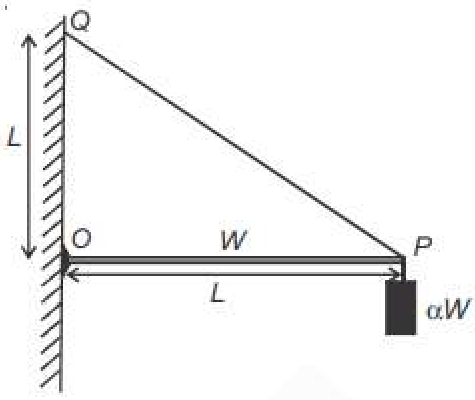
JEE Advanced 2021 (Paper 2)

Physics

Question Paper

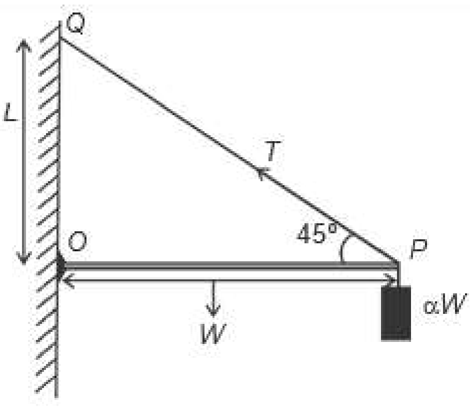
Question 1: One end of a horizontal uniform beam of weight W and length L is hinged on a vertical wall at point O and its other end is supported by a light inextensible rope. The other end of the rope is fixed at point Q, at a height L above the hinge at point O. A block of weight αW is attached at point P of the beam, as shown in the figure (not to scale). The rope can sustain a maximum tension of (2√2) W Which of the following statement(s) is(are) correct?



a. The vertical component of the reaction force at O does not depend on α
b. The horizontal component of the reaction force at O is equal to W for α = 0.5
c. The tension in the rope is 2W for α = 0.5
d. The rope breaks if α > 1.5

Solution:

Answer: (a, b, d)



W x (L/2) + αW x L = T x (1/√2) x L

$$\Rightarrow T = \sqrt{2}\left(\tfrac{1}{2} + \alpha\right) W$$

$$T \times \frac{1}{\sqrt{2}} + F_v = W + \alpha W$$

$$\tfrac{W}{2} + \alpha W + F_v = W + \alpha W$$

F_V = W/2

At α = ½, $T = \sqrt{2}\left(\tfrac{1}{2} + \tfrac{1}{2}\right) W$ = √2W

F_H (at α = ½) = √2W x ½ = W

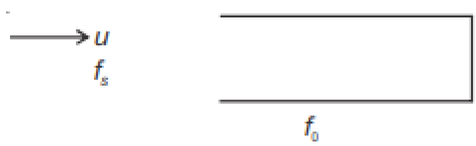
At α = 1.5, T = √2 x (½ + 3/2) W = 2√2W

Question 2: A source, approaching with speed u towards the open end of a stationary pipe of length L, is emitting a sound of frequency f_S. The farther end of the pipe is closed. The speed of sound in air is v and f_0 is the fundamental frequency of the pipe. For which of the following combination(s) of u and f s, will the sound reaching the pipe lead to a resonance?

a. u = 0.8v and $f_S = f_0$
b. u = 0.8v and $f_S = 2f_0$
c. u = 0.8v and $f_S = 0.5f_0$
d. u = 0.5v and $f_S = 1.5f_0$

Solution:

Answer: (a, d)



$$\frac{v}{v-u} \times f_s = (odd) \times f_0$$

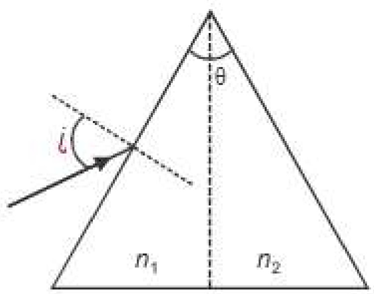
(a) $\frac{v}{v-0.8v} \times f_0 = 5f_0$

(b) $\frac{v}{v-0.8v} \times 2f_0 = 10f_0$

(c) $\frac{v}{v-0.8v} \times \frac{f_0}{2} = (5/2)f_0$

(d) $\frac{v}{v-0.5v} \times \frac{3f_0}{2} = 3f_0$

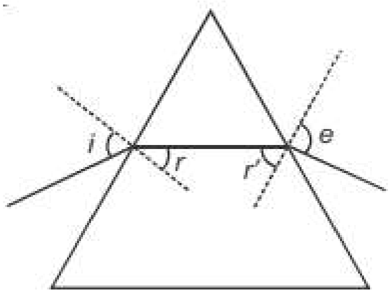
Question 3: For a prism of prism angle θ = 60°, the refractive indices of the left half and the right half are, respectively, n_1 and n_2 ($n_2 \geq n_1$) as shown in the figure. The angle of incidence is chosen such that the incident light rays will have minimum deviation if $n_1 = n_2 = n = 1.5$. For the case of unequal refractive indices, $n_1 = n$ and $n_2 = n + \Delta n$ (where $\Delta n << n$), the angle of emergence $e = i + \Delta e$. Which of the following statement(s) is(are) correct?



a. The value of Δe (in radians) is greater than that of Δn
b. Δe is proportional to Δn
c. Δe lies between 2.0 and 3.0 milliradians if $\Delta n = 2.8 \times 10^{-3}$
d. Δe lies between 1.0 and 1.6 milliradians if $\Delta n = 2.8 \times 10^{-3}$

