FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Monday 25th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

PHYSICS **TEST PAPER WITH SOLUTION**

SECTION-A

1. If momentum [P], area [A] and time [T] are taken as fundamental quantities, then the dimensional formula for coefficient of viscosity is :

(A) $[P\ A^{-1}\ T^0]$ (B) $[P\ A\ T^{-1}]$

(C) $[P\ A^{-1}\ T]$ (D) $[P\ A^{-1}\ T^{-1}]$

Official Ans. by NTA (A)

Sol. Viscosity = pascal.second

$P^x\ A^y\ T^z = [M^1\ L^{-1}\ T^{-1}]$

$[M^1\ L^{+1}\ T^{-1}]^x\ [L^2]^y[T^1]^z = M^1L^{-1}\ T^{-1}$

$M^xL^{+x+2y}\ T^{-x+z} = M^1\ L^{-1}\ T^{-1}$

$x = 1 \quad x + 2y = -1 \quad -x + z = -1$

$y = -1$

$z = 0$

Viscosity $= P^1\ A^{-1}\ T^0$

2. Which of the following physical quantities have the same dimensions ?

(A) Electric displacement $(\vec{D})$ and surface charge density

(B) Displacement current and electric field

(C) Current density and surface charge density

(D) Electric potential and energy

Official Ans. by NTA (A)

Sol. Electric displacement

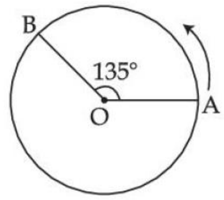
$\vec{D} = \epsilon_0 \vec{E}$

$$[D] = [\epsilon_0\ E] = \left[\epsilon_0 \frac{\sigma}{\epsilon_0}\right]$$

$[D] = [\sigma]$

$\rightarrow$ Surface change density = σ.

3. A person moved from A to B on a circular path as shown in figure. If the distance travelled by him is 60 m, then the magnitude of displacement would be : (Given cos135° = - 0.7)



(A) 42 m (B) 47 m

(C) 19 m (D) 40 m

Official Ans. by NTA (B)

Sol. $d = R\theta$

$$60 = R\left(\frac{3\pi}{4}\right)$$

$$R = \frac{60\times 4}{3\pi} = \frac{80}{\pi} m$$

Displacement $= \sqrt{R^2 + R^2 - 2R^2 \cos 135}$

$$\Rightarrow \sqrt{2R^2 - 2R^2(-0.7)}$$

$$\Rightarrow \sqrt{3.4R^2} = \sqrt{3.4\left(\frac{80}{\pi}\right)^2}$$

≈ 47 m

4. A body of mass 0.5 kg travels on straight line path with velocity $v = (3x^2 + 4)$m/s. The net workdone by the force during its displacement from x = 0 to x = 2 m is :

(A) 64 J (B) 60 J

(C) 120 J (D) 128 J

Official Ans. by NTA (B)

Sol. $v_i = 3(0^2) + 4 = 4 \quad \cong \quad x = 0$

$v_F = 3(2)^2 + 4 \quad \cong \quad x = 2$

$= 16$

$$W = \Delta K = \frac{1}{2}m\left(16^2 - 4^2\right)$$

$$= \frac{1}{2}\times\frac{1}{2}(256-16)$$

$$= \frac{240}{4} = 60J$$

5. A solid cylinder and a solid sphere, having same mass M and radius R, roll down the same inclined plane from top without slipping. They start from rest. The ratio of velocity of the solid cylinder to that of the solid sphere, with which they reach the ground, will be :

(A) $\sqrt{\frac{5}{3}}$ (B) $\sqrt{\frac{4}{5}}$

(C) $\sqrt{\frac{3}{5}}$ (D) $\sqrt{\frac{14}{15}}$

Official Ans. by NTA (D)

Sol. $V=\sqrt{\frac{2gH}{1+k^2/R^2}}$

$$\frac{V_{cylinder}}{V_{sphere}}=\sqrt{\frac{(1+k^2/R^2)_{sphere}}{(1+k^2/R^2)_{cylinder}}}$$

$$=\sqrt{\frac{1+2/5}{1+1/2}}=\sqrt{\frac{7}{5}\times\frac{2}{3}}=\sqrt{\frac{14}{15}}$$

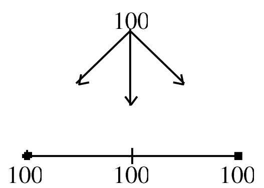
6. Three identical particle A, B and C of mass 100 kg each are placed in a straight line with AB = BC = 13 m. The gravitational force on a fourth particle P of the same mass is F, when placed at a distance 13 m from the particle B on the perpendicular bisector of the line AC. The value of F will be approximately :

(A) 21 G (B) 100 G

(C) 59 G (D) 42 G

Official Ans. by NTA (B)

Sol.



$$F=\frac{GMM}{r^2}+\sqrt{2}\frac{GMM}{(\sqrt{2}r)^2}$$

$$=\frac{GMM}{r^2}\left(1+\frac{1}{\sqrt{2}}\right)$$

$$=\frac{G\times10^4}{13^2}\left(1+\frac{1}{\sqrt{2}}\right)$$

$F\simeq100G$

7. A certain amount of gas of volume V at 27°C temperature and pressure 2×10^7 Nm^{-2} expands isothermally until its volume gets doubled. Later it expands adiabatically until its volume gets redoubled. The final pressure of the gas will be (Use $\gamma = 1.5$)

(A) 3.536×10^5 Pa (B) 3.536×10^6 Pa

(C) 1.25×10^6 Pa (D) 1.25×10^5 Pa

Official Ans. by NTA (B)

Sol. $P_1=2\times10^7\,\text{Pa}$

$P_1V_1 = P_2V_2$

Since $V_2 = 2V_1$ Hence $P_2 = P_1/2$ (isothermal expansion)

$P_2=1\times10^7\,\text{Pa}$

$P_2(V_2)^\gamma=P_3(2V_2)^\gamma$

$$P_3=\frac{1\times10^7}{2^{1.5}}=3.536\times10^6$$

8. Following statements are given :

(1) The average kinetic energy of a gas molecule decreases when the temperature is reduced.

(2) The average kinetic energy of a gas molecule increases with increase in pressure at constant temperature.

(3) The average kinetic energy of a gas molecule decreases with increases in volume.

(4) Pressure of a gas increases with increase in temperature at constant pressure.

(5) The volume of gas decreases with increase in temperature.

Choose the correct answer from the options given below :

(A) (1) and (4) only (B) (1), (2) and (4) only

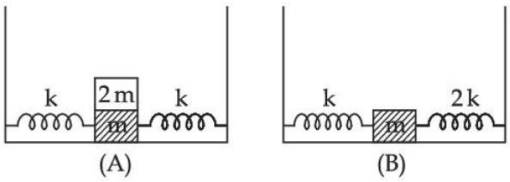
(C) (2) and (4) only (D) (1), (2) and (5) only

Sol. avg 2

$P=\frac{1}{3}\rho V_{rms}^2$

Note : Statement (4) is correct only if we consider it at constant volume and not constant pressure. Ideally, this question must be bonus but most appropriate answer is option (A)

9. In figure (A), mass '2 m' is fixed on mass 'm' which is attached to two springs of spring constant k. In figure (B), mass 'm' is attached to two spring of spring constant 'k' and '2k'. If mass 'm' in (A) and (B) are displaced by distance 'x' horizontally and then released, then time period T_1 and T_2 corresponding to (A) and (B) respectively follow the relation.



(A) $\frac{T_1}{T_2}=\frac{3}{\sqrt{2}}$ (B) $\frac{T_1}{T_2}=\sqrt{\frac{3}{2}}$

(C) $\frac{T_1}{T_2}=\sqrt{\frac{2}{3}}$ (D) $\frac{T_1}{T_2}=\frac{\sqrt{2}}{3}$

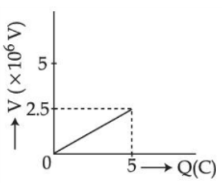
Official Ans. by NTA (A)

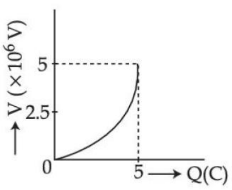
Sol. $T_1=2\pi\sqrt{\frac{3m}{2k}}$

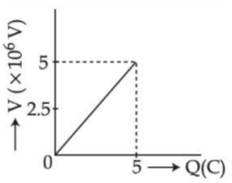
$T_2=2\pi\sqrt{\frac{m}{3k}}$

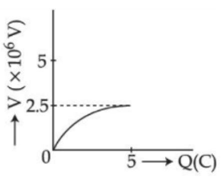
$$\frac{T_1}{T_2}=\frac{2\pi\sqrt{\frac{3m}{2k}}}{2\pi\sqrt{\frac{m}{3k}}}=\frac{3}{\sqrt{2}}$$

10. A condenser of 2 μF capacitance is charged steadily from 0 to 5C. Which of the following graph represents correctly the variation of potential difference (V) across it's plates with respect to the charge (Q) on the condenser ?

