$ $=k+6\sqrt{3}+8\sqrt{6},\ k>0$, touch internally at the point $P(\alpha, \beta)$, then $(\alpha+\sqrt{3})^2+(\beta+\sqrt{6})^2$ is equal to ___________

Official Ans. by NTA (25)

Sol. $C_1\ (-3, -4)$

$$r_1=\sqrt{25-16}=3$$

$$C_2 = (-3+\sqrt{3},-4+\sqrt{6})$$

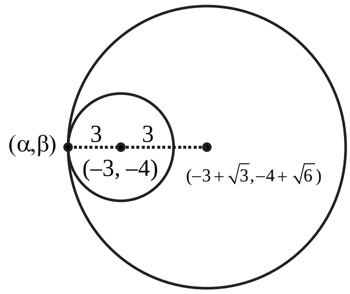
$$r_2=\sqrt{34+k}$$

$C_1C_2 = |r_1 - r_2|$

$$C_1C_2=\sqrt{3+6}=3$$

$$3=|3-\sqrt{34+k}\,| \Rightarrow k=2$$

$r_2 = 6$



$$(\alpha,\beta)=(-\sqrt{3}-3,-4-\sqrt{6})$$

$$(\alpha+\sqrt{3})^2+(\beta+\sqrt{6})^2=9+16=25$$

9. Let the area enclosed by the x-axis, and the tangent and normal drawn to the curve $4x^3 - 3xy^2 + 6x^2 - 5xy - 8y^2 + 9x + 14 = 0$ at the point $(-2, 3)$ be A. Then 8A is equal to _____

Official Ans. by NTA (170)

Sol. $4x^3 - 3xy^2 + 6x^2 - 5xy - 8y^2 + 9x + 14 = 0$ at $P(-2, 3)$

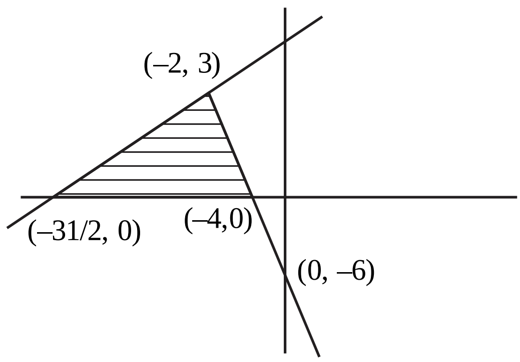
$12x^2 - 3(y^2 + 2yxy') + 12x - 5(xy' + y) - 16yy' + 9 = 0$

$48 - 3(9 - 12y') - 24 - 5(-2y' + 3) - 48y' + 9 = 0$

$y' = -9/2$

Tangent $y-3=-\frac{9}{2}(x+2) \Rightarrow 9x + 2y = -12$

Normal : $y-3=\frac{2}{9}(x+2) \Rightarrow 9y-2x=31$



Area = $\frac{1}{2}\left(\frac{31}{2}-4\right)\times 3 = \frac{85}{4}$

$8A = 170$

10. Let $x=\sin(2\tan^{-1}\alpha)$ and $y=\sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$. If $S=\{\alpha\in R: y^2=1-x\}$, then $\sum_{\alpha\in S}16\alpha^3$ is equal to ________

Official Ans. by NTA (130)

Sol. $x=\sin(2\tan^{-1}\alpha)=\frac{2\tan\theta}{1+\tan^2\theta}=\frac{2\alpha}{1+\alpha^2}$

$\tan^{-1}\alpha=\theta\Rightarrow\tan\theta=\alpha$

$y^2=\sin^2\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)=\frac{1}{5}$

$y^2+x=1\Rightarrow\frac{1}{5}+\frac{2\alpha}{1+\alpha^2}=1$

$\frac{2\alpha}{1+\alpha^2}=\frac{4}{5}$

$(2\alpha-1)(\alpha-2)=0$

$\Rightarrow 2\alpha^2-5\alpha+2=0$

$\therefore \alpha=2$ or $\frac{1}{2}$

$S=\left\{2,\frac{1}{2}\right\}$

$\sum_{\alpha\in S}16\alpha^3=16\left(8+\frac{1}{8}\right)=130$