Solution:

Answer: (b, c)



For $n_1 = n_2 = n = 1.5$,

$r = 30^o$

Therefore, $\sin i = 1.5 \times \sin (30^o) = 3/4$

$\Rightarrow \sin e = 3/4$ for $n_1 = n_2$

Now, $r' = 30^o$ and $n_2 = n + \Delta n$

$n_2 \times \sin (r') = 1 \times \sin e$

$\Rightarrow \Delta n_2 \times \sin 30^0 = \cos e \times \Delta e$

$$\Rightarrow \ \Delta e = \frac{(\Delta n)\times\frac{1}{2}}{\sqrt{1-\frac{9}{16}}} = \frac{2}{\sqrt{7}}\Delta n$$

$\Rightarrow \Delta e < \Delta n$ and, $\Delta e \propto \Delta n$

At $\Delta n = 2.8 \times 10^{-3}$, $\Delta e = 2.12 \times 10^{-3}$ rad

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Question 4: A physical quantity $\vec{S}$ is defined as $\vec{S} = \ (\vec{E} \times \ \vec{B})/\mu_0$, where $\vec{E}$ is electric field, $\vec{B}$ is magnetic field and μ0 is the permeability of free space. The dimensions of $\vec{S}$ are the same as the dimensions of which of the following quantity(ies)?

a. Energy /(Charge x Current)
b. Force/ (Length x Time)
c. Energy/Volume
d. Power/Area

Solution:

Answer: (b, d)

$$\vec{S} = \ (\vec{E} \times \ \vec{B})/\mu_0$$

$\vec{S}$ is known as poynting vector and represents intensity of electromagnetic waves

$$\left[\vec{S}\right] = \left[MT^{-3}\right] = \left[\frac{Power}{Area}\right] = \left[\frac{Force}{Length \times Time}\right]$$

Question 5: A heavy nucleus N, at rest, undergoes fission N → P + Q, where P and Q are two lighter nuclei. Let δ = $M_N - M_P - M_Q$, where M_P, M_Q and M_N are the masses of P, Q and N, respectively. E_P and E_Q are the kinetic energies of P and Q, respectively. The speeds of P and Q are V_P and V_Q, respectively. If c is the speed of light, which of the following statement(s) is(are) correct?

a. $E_P + E_Q = c^2\delta$

b. $E_p = \left(\frac{M_P}{M_P+M_Q}\right) c^2 \delta$

c. $\frac{V_P}{V_Q} = \frac{M_Q}{M_P}$

d. The magnitude of momentum for P as well as Q is $c\sqrt{2\mu\delta}$, where $\mu = \frac{M_P M_Q}{M_P+M_Q}$

Solution:

Answer: (a, c, d)

$E_P + E_Q = \delta c^2$ (Q-value of nuclear reaction)

$$\sqrt{2M_P E_P} = \sqrt{2M_Q E_Q}$$

$M_P V_P = M_Q V_Q$

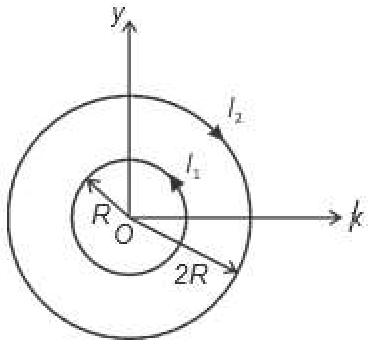
$$\Rightarrow \frac{E_P}{E_Q} = \frac{M_Q}{M_P}$$

$$\Rightarrow E_P = \frac{M_Q}{M_P+M_Q}\delta c^2$$

⇒ Momentum of P or

$$Q = \sqrt{\frac{2M_P M_Q}{M_P+M_Q}\delta c^2}$$

Question 6: Two concentric circular loops, one of radius R and the other of radius 2R lie in the xy-plane with the origin as their common centre, as shown in the figure. The smaller loop carries current I_1 in the anti-clockwise direction and the larger loop carries current I_2 in the clockwise direction, with $I_2 > 2I_1$. $\vec{B}(x, y)$ denotes the magnetic field at a point (x, y) in the xy-plane. Which of the following statement(s) is(are) correct?



a. $\vec{B}(x, y)$ is perpendicular to the xy-plane at any point in the plane

b. $\left|\vec{B}(x, y)\right|$ depends on x and y only through the radial distance $r = \sqrt{x^2 + y^2}$

c. $\left|\vec{B}(x, y)\right|$ is non-zero at all points for r < R

d. $\vec{B}(x, y)$ points normally outward from the xy-plane for all the points between the two loops

Solution:

Answer: (a, b)

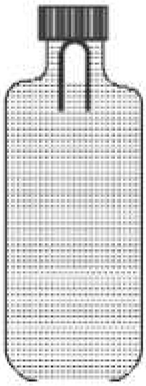
A magnetic field due to a circular loop at any point in its plane will be perpendicular to the plane. Due to symmetry, it will depend only on the distance from the centre. The field will be in opposite direction inside and outside the loop. The field may be non-zero for r < R, as it is in opposite direction due to both the loops.