(A)


(B)


(C)


(D)


Official Ans. by NTA (A)

Sol. $Q=CV$

$V=\frac{1}{C}Q$

Straight line with slope $=\frac{1}{C}$

Slope $=\frac{1}{C}=\frac{1}{2\times10^{-6}}=5\times10^{5}$

11. Two charged particles, having same kinetic energy, are allowed to pass through a uniform magnetic field perpendicular to the direction of motion. If the ratio of radii of their circular paths is 6 : 5 and their respective masses ratio is 9 : 4. Then, the ratio of their charges will be :

(A) 8 : 5 (B) 5 : 4

(C) 5 : 3 (D) 8 : 7

Official Ans. by NTA (B)

Sol. Radius of circular path $R=\frac{\sqrt{2mk}}{qB}$

$q=\frac{\sqrt{2mk}}{RB}$

$$\frac{q_1}{q_2}=\sqrt{\frac{m_1}{m_2}}\times\frac{R_2}{R_1}=\sqrt{\frac{9}{4}}\times\frac{5}{6}=\frac{5}{4}$$

12. To increase the resonant frequency in series LCR circuit,

(A) Source frequency should be increased

(B) Another resistance should be added in series with the first resistance.

(C) Another capacitor should be added in series with the first capacitor

(D) The source frequency should be decreased

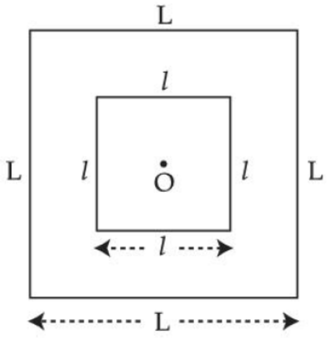
Official Ans. by NTA (C)

Sol. $f=\frac{1}{2\pi\sqrt{LC}}$

To increase the resonating frequency product of L and C should decrease.

By joining capacitor in series, capacitor will decrease

13. A small square loop of wire of side l is placed inside a large square loop of wire L ($L >> l$). Both loops are coplanar and their centres coincide at point O as shown in figure. The mutual inductance of the system is :



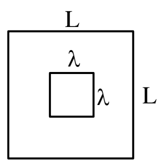
(A) $\frac{2\sqrt{2}\mu_0 L^2}{\pi \ell}$ (B) $\frac{\mu_0 \ell^2}{2\sqrt{2}\pi L}$

(C) $\frac{2\sqrt{2}\mu_0 \ell^2}{\pi L}$ (D) $\frac{\mu_0 L^2}{2\sqrt{2}\pi \ell}$

Official Ans. by NTA (C)

Sol. Assuming current I in outer loop magnetic field at

centre $= 4 \times \frac{\mu_0 i}{4\pi \times \frac{L}{2}} \times (2\sin 45°) = \frac{2\sqrt{2}\mu_0 i}{\pi L}$



$$M = \frac{\text{Flux through inner loop}}{i}$$

$$M = \frac{2\sqrt{2}\mu_0 \ell^2}{\pi L}$$

14. The rms value of conduction current in a parallel plate capacitor is 6.9 μA. The capacity of this capacitor, if it is connected to 230 V ac supply with an angular frequency of 600 rad/s, will be :

(A) 5 pF (B) 50 pF

(C) 100 pF (D) 200 pF

Official Ans. by NTA (B)

Sol. Current in capacitor $I = \frac{V}{X_C}$

$$I = (V) \times (\omega C)$$

$$C = \frac{I}{V\omega} = \frac{6.9 \times 10^{-6}}{230 \times 600} = 50\text{pF}$$

15. Which of the following statement is correct ?

(A) In primary rainbow, observer sees red colour on the top and violet on the bottom

(B) In primary rainbow, observer sees violet colour on the top and red on the bottom

(C) In primary rainbow, light wave suffers total internal reflection twice before coming out of water drops

(D) Primary rainbow is less bright than secondary rainbow.

Official Ans. by NTA (A)

Sol. In primary rainbow, red colour is at top and violet is at bottom.

Intensity of secondary rainbow is less in comparison to primary rainbow.

16. Time taken by light to travel in two different materials A and B of refractive indices μ_A and μ_B of same thickness is t_1 and t_2 respectively. If $t_2 - t_1 = 5 \times 10^{-10}$ s and the ratio of μ_A to μ_B is 1 : 2. Then the thickness of material, in meter is : (Given v_A and v_B are velocities of light in A and B materials respectively).

(A) $5 \times 10^{-10}\ v_a$ m (B) 5×10^{-10} m

(C) 1.5×10^{-10} m (D) $5 \times 10^{-10}\ v_B$ m

Official Ans. by NTA (A)

Sol. $\frac{\mu_A}{\mu_B} = \frac{c / V_A}{c / V_B} = \frac{V_B}{V_A} = \frac{1}{2}$

Let the thickness is d

$$\frac{d}{v_B} - \frac{d}{v_A} = 5 \times 10^{-10}$$

$$d = \frac{5 \times 10^{-10} \times v_A v_B}{v_A - v_B}$$

As $v_A = 2v_B \Rightarrow d = 5 \times 10^{-10} \times 2v_B$

Or $d = 5 \times 10^{-10} \times v_A$

17. A metal exposed to light of wavelength 800 nm and emits photoelectrons with a certain kinetic energy. The maximum kinetic energy of photo-electron doubles when light of wavelength 500 nm is used. The work function of the metal is (Take hc = 1230 eV-nm).

(A) 1.537 eV (B) 2.46 eV

(C) 0.615 eV (D) 1.23 eV

Official Ans. by NTA (C)

Sol. $k_1 = \frac{1230}{800} - \phi$ ...(1)

$k_2 = 2k_1 = \frac{1230}{500} - \phi$ ...(2)

Eliminating k_1 from (1) and (2) we get

$0 = \frac{1230}{500} - \frac{1230}{400} + \phi$

$\phi = 0.615 eV$

18. The momentum of an electron revolving in n^{th} orbit is given by : (Symbols have their usual meanings)

(A) $\frac{nh}{2\pi r}$ (B) $\frac{nh}{2r}$

(C) $\frac{nh}{2\pi}$ (D) $\frac{2\pi r}{nh}$

Official Ans. by NTA (A)

Sol. Angular momentum is integral multiple of $\frac{h}{2\pi}$

$mvr = \frac{nh}{2\pi}$

So momentum $mv = \frac{nh}{2\pi r}$

19. The magnetic moment of an electron (e) revolving in an orbit around nucleus with an orbital angular momentum is given by :

(A) $\vec{\mu}_L = \frac{e\vec{L}}{2m}$ (B) $\vec{\mu}_L = -\frac{e\vec{L}}{2m}$

(C) $\vec{\mu}_l = -\frac{e\vec{L}}{m}$ (D) $\vec{\mu}_l = \frac{2e\vec{L}}{m}$

Official Ans. by NTA (B)

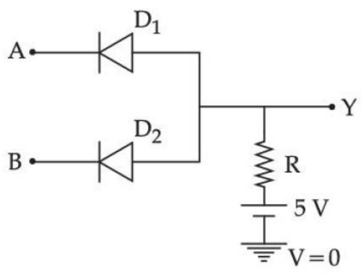
Sol. Ratio of magnetic moment and angular momentum

$\frac{\vec{\mu}}{\vec{L}} = \frac{q}{2m}$

For e^-

$\vec{\mu} = -\frac{e}{2m}\vec{L}$

20. In the circuit, the logical value of A = 1 or B = 1 when potential at A or B is 5V and the logical value of A = 0 or B = 0 when potential at A or B is 0 V.



The truth table of the given circuit will be :

(A)

A	B	Y
0	0	0
1	0	0
0	1	0
1	1	1

(B)

A	B	Y
0	0	0
1	0	1
0	1	1
1	1	1

(C)

A	B	Y
0	0	0
1	0	0
0	1	0
1	1	0

(D)

A	B	Y
0	0	1
1	0	1
0	1	1
1	1	0

Official Ans. by NTA (A)

Sol. When both A and B have logical value '1' both diode are reverse bias and current will flow in resistor hence output will be 5 volt i.e. logical value '1'.

In all other case conduction will take place, hence output will be zero volt i.e. logical value '0'.

So truth table is

A	B	Y	
0	0	0	
0	1	0	(AND gate)
1	0	0	
1	1	1	

SECTION-B

1. A car is moving with speed of 150 km/h and after applying the brake it will move 27 m before it stops. If the same car is moving with a speed of one third the reported speed then it will stop after travelling ________ m distance.

Official Ans. by NTA (3)

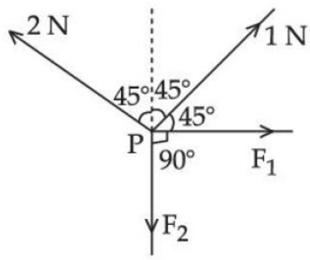
Sol. Stopping distance $= \frac{v^2}{2a} = d$

If speed is made $\frac{1}{3}$rd

$d' = \frac{1}{9}d$. $d' = \frac{27}{9} = 3$.

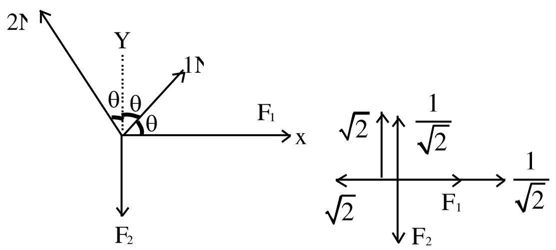
Braking acceleration remains same

2. Four forces are acting at a point P in equilibrium as shown in figure. The ratio of force F_1 to F_2 is 1 : x where x = _____ .



Official Ans. by NTA (3)

Sol.



$\theta = 45°$

Taking components along x & y

$F_1 = \sqrt{2} - \frac{1}{\sqrt{2}} = \frac{2-1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$F_2 = \sqrt{2} + \frac{1}{\sqrt{2}} = \frac{2+1}{\sqrt{2}} = \frac{3}{\sqrt{2}}$

$F_1 : F_2 = 1 : 3$

$x = 3$

3. A wire of length L and radius r is clamped rigidly at one end. When the other end of the wire is pulled by a force F, its length increases by 5 cm. Another wire of the same material of length 4L and radius 4r is pulled by a force 4F under same conditions. The increase in length of this wire is ______ cm.