Question Stem for Question Nos. 7 and 8

A soft plastic bottle, filled with water of density 1 gm/cc, carries an inverted glass test tube with some air (ideal gas) trapped as shown in the figure. The test tube has a mass of 5 gm, and it is made of a thick glass of density 2.5 gm/cc. Initially, the bottle is sealed at atmospheric pressure $p_0 = 10^5$ Pa so that the volume of the trapped air is $V_0 = 3.3$ cc. When

the bottle is squeezed from outside at a constant temperature, the pressure inside rises and the volume of the trapped air reduces. It is found that the test tube begins to sink at pressure $p_0 + \Delta p$ without changing its orientation. At this pressure, the volume of the trapped air is $V_0 - \Delta V$.

Let ΔV = X cc and $\Delta p = Y \times 10^3$ Pa.



Question 7: The value of X is ________.

Solution:

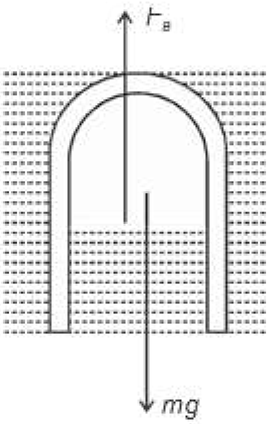
Answer: (0.30)

Question 8: The value of Y is ________.

Solution:

Answer: (10.00)

Solution of Q. Nos. 7 & 8



When buoyant force on (tube + air) system will become equal to the weight of the tube then the tube will start sinking. (Here we can neglect weight of air as compared to weight of tube)

Now, Let volume of air in this case = V_{air}

$F_B = mg$

So, $\delta_w (V_{tube} + V_{air})\, g = mg$

$$\Rightarrow 1\left(\frac{5}{2.5}cm^3 + V_{air}\right) = 5$$

$\Rightarrow 2 + V_{air} = 5$

$V_{air} = 3\ cm^3$

As initial volume of air = 3.3 cm^3

So, $\Delta V = 0.3$ cc

So, X = 0.30

As temperature of air is constant

So, PV = constant

$P_0\ 3.3 = P_f\ 3$, P_f is final pressure of air

$\Rightarrow P_f = 1.1\ P_0 = P_0 + 0.1\ P_0$

So, $\Delta P = 10^4$ Pa

So, Y = 10

So, X = 0.30

Y = 10.00

Question Stem for Question Nos. 9 and 10

A pendulum consists of a bob of mass m = 0.1 kg and a massless inextensible string of length L = 1.0 m. It is suspended from a fixed point at height H = 0.9 m above a frictionless horizontal floor. Initially, the bob of the pendulum is lying on the floor at rest vertically below the point of suspension. A horizontal impulse P = 0.2 kg-m/s is imparted to the bob at some instant. After the bob slides for some distance, the string becomes taut and the bob lifts off the floor. The magnitude of the angular momentum of the pendulum about the point of suspension just before the bob lifts off is J kg-m^2/s. The kinetic energy of the pendulum just after the lift-off is K Joules.

Question 9: The value of J is _________.

Solution:

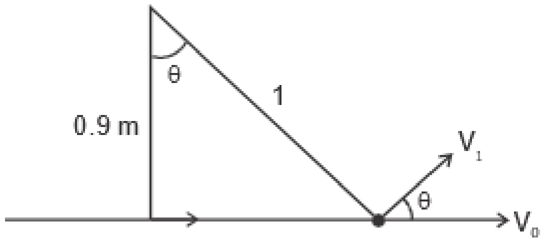
Answer: (0.18)

Question 10: The value of K is _________.

Solution:

Answer: (0.16)

Solution of Q. Nos. 9 and 10



V_0 =0.2/0.1 =2 m/s

9. L = P × H

= 0.2 × 0.9 = 0.18 kg m^2/s

10. V_1=V_0 cos θ = 2 x (0.9/1)

K = (½) x (0.1) x (2 x 0.9)^2

= 0.162 Joules

Question Stem for Question Nos. 11 and 12

In a circuit, a metal filament lamp is connected in series with a capacitor of capacitance C μF across a 200 V, 50 Hz supply. The power consumed by the lamp is 500 W while the voltage drop across it is 100 V. Assume that there is no inductive load in the circuit. Take rms values of the voltages. The magnitude of the phase angle (in degrees) between the current and the supply voltage is ϕ. Assume, π√3 = 5.

Question 11: The value of C is ___.

Solution:

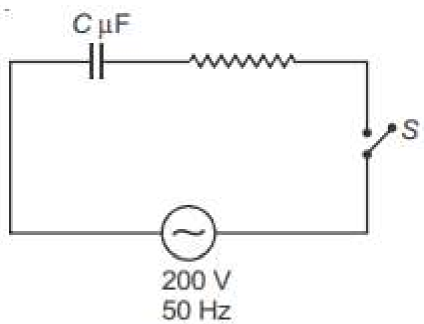
Answer: (100)

Question 12: The value of ϕ is ___.

Solution:

Answer: (60)

Solution of Q. Nos. 11 & 12



P = V^2/2

$\Rightarrow 500 = 100^2/R$

$\Rightarrow R = 20\ \Omega$

Now across resistance $500 = I \times 100$

$\Rightarrow I_{rms} = 5$ A

$V_{rms} = 200$ V,

$V_{rms/real} = 100$ V

$\cos\varphi = 100/200 = ½ \Rightarrow \Phi = 60^0$

$\tan\Phi = X_C/R = 1/\omega RC$

$\sqrt{3} = 1/100\pi(20)C$

$C = 1/(20\pi\sqrt{3} \times 100)$

$= 10^{-4}$ F

$= 100\ \mu F$

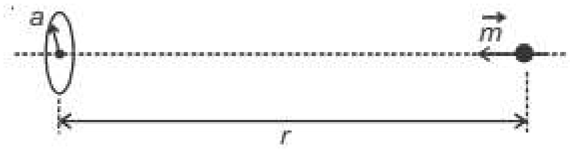
Question Stem for Question Nos. 13 and 14

A special metal S conducts electricity without any resistance. A closed wire loop, made of S, does not allow any change in flux through itself by inducing a suitable current to generate a compensating flux. The induced current in the loop cannot decay due to its zero resistance. This current gives rise to a magnetic moment which in turn repels the source of magnetic field or flux. Consider such a loop, of radius a, with its centre at the origin. A magnetic dipole of moment m is brought along the axis of this loop from infinity to a point at distance r (>> a) from the centre of the loop with its north pole always facing the loop, as shown in the figure below.

The magnitude of the magnetic field of a dipole m, at a point on its axis at distance r, is

$\frac{\mu_0 m}{2\Pi r^3}$ **, where μ_0 is the permeability of free space. The magnitude of the force between two**

magnetic dipoles with moments, m_1 and m_2, separated by a distance r on the common axis, with their north poles facing each other, is km_1m_2/r^4, where k is a constant of appropriate dimensions. The direction of this force is along the line joining the two dipoles.



Question 13: When the dipole m is placed at a distance r from the centre of the loop (as shown in the figure), the current induced in the loop will be proportional to?

a. m/r^3
b. m^2 /r^2
c. m/r^2
d. m^2 /r

Solution:

Answer: (a)

Magnetic flux due to dipole through ring = $\frac{\mu_0}{2\Pi} \times \frac{m}{r^3} \times \Pi a^2$ for net magnetic flux through the loop to be zero.

Magnetic flux due to dipole = Magnetic flux due to induced current

⇒ $\frac{\mu_0}{2\Pi} \times \Pi a^2 \times \frac{m}{r^3} = l \times \Pi a^2 \times \frac{k}{a}$, where k is proportionality constant.

⇒ I∝ m/r^3

Question 14: The work done in bringing the dipole from infinity to a distance r from the centre of the loop by the given process is proportional to?

a. m/r^5
b. m^2 /r^5
c. m^2 /r^6
d. m^2 /r^7

Solution:

Answer: (c)

$$F = \frac{km_1m_2}{r^4} = k(l\pi a^2)\left(\frac{m}{r^4}\right)$$

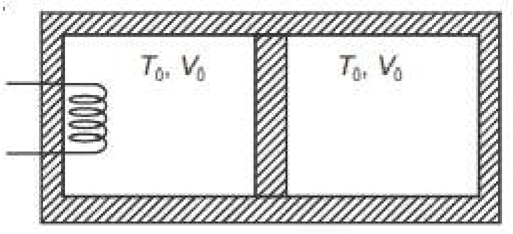
F = C(m^2/r^7) where C is a constant obtained by substituting the value of I from Q.13

$|W| = \int_{\infty}^{r} Fdr = Cm^2 \int_{\infty}^{r} \frac{dr}{r^7} = \frac{C'm^2}{r^6}$ where C' is a constant

$$|W| \propto \frac{m^2}{r^6}$$

Question Stem for Question Nos. 15 and 16

A thermally insulating cylinder has a thermally insulating and frictionless movable partition in the middle, as shown in the figure below. On each side of the partition, there is one mole of an ideal gas, with specific heat at constant volume, C_V = 2R. Here, R is the gas constant. Initially, each side has a volume V_0 and temperature T_0. The left side has an electric heater, which is turned on at very low power to transfer heat Q to the gas on the left side. As a result, the partition moves slowly towards the right, reducing the right side volume to V_0 /2. Consequently, the gas temperatures on the left and the right sides become T_L and T_R, respectively. Ignore the changes in the temperatures of the cylinder, heater and partition.



Question 15: The value of T_R/T_0 is

a. √2
b. √3
c. 2
d. 3

Solution:

Answer: (a)

$PV^\gamma = C$

$$\Rightarrow TV^{\gamma-1} = C$$

$$\Rightarrow T_0 V_0^{\gamma-1} = T_R\left(\frac{V_0}{2}\right)^{\gamma-1}$$

$$C_V = \frac{R}{\gamma-1}$$

$$\Rightarrow 2R = \frac{R}{\gamma-1}$$

$$\gamma - 1 = \frac{1}{2}$$

$$\gamma = \frac{3}{2}$$

$$\Rightarrow \frac{T_R}{T_0} = 2^{\gamma-1} = \sqrt{2}$$

Question 16: The value of Q/RT_0 is

a. 4(2√2 +1)
b. 4(2√2 -1)
c. (5√2 +1)
d. (5√2 -1)

Solution:

Answer: (b)

$Q = \Delta U_1 + \Delta U_2$

$\Delta U_1 = C_V\Delta T_1 = 2R(T_L - T_0)$

$\Delta U_2 = C_V\Delta T_2 = 2R(T_R - T_0)$

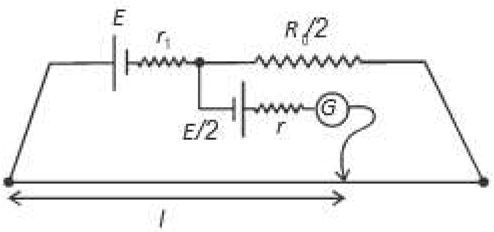
$T_L = 3\sqrt{2}T_0$, $T_R = \sqrt{2}T_0$