Official Ans. by NTA (5)

Sol. $\Delta\ell_1 = \frac{F\ell}{AY} = \frac{F\ell}{\pi r^2 Y} = 5\text{cm}$

$\Delta\ell_2 = \frac{4F4\ell}{\pi 16 r^2 Y} = \frac{F\ell}{\pi r^2 Y} = 5\text{cm}$

4. A unit scale is to be prepared whose length does not change with temperature and remains 20 cm, using a bimetallic strip made of brass and iron each of different length. The length of both components would change in such a way that difference between their lengths remains constant. If length of brass is 40 cm and length of iron will be ______cm.

($\alpha_{iron} = 1.2 \times 10^{-5}\ K^{-1}$ and $\alpha_{brass} = 1.8 \times 10^{-5}\ K^{-1}$).

Official Ans. by NTA (60)

Sol. $\ell_B(1+\alpha_B\Delta T) - \ell_i(1+\alpha_i\Delta T) = \ell_B - \ell_i$

$\alpha_B\ell_B = \ell_i\alpha_i$

$1.8\times10^{-5}\times40 = \ell_i\times1.2\times10^{-5}$

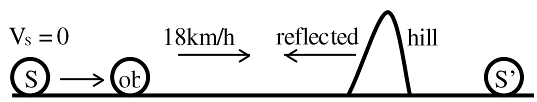
$\ell_i = \frac{1.8\times10^{-5}\times40}{1.2\times10^{-5}} = \frac{3\times40}{2} = 60$

$\ell_i = 60\text{cm}$

5. An observer is riding on a bicycle and moving towards a hill at 18 kmh^{-1}. He hears a sound from a source at some distance behind him directly as well as after its reflection from the hill. If the original frequency of the sound as emitted by source is 640 Hz and velocity of the sound in air is 320 m/s, the beat frequency between the two sounds heard by observer will be _____ Hz.

Official Ans. by NTA (20)

Sol.



$V_S = 0,\ V_{ob} = 5\text{m/s}$

$f_{direct} = \left(\frac{320-5}{320}\right)640 = 630\text{Hz}$

$f_{reflected} = \left(\frac{320+5}{320}\right)640 = 650\text{Hz}$

$f_{beat} = 650 - 630 = 20\text{Hz}$

6. The volume charge density of a sphere of radius 6 m is 2 $\mu C\ cm^{-3}$. The number of lines of force per unit surface area coming out from the surface of the sphere is ______ $\times 10^{10}\ NC^{-1}$.

[Given : Permittivity of vacuum

$\epsilon_0 = 8.85 \times 10^{-12}\ C^2\ N^{-1} - m^{-2}$]

Official Ans. by NTA (45)

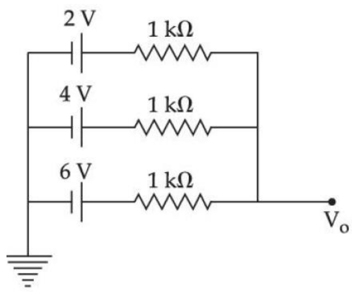
Sol. No. of electric field lines per unit area = electric field.

$$E = \frac{\rho r}{3\epsilon_0},\ \text{for } r = R$$

$$E = \frac{\rho R}{3\epsilon_0} = \frac{2\times 6}{3\times 8.85\times 10^{-12}} = 0.45\times 10^{12}\ NC^{-1}$$

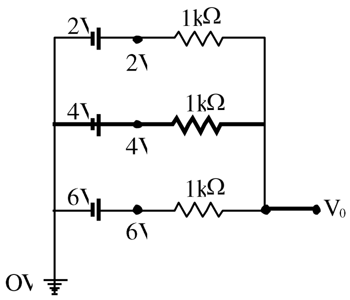
$$= 45\times 10^{10}\ N/C$$

7. In the given figure, the value of V_0 will be ____ V.



Official Ans. by NTA (4)

Sol.



By nodal analysis $\frac{V_0 - 2}{1k\Omega} + \frac{V_0 - 4}{1k\Omega} + \frac{V_0 - 6}{1k\Omega} = 0$

$$3V_0 - 12 = 0$$

$$V_0 = 4$$

8. Eight copper wire of length l and diameter d are joined in parallel to form a single composite conductor of resistance R. If a single copper wire of length $2l$ have the same resistance (R) then its diameter will be __________ d.

Official Ans. by NTA (4)

Sol. Each wire has resistance $= \rho \frac{4\ell}{\pi d^2} = r$

Eight wire in parallel, then equivalent resistance is

$$\frac{r}{8} = \frac{\rho \ell}{2\pi d^2}$$

Single copper wire of length $2l$ has resistance

$$R = \rho \frac{2\ell \times 4}{\pi d_1^2} = \frac{\rho \ell}{2\pi d^2}$$

$$\Rightarrow d_1 = 4d$$

9. The energy band gap of semiconducting material to produce violet (wavelength = 4000 Å) LED is _____ eV. (Round off to the nearest integer).

Official Ans. by NTA (3)

Sol. $E_g = \frac{hc}{\lambda} = \frac{1242}{\lambda(nm)} = \frac{1242}{400} = 3.105$

Answer rounded to 3 eV

10. The required height of a TV tower which can cover the population of 6.03 lakh is h. If the average population density is 100 per square km and the radius of earth is 6400 km, then the value of h will be ______ m.

Official Ans. by NTA (150)

Sol. $d = \sqrt{2Rh}$

$d = \sqrt{2\times 6400\times h\times 10^{-3}}$ (h in m)

Area $= \pi d^2$

$= (\pi \times 2 \times 6400 \times h \times 10^{-3})\ km^2$

$6.03 \times 100000 = 100 \times \pi \times 2 \times 6400 \times 10^{-3}\ h$

$$h = \frac{6.03\times 10^5}{10\times \pi\times 128}$$

h = 150 m

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Monday 25th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

CHEMISTRY **TEST PAPER WITH SOLUTION**

SECTION-A

1. SO_2Cl_2 on reaction with excess of water results into acidic mixture

$SO_2Cl_2 + 2H_2O \rightarrow H_2SO_4 + 2HCl$

16 moles of NaOH is required for the complete neutralisation of the resultant acidic mixture. The number of moles of SO_2Cl_2 used is :

(A) 16 (B) 8

(C) 4 (D) 2

Official Ans. by NTA (C)

Sol. Let $n(SO_2Cl_2) = x$ moles

$\therefore n(H_2SO_4) = x$, $n(HCl) = 2x$

$\Rightarrow n(H^+) = 4x$

For Neutralisation

$\Rightarrow n(H^+) = n(OH^-)$

$\Rightarrow 4x = 16$

$\Rightarrow \mathbf{x} = 4$

2. Which of the following sets of quantum numbers is not allowed ?

(A) $n = 3, l = 2, m_l = 0, s = +\frac{1}{2}$

(B) $n = 3, l = 2, m_l = -2, s = +\frac{1}{2}$

(C) $n = 3, l = 3, m_l = -3, s = -\frac{1}{2}$

(D) $n = 3, l = 0, m_l = 0, s = -\frac{1}{2}$

Official Ans. by NTA (C)

Sol. $l = 0, 1, 2 \ldots\ldots (n-1)$

$\therefore$ for $n = 3$

$l = 0, 1, 2$

$\Rightarrow l = 3,$

not possible for $n = 3$

3. The depression in freezing point observed for a formic acid solution of concentration 0.5 mL L^{-1} is 0.0405°C. Density of formic acid is 1.05 g mL^{-1}. The Van't Hoff factor of the formic acid solution is nearly : (Given for water k_f= 1.86 K kg mol^{-1})

(A) 0.8 (B) 1.1

(C) 1.9 (D) 2.4

Official Ans. by NTA (C)

Sol. $[HCOOH] = 0.5 \text{ ml } l^{-1}$

$\Rightarrow (0.5 \text{ ml} \times 1.05 \text{ g ml}^{-1})$ HCOOH in 1L

$\Rightarrow$ 0.525 g HCOOH in 1L

$$m = \frac{(0.525/46)}{1\text{kg}} \text{mol} \text{ [Assuming dilute solution]}$$

$$\therefore \Delta T_f = iK_f m \Rightarrow i = \frac{\Delta T_f}{k_f m} = \frac{0.0405 \times 46}{1.86 \times 0.525} = 1.9$$

4. 20 mL of 0.1 M NH_4OH is mixed with 40 mL of 0.05 M HCl. The pH of the mixture is nearest to:

(Given: $K_b(NH_4OH) = 1 \times 10^{-5}$, log 2 = 0.30, log 3 = 0.48, log 5 = 0.69, log 7 = 0.84, log 11 =1.04)

(A) 3.2 (B) 4.2

(C) 5.2 (D) 6.2

Official Ans. by NTA (C)

Sol. $NH_4OH + HCl \rightarrow NH_4Cl + H_2O$

mmole 2 2

\- - 2 mmole

$$\left[NH_4^+\right] = \frac{2\text{mmole}}{60\,\text{ml}} = \frac{1}{30} M$$

$$pH = \frac{pK_w - pK_b - \log C}{2} = \frac{14 - 5 + 1.48}{2} = 5.24$$

5.