$Q = 2R[3\sqrt{2} -1]T_0 + 2R(\sqrt{2} -1]T_0$

$Q = 4RT_0[2\sqrt{2} -1]$

$\Rightarrow Q/RT_0 = 4[2\sqrt{2} -1]$

Question 17:In order to measure the internal resistance r_1 of a cell of emf E, a meter bridge of wire resistance R_0 = 50 Ω, a resistance $R_0/2$, another cell of emf E/2 (internal resistance r) and a galvanometer G are used in a circuit, as shown in the figure. If the null point is found at l = 72 cm, then the value of r_1 = ___ Ω.



Solution:

Answer: (3)

Current will flow in the main circuit

$$I = \frac{E}{r_1 + \frac{3R_0}{2}}$$

$$+E - IR_0 \times 0.72 - Ir_1 - \frac{E}{2} = 0$$

$$\frac{E}{2} = \frac{2E}{2r_1+3R_0} \times [0.72R_0 + r_1]$$

$$2r_1 + 3R_0 = 4[0.72R_0 + r_1]$$

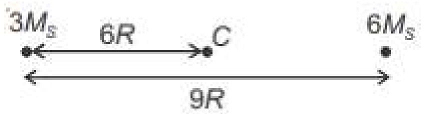
$0.12R_0 = 2r_1$

$r_1 = 3\Omega$

Question 18:The distance between two stars of masses $3M_S$ and $6M_S$ is 9R. Here R is the mean distance between the centres of the Earth and the Sun, and MS is the mass of the Sun. The two stars orbit around their common centre of mass in circular orbits with period nT, where T is the period of Earth's revolution around the Sun. The value of n is ___.

Solution:

Answer: (9)



Centre of mass of system lies at 6R from lighter mass

$$\left[3M_s\omega^2 \times 6R\right] = \frac{G(18M_s^2)}{81R^2}$$

$$\omega^2 R = \frac{GM}{81R^2}$$

$$T' = \sqrt{\frac{81R^3}{GM_s}}$$

T' = 9T

n = 09

Question 19: In a photoemission experiment, the maximum kinetic energies of photoelectrons from metals P, Q and R are E_P, E_Q and E_R, respectively, and they are related by $E_P = 2E_Q = 2E_R$. In this experiment, the same source of monochromatic light is used for

metals P and Q while a different source of monochromatic light is used for metal R. The work functions for metals P, Q and R are 4.0 eV, 4.5 eV and 5.5 eV, respectively. The energy of the incident photon used for metal R, in eV, is ___.

Solution:

Answer: (6)

$$\frac{hc}{\lambda_1} = \phi_p + E_p$$

$$\frac{hc}{\lambda_1} = \phi_Q + E_Q$$

$E_p = 2E_Q$

$E_P - E_Q = 0.5$

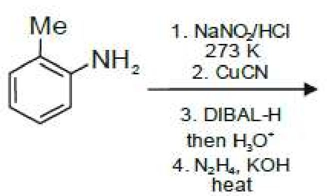
$\Rightarrow E_p = 1.0$ eV, $E_Q = 0.5$ eV

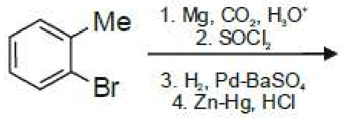
$E_R = 0.5$ eV

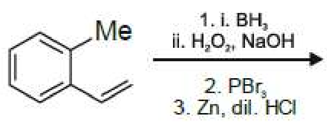
Energy of incident photon on R = $\varphi_R + E_R = 6$ eV

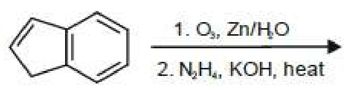
JEE Advanced 2021 Paper 2 Chemistry Question Paper

Question 1. The reaction sequence(s) that would lead to o-xylene as the major product is(are).

(a) 

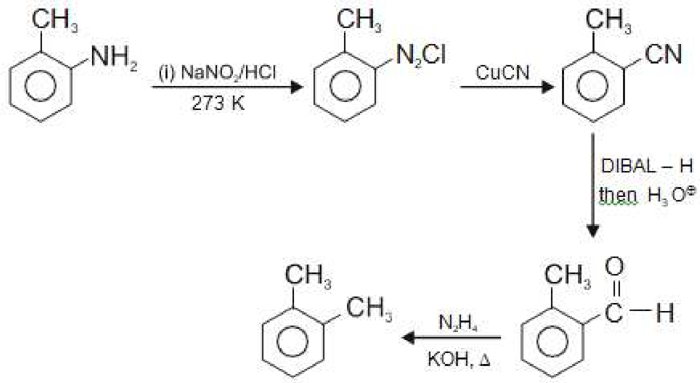
(b) 

(c) 

(d) 

Solution:

Answer: (a, b)

(a)


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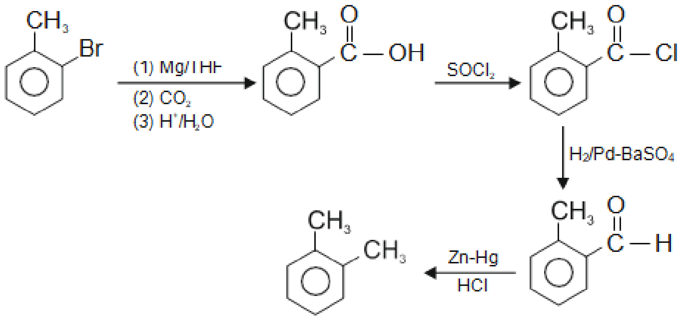