Match **List - I** with **List - II**

	List - I		List - II
(A)	$N_2(g)+3H_2(g) \rightarrow 2NH_3(g)$	(I)	Cu
(B)	$CO(g)+3H_2(g) \rightarrow CH_4(g)+H_2O(g)$	(II)	$Cu/ZnO-Cr_2O_3$
(C)	$CO(g)+H_2(g) \rightarrow HCHO(g)$	(III)	$Fe_xO_y+K_2O+Al_2O_3$
(D)	$CO(g)+2H_2(g) \rightarrow CH_3OH(g)$	(IV)	Ni

Choose the correct answer from the options given below :

(A) (A) - (II), (B) - (IV), (C) - (I), (D) - (III)

(B) (A) - (II), (B) - (I), (C) - (IV), (D) - (III)

(C) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

(D) (A) - (III), (B) - (I), (C) - (IV), (D) - (II)

Official Ans. by NTA (C)

Sol. Factual

6. The IUPAC nomenclature of an element with electronic configuration $[Rn]5f^{14}6d^17s^2$ is :

(A) Unnilbium (B) Unnilunium

(C) Unnilquadium (D) Unniltrium

Official Ans. by NTA (D)

Sol. Atomic Number 103

7. The compound(s) that is(are) removed as slag during the extraction of copper is :

(1) CaO (2) FeO

(3) Al_2O_3 (4) ZnO

(5) NiO

Choose the correct answer from the options given below :

(A) (3) (4) Only (B) (1), (2), (5) Only

(C) (1), (2) Only (D) (2) Only

Official Ans. by NTA (D)

Sol. $FeO + SiO_2 \rightarrow FeSiO_3$

8. The reaction of H_2O_2 with potassium permanganate in acidic medium leads to the formation of mainly:

(A) Mn^{2+} (B) Mn^{4+}

(C) Mn^{3+} (D) Mn^{6+}

Official Ans. by NTA (A)

Sol. $H_2O_2 + MnO_4^- \rightarrow Mn^{2+} + O_2$ (unbalanced)

9. Choose the correct order of density of the alkali metals :

(A) Li < K < Na < Rb < Cs

(B) Li < Na < K < Rb < Cs

(C) Cs < Rb < K < Na < Li

(D) Li < Na < K < Cs < Rb

Official Ans. by NTA (A)

Sol. Factual

10. The geometry around boron in the product 'B' formed from the following reaction is

$$BF_3 + NaH \xrightarrow{450K} A + NaF$$

$$A + NMe_3 \rightarrow B$$

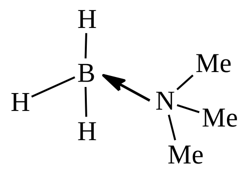
(A) trigonal planar (B) tetrahedral

(C) pyramidal (D) square planar

Official Ans. by NTA (B)

Sol. $$BF_3 + NaH \xrightarrow{450K} \underset{\text{(diborane)}}{B_2H_6} + NaF$$

$$B_2H_6 + NMe_3 \longrightarrow 2\underset{\text{symmetrical cleavage}}{\left[BH_3 \leftarrow NMe_3\right]}$$



11. The interhalogen compound formed from the reaction of bromine with excess of fluorine is a :

(A) hypohalite (B) halate

(C) perhalate (D) halite

Official Ans. by NTA (B)

Sol. $Br_2 + 5\underset{\text{(excess)}}{F_2} \longrightarrow 2\overset{+5}{Br}F_5 \xrightarrow{H_2O} HBrO_3$ (Forms bromate)

12. The photochemical smog does not generally contain :

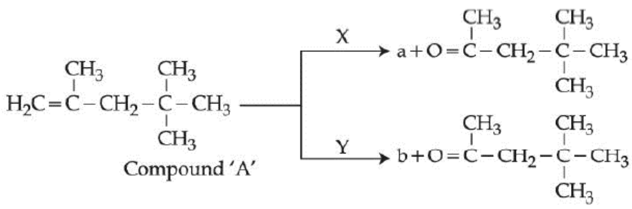
(A) NO (B) NO_2

(C) SO_2 (D) HCHO

Official Ans. by NTA (C)

Sol. Factual

13. A compound 'A' on reaction with 'X' and 'Y produces the same major product but different by product 'a' and 'b'. Oxidation of 'a' gives a substance produced by ants.



'X' and 'Y' respectively are :

(A) $KMnO_4/H^+$ and dil. $KMnO_4$, 273 K

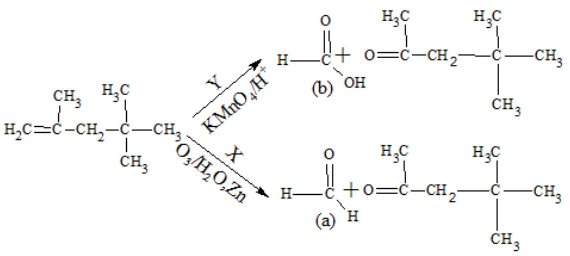
(B) $KMnO_4$,(dilute), 273 K and $KMnO_4/H^+$

(C) $KMnO_4/H^+$ and O_3, H_2O/Zn

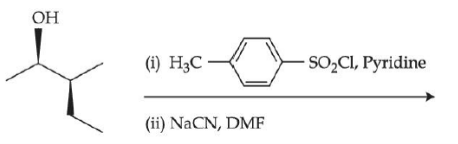
(D) O_3, H_2O/Zn and $KMnO_4/H^+$

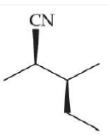
Official Ans. by NTA (D)

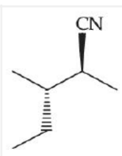
Sol.

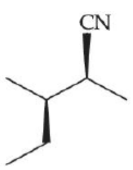


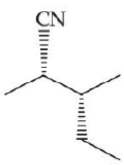
14. Most stable product of the following reaction is:



(A) 

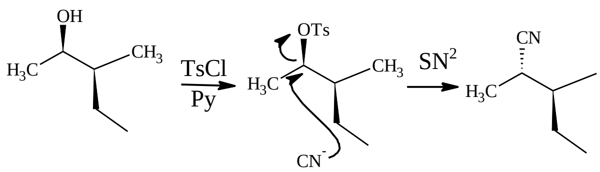
(B) 

(C) 

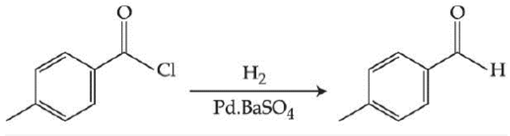
(D) 

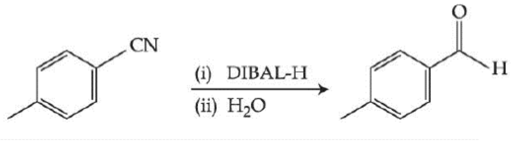
Official Ans. by NTA (B)

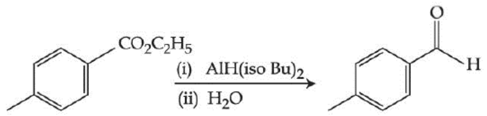
Sol.

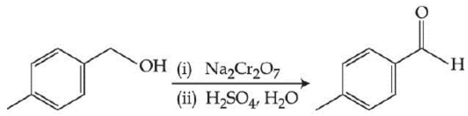


15. Which one of the following reactions does not represent correct combination of substrate and product under the given conditions ?

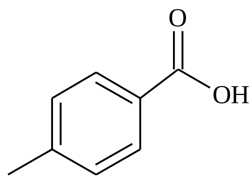
(A) 

(B) 

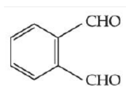
(C) 

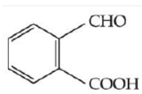
(D) 

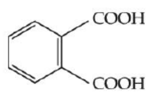
Official Ans. by NTA (D)

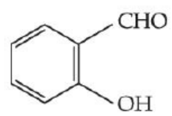
Sol.  is formed

16. An organic compound 'A' on reaction with NH_3 followed by heating gives compound B. Which on further strong heating gives compound C ($C_8H_5NO_2$). Compound C on sequential reaction with ethanolic KOH, alkyl chloride and hydrolysis with alkali gives a primary amine. The compound A is :

(A) 

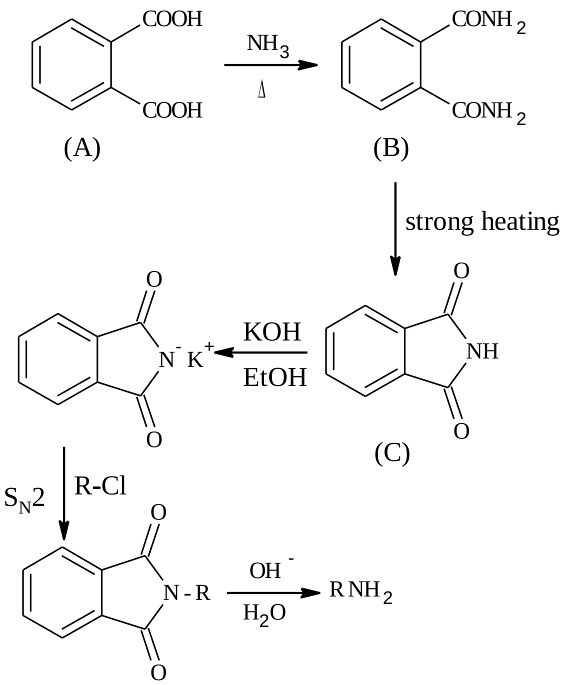
(B) 

(C) 

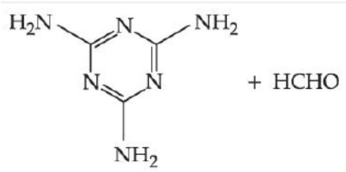
(D) 