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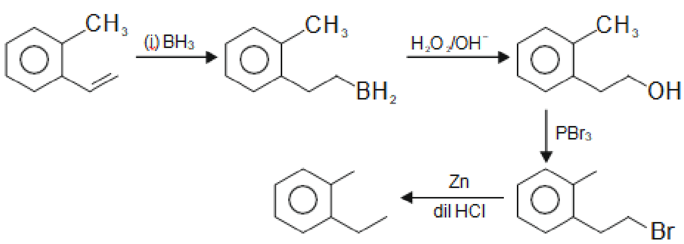

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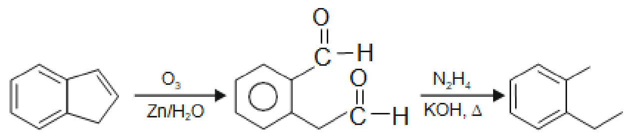
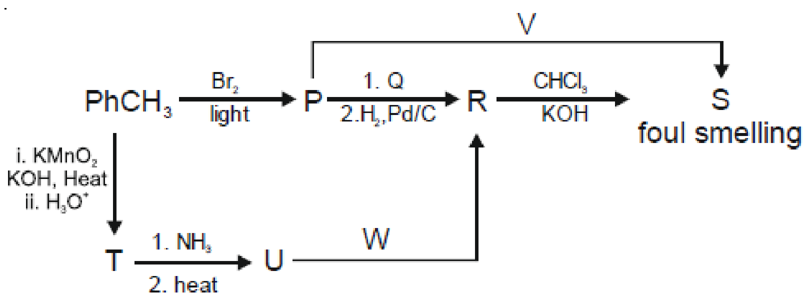


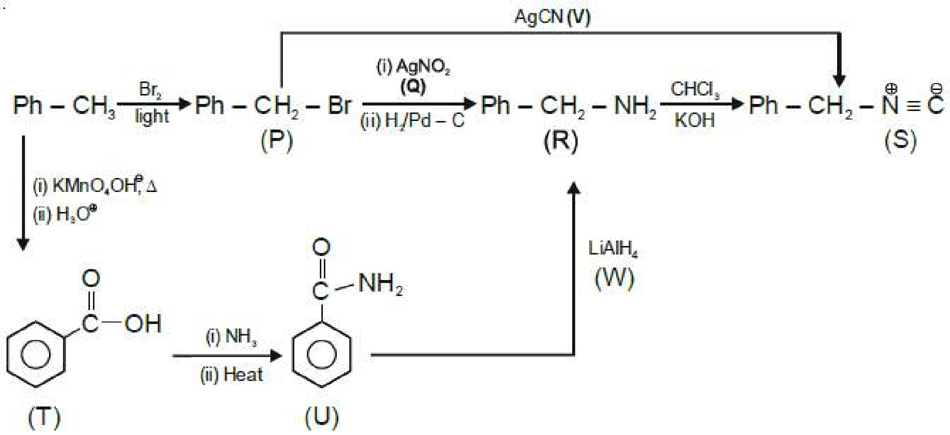
Question 2. Correct option(s) for the following sequence of reactions is(are)



a. Q = KNO2, W = LiAlH4
b. R = benzenamine, V = KCN
c. Q = AgNO2, R = phenylmethanamine
d. W = LiAlH4, V = AgCN

Solution:

Answer: (c, d)



Therefore, correct options are

Q = $AgNO_2$, R = phenylmethanamine

W = $LiAlH_4$, V = AgCN

Question 3. For the following reaction;

$$2X + Y \xrightarrow{k} P$$

The rate of reaction is $\frac{d[P]}{dt} = k[X]$. Two moles of X are mixed with one mole of Y to make 1.0 L of solution. At 50 s, 0.5 mole of Y is left in the reaction mixture. The correct statement(s) about the reaction is(are). (Use: ln 2 = 0.693)

a. The rate constant, k, of the reaction is $13.86 \times 10^{-4}\ s^{-1}$.

a. The rate constant, k, of the reaction is $13.86 \times 10^{-4}\ s^{-1}$.
b. Half-life of X is 50 s.
c. At 50 s, $-d[X]/dt = 13.86 \times 10^{-3}\ mol\ L^{-1}\ s^{-1}$.
d. At 100 s, $d[Y]/dt = 3.46 \times 10^{-3}\ mol\ L^{-1}\ s^{-1}$.