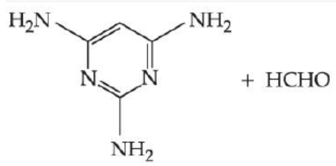
Official Ans. by NTA (C)

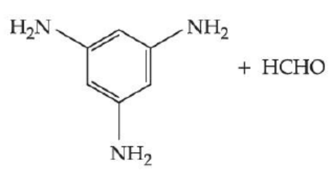
Sol. Gabriel Pthalimide reaction

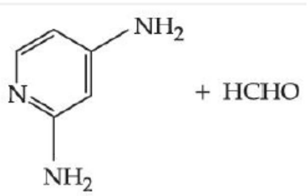


17. Melamine polymer is formed by the condensation of :

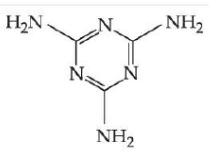
(A) 

(B) 

(C) 

(D) 

Official Ans. by NTA (A)

Sol. Melamine : 

Formaldehyde HCHO

Melamine formaldehyde Resin is melamine polymer

18. During the denaturation of proteins, which of these structures will remain intact ?

(A) Primary

(B) Secondary

(C) Tertiary

(D) Quaternary

Official Ans. by NTA (A)

Sol. Primary structure remains intact during denaturation of proteins

19. Drugs used to bind to receptors, inhibiting its natural function and blocking a message are called :

(A) Agonists

(B) Antagonists

(C) Allosterists

(D) Anti histaminists

Official Ans. by NTA (B)

Sol. Factual

20. Given below are two statements :

Statement I : On heating with $KHSO_4$, glycerol is dehydrated and acrolein is formed.

Statement II : Acrolein has fruity odour and can be used to test glycerol's presence.

Choose the correct option.

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Official Ans. by NTA (B)

Sol. Acrolein has a pungent, suffocating odour.

Acrolein is used to detect presence of glycerol

SECTION-B

1. Among the following species

$N_2, N_2^+, N_2^-, N_2^{2-}, O_2, O_2^+, O_2^-, O_2^{2-}$

the number of species showing diamagnetism is

Official Ans. by NTA (2)

Sol. Diamagnetic species are: N_2, O_2^{2-}

2. The enthalpy of combustion of propane, graphite and dihydrogen at 298 K are: –2220.0 kJ mol^{-1}, –393.5 kJ mol^{-1} and –285.8 kJ mol^{-1} respectively. The magnitude enthalpy of formation of propane (C_3H_8) is……..kJ mol^{-1}. (Nearest integer)

Official Ans. by NTA (104)

Sol. $3C_{(gr)} + 4H_{2(g)} \rightarrow C_3H_{8(g)}$

$= -103.7\ \text{kJ mol}^{-1}$

3. The pressure of a moist gas at 27°C is 4 atm. The volume of the container is doubled at the same temperature. The new pressure of the moist gas is …. $\times 10^{-1}$ atm. (Nearest integer)

(Given : The vapour pressure of water at 27°C is 0.4 atm)

Official Ans. by NTA (22)

Sol. $\left[P_{gas}\right]_0 + V.P. = 4$

$\left[P_{gas}\right]_0 = 4 - 0.4 = 3.6$

As volume is doubled, $\left[P_{gas}\right]_{new} = 1.8$ atm

New Total Pressure = 1.8 + 0.4 = 2.2 atm

4. The cell potential for $Zn|Zn^{2+}(aq)||Sn^{x+}|Sn$ is 0.801 V at 298 K. The reaction quotient for the above reaction is 10^{-2}. The number of electrons involved in the given electrochemical cell reaction is. . . . . .

(Given $E^0_{Zn^{2+}|Zn} = -0.763\text{V}$, $E^0_{Sn^{x+}|Sn} = +0.008\text{V}$ and $\dfrac{2.303RT}{F} = 0.06\text{V}$)

Official Ans. by NTA (4)

Sol. $E = E^0 - \dfrac{2.303\,RT}{nF}\log Q$

Here, $E = +0.801\text{V}$, $E^0 = 0.008 - (-0.763)$

$= +0.771$ V

$\therefore 0.801 = +0.771 - \dfrac{0.06}{n}\ \log 10^{-2}$

$\Rightarrow n = 4$

5. The half life for the decomposition of gaseous compound A is 240 s when the gaseous pressure was 500 Torr initially. When the pressure was 250 Torr, the half life was found to be 4.0 min. The order of the reaction is……. (Nearest integer)

Official Ans. by NTA (1)

Sol. $(t_{1/2})_{500\ torr} = 240$ sec = 4 min.

$(t_{1/2})_{250\ torr} =$ 4 min.

$t_{1/2} \propto a^{1-n}$

As $t_{1/2}$ is independent of initial pressure. Hence, order is 1st order.

6. Consider the following metal complexes :

$[Co(NH_3)]^{3+}$

$[CoCl(NH_3)_5]^{2+}$

$[Co(CN)_6]^{3-}$

$[Co(NH_3)_5(H_2O)]^{3+}$

The spin-only magnetic moment value of the complex that absorbs light with shortest wavelength is B.M. (Nearest integer)

Official Ans. by NTA (0)

Sol. $\Delta_0 \propto \dfrac{1}{\lambda}$

Here, CN^- being SFL will have maximum CFSE

So, $[Co(CN)_6]^{3-}$ will be d^2sp^3, $\mu = 0$

7. Among Co^{3+}, Ti^{2+}, V^{2+} and Cr^{2+}ions, one if used as a reagent cannot liberate H_2 from dilute mineral acid solution, its spin-only magnetic moment in gaseous state is ……B.M. (Nearest integer)

Official Ans. by NTA (5)

Sol. Co^{3+} can't liberate H_2.

It has d^6 configuration,

Number of unpaired electrons = 4

$\mu = \sqrt{4 \times 6} = 4.92$ B.M.

8. While estimating the nitrogen present in an organic compound by Kjeldahl's method, the ammonia evolved from 0.25 g of the compound neutralized 2.5 mL of 2 M H_2SO_4. The percentage of nitrogen present in organic compound is ....

Official Ans. by NTA (56)

Sol. $$\%N = \frac{1.4(N_1V_1)}{\text{mass of organic compound}}$$

$$\%N = \frac{1.4(2.5\times2\times2)}{0.25} = 56$$

9. The number of sp^3 hybridised carbons in an acyclic neutral compound with molecular formula C_4H_5N is :

Official Ans. by NTA (1)

Sol. $$DU = 4+1-\left(\frac{5-1}{2}\right) = 3$$

$H_3C-CH=CH-C\equiv N$

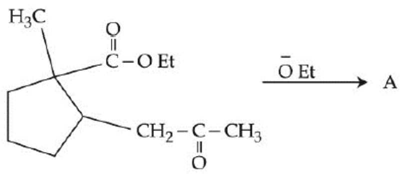
sp^3

or

$CH_2=C=CH=CH=NH$

Zero sp^3 carbon

10. In the given reaction

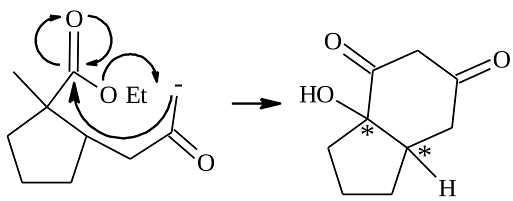


(Where Et is $-C_2H_5$)

The number of chiral carbon/s in product A is

Official Ans. by NTA (2)

Sol.



2 chiral carbons

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Monday 25th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. The total number of functions,

f : {1, 2, 3, 4} • {1, 2, 3, 4, 5, 6}

such that f(1) + f(2) = f(3), is equal to :

(A) 60 (B) 90

(C) 108 (D) 126

Official Ans. by NTA (B)

Sol. A = {1, 2, 3, 4}

B = {1, 2, 3, 4, 5, 6}

Here f(3) can be 2, 3, 4, 5, 6

f(3) = 2, (f(1), f(2)) $\rightarrow$ (1,1) $\rightarrow$ 6 cases

f(3) = 3, (f(1), f(2)) $\rightarrow$ (1,2), (2,1)

$\rightarrow$ 2 × 6 = 12 cases

f(3) = 4, (f(1), f(2)) $\rightarrow$ (1,3), (3,1), (2,2)

$\rightarrow$ 3 × 6 = 18 cases

f(3) = 5, (f(1), f(2)) $\rightarrow$ (1,4), (4,1), (2,3), (3,2)

$\rightarrow$ 4 × 6 = 24 cases

f(3) = 6, (f(1), f(2)) $\rightarrow$ (1,5),(5,1),(2,4),(4,2),(3,3)

$\rightarrow$ 5 × 6 = 30 cases

Total number of cases = 6+ 12 + 18 + 24 + 30 = 90

2. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 + x^3 + x^2 + x + 1 = 0$, then

$\alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021}$ is equal to .