Solution:

Answer: (b, c, d)

rate = $\frac{d[P]}{dt} = k[X]$

2X + Y → P

2 mole 1 mole

1 mole 0.5 mole 0.5 mole

$-d[X]/dt = k_1[X] = 2k[X] \Rightarrow 2k = k_1$

$-d[Y]/dt = k_2[X] = 2k[X] \Rightarrow k_2 = k$

2K = 1/50 In2

K = 1 / 100 In2 = 0.6 93 / 100 = 6.93 × 10^{-3} × s^{-1} = 50 sec

At 50 sec,

d[X] / dt = 2k[X] = 2 × 0.693 / 100 × 1

= 13.86 × 10^{-3} mol L^{-1} s^{-1}

At 100 sec

-d[Y] / dt = k_2[X] = k[X] × 0.693 / 100 × ½

(Concentration of X after 2 half-lives = ½ M)

= 3.46 × 10^{-3} mol L^{-1} s^{-1}

Question 4. Some standard electrode potentials at 298 K are given below:

Pb^{2+}/Pb −0.13 V

Ni^{2+}/Ni −0.24 V

C^{2+}/Cd −0.40 V

Fe^{2+}/Fe −0.44 V

To a solution containing 0.001 M of X^{2+} and 0.1 M of Y^{2+}, the metal rods X and Y are inserted (at 298 K) and connected by a conducting wire. This resulted in the dissolution of X.

The correct combination(s) of X and Y, respectively, is(are)

(Given: Gas constant, R = 8.314 J K^{-1} mol^{-1}, Faraday constant, F = 96500 C mol^{-1})

a. Cd and Ni
b. Cd and Fe
c. Ni and Pb
d. Ni and Fe

Solution:

Answer: (a, b, c)

X + Y^{2+} X^{2+} + Y

$$E = E^0 - \frac{0.06}{2} log_{10}\left(\frac{10^{-3}}{10^{-1}}\right)$$

E = E^o + 0.06

(a) E° = −(− .4) + (− .24) = .16 > 0

(b) E° = −(− .4) + (− .44) = −.04 < 0 and E_{cell} = −0.04 + 0.06 = +0.02 > 0

(c) E° = −(− .24) + (− .13) = .11 > 0

(d) $E° = -(-.24) + (-.44) = -.2 < 0$

$\therefore E_{cell} = -0.2 + 0.06 = -0.14 < 0$

$\therefore$ If $E_{cell} > 0$ then the cell construction is possible.

Question 5. The pair(s) of complexes wherein both exhibit tetrahedral geometry is(are) (Note: py = pyridine, Given: Atomic numbers of Fe, Co, Ni and Cu are 26, 27, 28 and 29, respectively)

a. $[FeCl4]^-$ and $[Fe(CO)4]^{2-}$
b. $[Co(CO)4]^-$ and $[CoCl4]^{2-}$
c. $[Ni(CO)4]$ and $[Ni(CN)4]^{2-}$
d. $[Cu(py)4]^+$ and $[Cu(CN)4]^{3-}$

Solution:

Answer: (a, b, d)

$[FeCl4]^- \rightarrow Fe^{3+}$, $3d^5$ (weak field ligand) = sp^3

$[Fe(CO)4]^{-2} \rightarrow Fe^{2-}$, $3d^{10} \rightarrow sp^3$

$[Co(CO)4]^- \rightarrow Co^-$, $3d^{10} \rightarrow sp^3$

$[CoCl4]^{2-} \rightarrow Co^{2+}$, $3d^7$ (weak field ligand) $\rightarrow sp^3$

$[Ni(CO)4] \rightarrow Ni$, $3d^{10} \rightarrow sp^3$

$[Ni(CN)4]^{2-} \rightarrow Ni^{2+}$, $3d^8$ (strong field ligand) $\rightarrow dsp^2$

$[Cu(py)4]^+ \rightarrow Cu^+$, $3d^{10} \rightarrow sp^3$

$[Cu(CN)4]^{3-} \rightarrow Cu^+$, $3d^{10} \rightarrow sp^3$

In $3d^{10}$ electronic configuration, only sp^3 hybridisation and tetrahedral geometry are possible.

Question 6. The correct statement(s) related to oxoacids of phosphorous is(are).

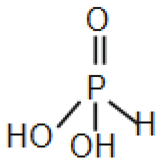
a. Upon heating, H_3PO_3 undergoes a disproportionation reaction to produce H_3PO_4 and PH_3.
b. While H_3PO_3 can act as a reducing agent, H_3PO_4 cannot.
c. H_3PO_3 is a monobasic acid.
d. The H atom of the P-H bond in H_3PO_3 is not ionizable in water.

Solution:

Answer: (a, b, d)

$$4H_3PO_3 \xrightarrow{\Delta} PH_3 + 3H_3PO_4$$

In H_3PO_4, phosphorous is present in the highest oxidation state, i.e., +5. So H3PO4 cannot act as a reducing agent. Structure of H_3PO_3,



It is a dibasic acid.

H atom present in the P–H bond is not ionizable.

These P-H bonds are not ionisable to give H^+ and do not play any role in basicity. Only those H atoms which are attached with oxygen in P-OH form are ionisable and cause the basicity. Thus, H_3PO_3 and H_3PO_4 are dibasic and tribasic, respectively as the structure of H_3PO_3 has two P – OH bonds and H_3PO_4 three.

Question Statement for Questions 7 and 8.

At 298 K, the limiting molar conductivity of a weak monobasic acid is 4×10^2 S cm^2 mol^{-1}. At 298 K, for an aqueous solution of the acid, the degree of dissociation is a and the molar conductivity is $y \times 10^2$ S cm^2 mol^{-1}. At 298 K, upon 20 times dilution with water, the molar conductivity of the solution becomes $3y \times 10^2$ S cm^2 mol^{-1}.

Question 7. The value of α is ______.

Solution:

Answer: (0.215)

Question 8. The value of y is ______.

Solution:

Answer: (0.86)

Solution for Questions 7 and 8.