(A) –4 (B) –1

(C) 1 (D) 4

Official Ans. by NTA (B)

Sol. $\alpha, \beta, \gamma, \delta$ root of the equation

$x^4 + x^3 + x^2 + x + 1 = 0$

Which are 5^{th} roots of unity except 1.

then $\alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021} =$

$\alpha + \beta + \gamma + \delta = -1$

3. For n $\in$ N, let $S_n = \left\{z \in C : |z - 3 + 2i| = \frac{n}{4}\right\}$ and

$$T_n = \left\{z \in C : |z - 2 + 3i| = \frac{1}{n}\right\}.$$

Then the number of elements in the set

$\{n \in N : S_n \cap T_n = \phi\}$ is :

(A) 0 (B) 2 (C) 3 (D) 4

Official Ans. by NTA (D)

Sol. $S_n : |z - (3 - 2i)| = \frac{n}{4}$ is a circle center $C_1(3,-2)$

and radius n/4

$T_n : |z - (2 - 3i)| = \frac{1}{n}$ is a circle center C_2 (2,–3)

and radius 1/n

Here $S_n \cap T_n = \phi$

Both circles do not intersect each other

Case-1 : $C_1C_2 > n/4 + 1/n$

$$\sqrt{2} > \frac{n}{4} + \frac{1}{n}$$

then n = 1, 2, 3, 4

Case-2 : $C_1C_2 < \left|\frac{n}{4} - \frac{1}{n}\right|$

$$\Rightarrow \sqrt{2} < \left|\frac{n^2 - 4}{4n}\right|$$

$\Rightarrow$ n has infinite solutions for n $\in$ **N**

4. The number of $\theta \in (0, 4\pi)$ for which the system of linear equations

3 (sin 3θ) x – y + z = 2

3 (cos 2 θ) x + 4y + 3z = 3

6x + 7y + 7z = 9

has no solution is :

(A) 6 (B) 7 (C) 8 (D) 9

Official Ans. by NTA (B)

Sol. The system of equation has no solution.

$$D = \begin{vmatrix} 3\sin 3\theta & -1 & 1 \\ 3\cos 2\theta & 4 & 3 \\ 6 & 7 & 7 \end{vmatrix} = 0$$

$21 \sin 3\theta + 42 \cos 2\theta - 42 = 0$

$\sin 3\theta + 2 \cos 2\theta - 2 = 0$

Number of solution is 7 in $(0,4\pi)$

5. If $\lim\limits_{n\to\infty}\left(\sqrt{n^2-n-1}+n\alpha+\beta\right)=0$ then $8(\alpha + \beta)$ is equal to :

(A) 4 (B) –8

(C) –4 (D) 8

Official Ans. by NTA (C)

Sol. $\lim\limits_{n\to\infty} n\left(1-\frac{n+1}{n^2}\right)^{\frac{1}{2}} + \alpha n + \beta = 0$

$$\lim_{n\to\infty} n\left\{1-\frac{1}{2}\left(\frac{n+1}{n^2}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{n+1}{n^2}\right)^2+....\right\}+\alpha n+\beta=0$$

$$\lim_{n\to\infty} n-\frac{1}{2}+\frac{1}{n}+....+n\alpha+\beta=0$$

$$\alpha=-1, \beta=\frac{1}{2}$$

$8(\alpha + \beta) = -4$

6. If the absolute maximum value of the function $f(x) = (x^2 - 2x + 7)\ e^{(4x^3-12x^2-180x+31)}$ in the interval [–3, 0] is $f(\alpha)$, then :

(A) $\alpha = 0$ (B) $\alpha = -3$

(C) $\alpha \in (-1,0)$ (D) $\alpha \in (-3,-1)$

Official Ans. by NTA (B)

Sol. $f'(x) = e^{(4x^3-12x^2-180x+31)}\left(12(x^2-2x+7)(x+3)(x-5)+2(x-1)\right)$

for $x \in [-3,0]$

$\Rightarrow f'(x) < 0$

f(x) is decreasing function on [–3,0]

The absolute maximum value of the function f(x) is at x = –3

$\Rightarrow \alpha = -3$

7. The curve $y(x) = ax^3 + bx^2 + cx + 5$ touches the x-axis at the point P(–2, 0) and cuts the y-axis at the point Q, where y' is equal to 3. Then the local maximum value of y(x) is :

(A) $\frac{27}{4}$ (B) $\frac{29}{4}$ (C) $\frac{37}{4}$ (D) $\frac{9}{2}$

Official Ans. by NTA (A)

Sol. $y(x) = ax^3 + bx^2 + cx + 5$ is passing through (-2,0) then $8a - 4b + 2c = 5$……(1)

$y'(x) = 3ax^2 + 2bx + c$ touches x-axis at (-2,0)

$12a - 4b + c = 0$ ……(2)

again, for $x = 0$, $y'(x) = 3$

$c = 3$ ………(3)

Solving eq. (1), (2) & (3) $a=-\frac{1}{2}$, $b=-\frac{3}{4}$

$$y'(x)=-\frac{3}{2}x^2-\frac{3}{2}x+3$$

y(x) has local maxima at x = 1

$y(1) = \frac{27}{4}$

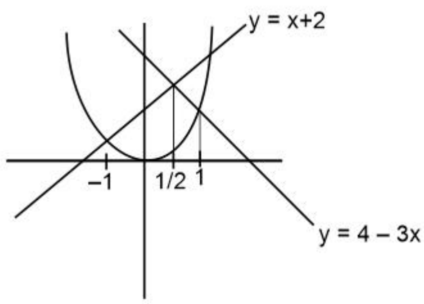
8. The area of the region given by $A=\{(x, y) : x^2 \le y \le \min \{x + 2, 4 - 3x\}\}$ is :

(A) $\frac{31}{8}$ (B) $\frac{17}{6}$ (C) $\frac{19}{6}$ (D) $\frac{27}{8}$

Official Ans. by NTA

(B)

Sol.



$$A=\int_{-1}^{\frac{1}{2}}(x+2-x^2)dx+\int_{\frac{1}{2}}^{1}(4-3x-x^2)dx=\frac{17}{6}$$

9. For any real number x, let [x] denote the largest integer less than equal to x. Let f be a real valued function defined on the interval [–10,10] by $f(x)=\begin{cases} x-[x], & \text{if}(x) \text{ is odd} \\ 1+[x]-x & \text{if}(x) \text{ is even}\end{cases}$

Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos\pi x dx$ is :

(A) 4 (B) 2

(C) 1 (D) 0

Official Ans. by NTA (A)

Sol. f(x) is periodic function whose period is 2

$$\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos\pi x dx=\frac{\pi^2}{10}\times 10\int_0^2 f(x)\cos\pi x dx$$

$$=\pi^2\left(\int_0^1(1-x)\cos\pi x dx+\int_1^2(x-1)\cos\pi x dx\right)$$

Using by parts

$$=\pi^2\times\frac{4}{\pi^2}=4$$

10. The slope of the tangent to a curve C : y =y(x) at any point [x, y) on it is $\frac{2e^{2x}-6e^{-x}+9}{2+9e^{-2x}}$. If C passes through the points $\left(0,\frac{1}{2}+\frac{\pi}{2\sqrt{2}}\right)$ and $\left(\alpha,\frac{1}{2}e^{2\alpha}\right)$ then e^{α} is equal to :

(A) $\frac{3+\sqrt{2}}{3-\sqrt{2}}$

(B) $\frac{3}{\sqrt{2}}\left(\frac{3+\sqrt{2}}{3-\sqrt{2}}\right)$

(C) $\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$

(D) $\frac{\sqrt{2}+1}{\sqrt{2}-1}$

Official Ans. by NTA

(B)

Sol. $\frac{dy}{dx}=\frac{2e^{2x}-6e^{-x}+9}{2+9e^{-2x}}$

$$\frac{dy}{dx}=e^{2x}-\frac{6e^{x}}{2e^{2x}+9}$$

$$y=\frac{e^{2x}}{2}-\tan^{-1}\left(\frac{\sqrt{2}e^{x}}{3}\right)+c$$

If C passes through the point $\left(0,\frac{1}{2}+\frac{\pi}{2\sqrt{2}}\right)$

$$c=-\frac{\pi}{4}-\tan^{-1}\frac{\sqrt{2}}{3}$$

Again C passes through the point $\left(\alpha,\frac{1}{2}e^{2\alpha}\right)$

then $e^{\alpha}=\frac{3}{\sqrt{2}}\left(\frac{3+\sqrt{2}}{3-\sqrt{2}}\right)$

11. The general solution of the differential equation $(x-y^2)dx+y(5x+y^2)dy=0$ is :

(A) $(y^2+x)^4=C|(y^2+2x)^3|$

(B) $(y^2+2x)^4=C|(y^2+x)^3|$

(C) $|(y^2+x)^3|=C(2y^2+x)^4$

(D) $|(y^2+2x)^3|=C(2y^2+x)^4$

Official Ans. by NTA

(A))

Sol. $(x-y^2)dx+y(5x+y^2)dy=0$

$$\frac{dy}{dx}=\frac{y^2-x}{y(5x+y^2)}.\ \text{Let } y^2=v$$

$$\frac{2ydy}{dx}=2\left(\frac{y^2-x}{5x+y^2}\right)$$

$$\frac{dv}{dx}=2\left(\frac{v-x}{5x+v}\right) \qquad v=kx$$

$$k+x\frac{dk}{dx}=2\left(\frac{kx-x}{5x+kx}\right)$$

$$x\frac{dk}{dx}=-\frac{(k^2+3k+2)}{k+5}$$

$$\int\frac{(5+k)}{(k+1)(k+2)}dk=\int-\frac{dx}{x}$$

$$\int\left(\frac{4}{k+1}-\frac{3}{k+2}\right)dk=-\int\frac{dx}{x}$$

$4\ln(k+1)-3\ln(k+2)=-\ln x+\ln c$

$$\frac{(k+1)^4}{(k+2)^3}=-\ln x+\ln c$$

$$c(y^2+2x)^3=(y^2+x)^4$$

12. A line, with the slope greater than one, passes through the point A(4, 3) and intersects the line $x - y - 2 = 0$ at the point B. If the length of the line segment AB is $\frac{\sqrt{29}}{3}$, then B also lies on the line :

(A) $2x + y = 9$ (B) $3x - 2y = 7$
(C) $x + 2y = 6$ (D) $2x - 3y = 3$

Official Ans. by NTA (C)

Sol. Let $B(x_1, x_1 - 2)$

$$\sqrt{(x_1 - 4)^2 + (x_1 - 2 - 3)^2} = \frac{\sqrt{29}}{3}$$

Squaring on both side

$18x_1^2 - 162x_1 + 340 = 0$

$x_1 = \frac{51}{9}$ or $x_1 = \frac{10}{3}$

$y_1 = \frac{33}{9}$ or $y_1 = \frac{4}{3}$

Option (C) will satisfy $\left(\frac{10}{3}, \frac{4}{3}\right)$

13. Let the locus of the centre (α, β), $\beta > 0$, of the circle which touches the circle $x^2 + (y - 1)^2 = 1$ externally and also touches the x-axis be L. Then the area bounded by L and the line $y = 4$ is :

(A) $\frac{32\sqrt{2}}{3}$ (B) $\frac{40\sqrt{2}}{3}$ (C) $\frac{64}{3}$ (D) $\frac{32}{3}$

Official Ans. by NTA (C)

Sol. $(\alpha - 0)^2 + (\beta - 1)^2 = (\beta + 1)^2$

$\alpha^2 = 4\beta$

$x^2 = 4y$

$$A = 2\int_0^4 \left(4 - \frac{x^2}{4}\right) dx = \frac{64}{3}$$

14. Let P be the plane containing the straight line $\frac{x-3}{9} = \frac{y+4}{-1} = \frac{z-7}{-5}$ and perpendicular to the plane containing the straight lines $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$ and $\frac{x}{3} = \frac{y}{7} = \frac{z}{8}$. If d is the distance of P from the point (2, –5, 11), then d^2 is equal to :

(A) $\frac{147}{2}$ (B) 96 (C) $\frac{32}{3}$ (D) 54

Official Ans. by NTA (D)

Sol. $a(x - 3) + b(y + 4) + c(z - 7) = 0$

P : $9a - b - 5c = 0$

$-11a - b + 5c = 0$

After solving DR's $\propto (1, -1, 2)$

Equation of plane

$x - y + 2z = 21$

$d = \frac{8}{\sqrt{6}}$

$d^2 = \frac{32}{3}$

15. Let ABC be a triangle such that $\overrightarrow{BC} = \vec{a}$, $\overrightarrow{CA} = \vec{b}$, $\overrightarrow{AB} = \vec{c}$, $|\vec{a}| = 6\sqrt{2}$, $|\vec{b}| = 2\sqrt{3}$ and $\vec{b}\cdot\vec{c} = 12$

Consider the statements :

(S1) : $|(\vec{a}\times\vec{b}) + (\vec{c}\times\vec{b})| - |\vec{c}| = 6(2\sqrt{2} - 1)$

(S2) : $\angle ABC = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$. Then

(A) both (S1) and (S2) are true
(B) only (S1) is true
(C) only (S2) is true
(D) both (S1) and (S2) are false

Official Ans. by NTA (D)

Sol. $\vec{a} + \vec{b} + \vec{c} = 0$

$\vec{b} + \vec{c} = -\vec{a}$

$|\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b}\cdot\vec{c} = |\vec{a}|^2$

$|\vec{c}|^2 = 36$

$|\vec{c}| = 6$

S1 : $|\vec{a}\times\vec{b} + \vec{c}\times\vec{b}| - |\vec{c}|$

$|(\vec{a} + \vec{c})\times\vec{b}| - |\vec{c}|$

$|-\vec{b}\times\vec{b}| - |\vec{c}|$

$0 - 6 = -6$

S2 : $\vec{a} + \vec{b} + \vec{c} = 0$

$\vec{b} + \vec{c} = -\vec{a}$

$|\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos(\angle ACB) = |\vec{c}|^2$

$\cos(\angle ACB) = \sqrt{\frac{2}{3}}$

16. If the sum and the product of mean and variance of a binomial distribution are 24 and 128 respectively, then the probability of one or two successes is :

(A) $\frac{33}{2^{32}}$ (B) $\frac{33}{2^{29}}$ (C) $\frac{33}{2^{28}}$ (D) $\frac{33}{2^{27}}$

Official Ans. by NTA (C)

Sol. $np + npq = 24$ ...(1)

$np \cdot npq = 128$ ...(2)

Solving (1) and (2) :

We get $p = \frac{1}{2}, q = \frac{1}{2}, n = 32.$

Now,

$P(X = 1) + P(X = 2)$

$= {}^{32}C_1\, pq^{31} + {}^{32}C_2\, p^2 q^{30}$

$= \frac{33}{2^{28}}$

17. If the numbers appeared on the two throws of a fair six faced die are α and β, then the probability that $x^2 + \alpha x + \beta > 0$, for all $x \in R$, is :

(A) $\frac{17}{36}$ (B) $\frac{4}{9}$ (C) $\frac{1}{2}$ (D) $\frac{19}{36}$

Official Ans. by NTA (A)

Sol. $x^2 + \alpha x + \beta > 0, \forall\, x \in R$

$D = \alpha^2 - 4\beta < 0$

$\alpha^2 < 4\beta$

Total cases = $6 \times 6 = 36$

Fav. cases = $\beta = 1, \alpha = 1$

$\beta = 2, \alpha = 1, 2$

$\beta = 3, \alpha = 1, 2, 3$

$\beta = 4, \alpha = 1, 2, 3$

$\beta = 5, \alpha = 1, 2, 3, 4$

$\beta = 6, \alpha = 1, 2, 3, 4$

Total favourable cases = 17

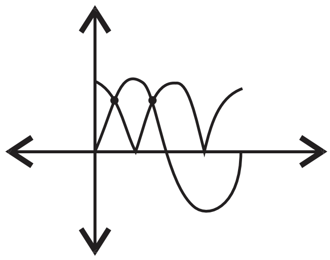
$P(x) = \frac{17}{36}$

18. The number of solutions of $|\cos x| = \sin x$, such that $-4\pi \le x \le 4\pi$ is :

(A) 4 (B) 6 (C) 8 (D) 12

Official Ans. by NTA (C)

Sol.



2 solutions in $(0, 2\pi)$

So, 8 solutions in $[-4\pi, 4\pi]$

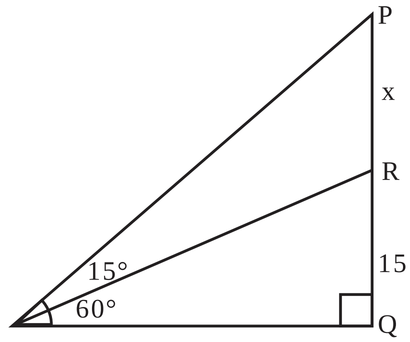
19. A tower PQ stands on a horizontal ground with base Q on the ground. The point R divides the tower in two parts such that QR = 15 m. If from a point A on the ground the angle of elevation of R is 60° and the part PR of the tower subtends an angle of 15° at A, then the height of the tower is :

(A) $5(2\sqrt{3}+3)$ m (B) $5(\sqrt{3}+3)$ m

(C) $10(\sqrt{3}+1)$ m (D) $10(2\sqrt{3}+1)$ m

Official Ans. by NTA (A)

Sol.



$\frac{15}{AQ} = \tan 60°$ ...(1)

$\frac{15+x}{AQ} = \tan 75°$ ...(2)

$\frac{(1)}{(2)} \Rightarrow x = 10\sqrt{3}$

So, PQ = $5(2\sqrt{3}+3)$ m

20. Which of the following statements is a tautology ?

(A) $((\sim p) \vee q) \Rightarrow p$ (B) $p \Rightarrow ((\sim p) \vee q)$

(C) $((\sim p) \vee q) \Rightarrow q$ (D) $q \Rightarrow ((\sim p) \vee q)$

Official Ans. by NTA (D)

Sol.

p	q	~p	~q	~pvq
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

options	1	2	3	4
	T	T	T	T
	T	F	T	T
	F	T	T	T
	F	T	F	T

SECTION-B

1. Let $A = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$ and $B = A - I$. If $\omega = \dfrac{\sqrt{3}i - 1}{2}$, then the number of elements in the set $\{n \in \{1, 2, \ldots, 100\} : A^n + (\omega B)^n = A + B\}$ is equal to _______.

Official Ans. by NTA (17)

Sol. $A = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix} \Rightarrow A^2 = A \Rightarrow A^n = A.$

$\forall\, n \in \{1, 2, \ldots, 100\}$

Now, $B = A - I = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$

$B^2 = -B$

$\Rightarrow B^3 = -B^2 = B$

$\Rightarrow B^5 = B$

$\Rightarrow B^{99} = B$

Also, $\omega^{3k} = 1$

So, n = common of $\{1, 3, 5, \ldots, 99\}$ and $\{3, 6, 9, \ldots, 99\} = 17$

2. The letters of the word 'MANKIND' are written in all possible orders and arranged in serial order as in an English dictionary. Then the serial number of the word 'MANKIND' is _______.

Official Ans. by NTA (1492)

Sol.

M	A	N	K	I	N	D

$\left(\dfrac{4\times 6!}{2!}\right) + (5!\times 0) + \left(\dfrac{4!\times 3}{2!}\right) + (3!\times 2) + (2!\times 1) + (1!\times 1) + (0!\times 0) + 1 = 1492$

3. If the maximum value of the term independent of t in the expansion of $\left(t^2 x^{\frac{1}{5}} + \dfrac{(1-x)^{\frac{1}{10}}}{t}\right)^{15}$, $x \geq 0$, is K, then 8K is equal to _______.

Official Ans. by NTA (6006)

Sol. $\left(t^2 x^{\frac{1}{5}} + \dfrac{(1-x)^{\frac{1}{10}}}{t}\right)^{15}$

$T_{r+1} = {}^{15}C_r \left(t^2 x^{\frac{1}{5}}\right)^{15-r} \cdot \dfrac{(1-x)^{\frac{r}{10}}}{t^r}$

For independent of t,

$30 - 2r - r = 0$

$\Rightarrow r = 10$

So, Maximum value of ${}^{15}C_{10}\, x(1 - x)$ will be at $x = \dfrac{1}{2}$

i.e. 6006

4. Let a, b be two non-zero real numbers. If p and r are the roots of the equation $x^2 - 8ax + 2a = 0$ and q and s are the roots of the equation $x^2 + 12bx + 6b = 0$, such that $\dfrac{1}{p}, \dfrac{1}{q}, \dfrac{1}{r}, \dfrac{1}{s}$ are in A.P., then $a^{-1} - b^{-1}$ is equal to _______.

Official Ans. by NTA (38)

Sol. $x^2 - 8ax + 2a = 0$ $\quad$ $x^2 + 12bx + 6b = 0$

$p + r = 8a$ $\quad$ $q + s = -12b$

$pr = 2a$ $\quad$ $qs = 6b$

$\frac{1}{p}+\frac{1}{r} = 4$ $\quad$ $\frac{1}{q}+\frac{1}{s} = -2$

$\frac{2}{q} = 4$ $\quad$ $\frac{2}{r} = -2$

$q = \frac{1}{2}$ $\quad$ $r = -1$

$p = \frac{1}{5}$ $\quad$ $s = \frac{-1}{4}$

Now, $\frac{1}{a}-\frac{1}{b}=\frac{2}{pr}-\frac{6}{qs} = 38$

5. Let $a_1 = b_1 = 1$, $a_n = a_{n-1} + 2$ and $b_n = a_n + b_{n-1}$ for every natural number $n \ge 2$. Then $\sum_{n=1}^{15} a_n \cdot b_n$ is equal to ________.

Official Ans. by NTA (27560)

Sol. $a_1 = b_1 = 1$

$a_2 = a_1 + 2 = 3$

$a_3 = a_2 + 2 = 5$

$a_4 = a_2 + 2 = 7$

$\Rightarrow a_n = 2n - 1$

$b_2 = a_1 + b_1 = 4$

$b_3 = a_3 + b_2 = 9$

$b_4 = a_4 + b_3 = 16$

$b_n = n^2$

$$\sum_{n=1}^{15} a_n b_n$$

$$\sum_{n=1}^{15} (2n-1)n^2$$

$$\sum_{n=1}^{15} (2n^3 - n^2)$$

$$= 2\frac{n^2(n+1)^2}{4} - \frac{n(n+1)(2n+1)}{6}$$

Put n = 15

$$= \frac{2\times225\times16\times16}{4} - \frac{15\times16\times31}{6}$$

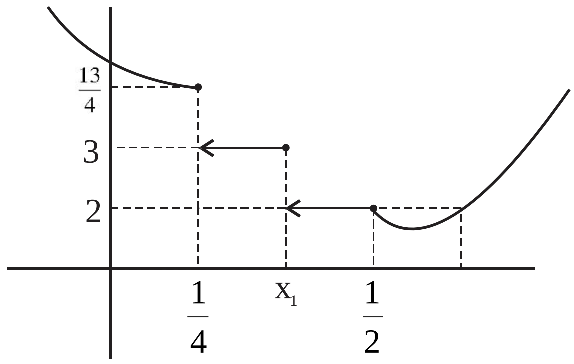
$= 27560$

6. Let $f(x) = \begin{cases} |4x^2-8x+5|, \text{ if } 8x^2-6x+1 \ge 0 \\ [4x^2-8x+5], \text{ if } 8x^2-6x+1 < 0 \end{cases}$,

where $[\alpha]$ denotes the greatest integer less than or equal to α. Then the number of points in R where f is not differentiable is ________.

Official Ans. by NTA (3)

Sol.



ND at $\frac{1}{4}, x_1, \frac{1}{2}$

7. If $\lim_{n\to\infty} \frac{(n+1)^{k-1}}{n^{k+1}}$ [(nk + 1) + (nk + 2) + … + (nk + n)] = 33. $\lim_{n\to\infty} \frac{1}{n^{k+1}} \cdot [1^k + 2^k + 3^k + ... + n^k]$, then the integral value of k is equal to ________.

Official Ans. by NTA (5)

Sol. LHS

$$\lim_{n\to\infty} \frac{(n+1)^{k-1}}{n^{k+1}}[nk \cdot n + 1 + 2 + ... + n]$$

$$= \lim_{n\to\infty} \frac{(n+1)^{k-1}}{n^{k+1}} \cdot \left[n^2 k + \frac{n(n+1)}{2}\right]$$

$$= \lim_{n\to\infty} \frac{(n+1)^{k-1} \cdot n^2 \left(k + \frac{\left(1+\frac{1}{n}\right)}{2}\right)}{n^{k+1}}$$

$$\Rightarrow \lim_{n\to\infty} \left(1+\frac{1}{n}\right)\left(k + \frac{\left(1+\frac{1}{n}\right)}{2}\right)$$

$$\Rightarrow \left(k + \frac{1}{2}\right)$$

RHS

$\Rightarrow \lim_{n\to\infty}\frac{1}{n^{k+1}}(1^k+2^k+...+n^k)=\frac{1}{k+1}$

LHS = RHS

$\Rightarrow k+\frac{1}{2}=33\cdot\frac{1}{k+1}$

$\Rightarrow (2k + 1) (k + 1) = 66$

$\Rightarrow (k - 5) (2k + 13) = 0$

$\Rightarrow k = 5$ or $-\frac{13}{2}$

8. Let the equation of two diameters of a circle $x^2 + y^2 - 2x + 2fy + 1 = 0$ be $2px - y = 1$ and $2x + py = 4p$. Then the slope $m \in (0, \infty)$ of the tangent to the hyperbola $3x^2 - y^2 = 3$ passing through the centre of the circle is equal to _______.

Official Ans. by NTA (2)

Sol. $2p + f - 1 = 0$ ...(1)

$2 - pf - 4p = 0$ ...(2)

$2 = p(f + 4)$

$p=\frac{2}{f+4}$

$2p = 1 - f$

$\frac{4}{f+4}=1-f$

$f^2 + 3f = 0$

$f = 0$ or -3

Hyperbola $3x^2 - y^2 = 3$, $x^2-\frac{y^2}{3}=1$

$y=mx\pm\sqrt{m^2-3}$

It passes (1, 0)

$o=m\pm\sqrt{m^2-3}$

m tends ∞

It passes (1, 3)

$3=m\pm\sqrt{m^2-3}$

$(3 - m)^2 = m^2 - 3$

$m = 2$

9. The sum of diameters of the circles that touch (i) the parabola $75x^2 = 64(5y - 3)$ at the point $\left(\frac{8}{5},\frac{6}{5}\right)$ and (ii) the y-axis, is equal to _______.

Official Ans. by NTA (10)

Allen Ans. (10)

Sol. $x^2=\frac{64.5}{75}\left(y-\frac{3}{5}\right)$

equation of tangent at $\left(\frac{8}{5},\frac{6}{5}\right)$

$x\cdot\frac{8}{5}=\frac{64}{15}\left(\frac{y+\frac{6}{5}}{2}-\frac{3}{5}\right)$

$3x - 4y = 0$

equation of family of circle is

$\left(x-\frac{8}{5}\right)^2+\left(y-\frac{6}{5}\right)^2+\lambda(3x-4y)=0$

It touches y axis so $f^2 = c$

$x^2+y^2+x\left(3\lambda-\frac{16}{5}\right)+y\left(-4\lambda-\frac{12}{5}\right)+4=0$

$\frac{\left(4\lambda+\frac{12}{5}\right)^2}{4}=4$

$\lambda=\frac{2}{5}$ or $\lambda=-\frac{8}{5}$

$\lambda=\frac{2}{5}$, $r = 1$

$\lambda=-\frac{8}{5}$, $r = 4$

$d_1 + d_2 = 10$

10. The line of shortest distance between the lines $\frac{x-2}{0}=\frac{y-1}{1}=\frac{z}{1}$ and $\frac{x-3}{2}=\frac{y-5}{2}=\frac{z-1}{1}$ makes an angle of $\cos^{-1}\left(\sqrt{\frac{2}{27}}\right)$ with the plane P : $ax - y - z = 0$, $(a > 0)$. If the image of the point (1, 1, –5) in the plane P is (α, β, γ), then $\alpha + \beta - \gamma$ is equal to _______.

Official Ans. by NTA (3)

Sol. DR's of line of shortest distance

$\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\0&1&1\\2&2&1\end{vmatrix}=-\hat{i}+2\hat{j}-2\hat{k}$

angle between line and plane is $\cos^{-1}\sqrt{\frac{2}{27}}=\alpha$

$\cos\alpha=\sqrt{\frac{2}{27}}$, $\sin\alpha=\frac{5}{3\sqrt{3}}$

DR's normal to plane (1, –1, –1)

$\sin\alpha=\left|\frac{-a-2+2}{\sqrt{4+4+1}\sqrt{a^2+1+1}}\right|=\frac{5}{3\sqrt{3}}$

$\sqrt{3}\,|a|=5\sqrt{a^2+2}$

$3a^2 = 25a^2 + 50$

No value of (a)