Molar conductivity of HX at infinite dilution

Λ_m^∞ = 4×10^2 S cm^2 mol^{-1}

Molar conductivity of HX at conc. $c_1 = y \times 10^2$ S cm^2 mol^{-1}

$$\alpha_1 = \frac{\Lambda_m^{c_1}}{\Lambda_m^{\infty}} = \frac{y \times 10^2}{4 \times 10^2} = \frac{y}{4}$$

On 20 times dilution of the solution of HX

$$\alpha_2 = \frac{\Lambda_m^{c_2}}{\Lambda_m^{\infty}} = \frac{3y \times 10^2}{4 \times 10^2} = \frac{3y}{4} \qquad \left[c_2 = \frac{c_1}{20}\right]$$

$$\frac{\alpha_1}{\alpha_2} = \frac{1}{3} \qquad \Rightarrow \alpha_2 = 3\alpha_1$$

$$\begin{array}{cccc} HX & \rightleftharpoons & H^+ & + \ X^- \\ c_1(1-\alpha_1) & & c_1\alpha_1 & c_1\alpha_1 \end{array}$$

$$K_a = \frac{c_1\alpha_1^2}{1-\alpha_1} = \frac{c_2\alpha_2^2}{1-\alpha_2} = \frac{c_1(3\alpha_1)^2}{20(1-3\alpha_1)}$$

$$\frac{1}{1-\alpha_1} = \frac{9}{20(1-3\alpha_1)}$$

$20 - 60\alpha_1 = 9 - 9\alpha_1$

$\Rightarrow \alpha_1 = 11/51 = 0.215$

$Y = 4\alpha_1 = 0.086$

Question Statement for Questions 9 and 10.

The reaction of x g of Sn with HCl quantitatively produced a salt. The entire amount of the salt reacted with y g of nitrobenzene in the presence of the required amount of HCl to produce 1.29 g of an organic salt (quantitatively).

(Use Molar masses (in g mol^{-1}) of H, C, N, O, Cl and Sn as 1, 12, 14, 16, 35 and 119, respectively).

Question 9. The value of x is _______.

Solution:

Answer: (3.57)

$Sn + HCl \rightarrow SnCl_2$

$\Rightarrow$ Moles of ammonium salt = 1.29 / 129 = 0.01

$\Rightarrow$ Moles of nitrobenzene = 0.01

No. of eq. of nitrobenzene = No. of eq. of $SnCl_2$

$6 \times (0.01) = 2 \times n_{SnCl_2}$

$n_{SnCl_2} = 0.03$

$\Rightarrow n_{Sn} = 0.03$

$w_{Sn} = 0.03 \times 119$

x = 3.57

Question 10. The value of y is ________.

Colution:

Answer: (1.23)

Solution of Question Nos. 9 and 10

$Sn + HCl \rightarrow SnCl_2$

⇒ Moles of ammonium salt = 1.29 / 129 = 0.01

⇒ Moles of nitrobenzene = 0.01

⇒ y = 0.01 × Molar mass of nitrobenzene

= 0.01 × 123

= y = 1.23

Question Statement for Questions 11 and 12.

A sample (5.6 g) containing iron is completely dissolved in cold dilute HCl to prepare a 250 mL of solution. Titration of 25.0 mL of this solution requires 12.5 mL of 0.03 M KMnO4 solution to reach the endpoint. Number of moles of Fe^{2+} present in 250 mL solution is $x \times 10^{-2}$ (consider complete dissolution of FeCl2). The amount of iron present in the sample is y% by weight.

(Assume: $KMnO_4$ reacts only with Fe^{2+} in the solution Use: Molar mass of iron as 56 g mol^{-1})

Question 11. The value of x is ________.

a.swer: (1.875)

Solution:

Question 12. The value of y is ________.

Solution:

Answer: (18.75)

Solution of Question Nos. 11 and 12

$8H^+ + 5Fe^{2+} + MnO^{-} \rightarrow 5Fe^{3+} + Mn^{2+} + 4H_2O$

For 25 ml,

meq of Fe^{2+} = meq of MnO^{-}

$= 12.5 \times 0.03 \times 5$

For 250 ml,

mmoles of $Fe^{2+} = 12.5 \times 0.03 \times 5 \times 250 / 25$

moles of $Fe^{2+} = 18.75 / 1000$ mol

$= 18.75 \times 10^{-3}$ mol

$= 1.875 \times 10^{-2}$ mol

$x = 1.875$

Weight of $Fe^{2+} = 1.875 \times 10^{-2} \times 56 = 1.05$ g

% purity of Fe^{2+} $y = 18.75\%$

$= 1.05 / 5.6 \times 100$

Statement: The amount of energy required to break a bond is the same as the amount of energy released when the same bond is formed. In a gaseous state, the energy required for homolytic cleavage of a bond is called Bond Dissociation Energy (BDE) or Bond Strength. BDE is affected by the s-character of the bond and the stability of the radicals formed. Shorter bonds are typically stronger bonds. BDEs for some bonds are given below:

$$H_3C - H(g) \longrightarrow H_3\dot{C}(g) + \dot{H}(g) \quad \Delta H^\circ = 105 \text{ kcal mol}^{-1}$$

$$Cl - Cl(g) \longrightarrow \dot{Cl}(g) + \dot{Cl}(g) \quad \Delta H^\circ = 58 \text{ kcal mol}^{-1}$$

$$H_3C - Cl(g) \longrightarrow H_3\dot{C}(g) + \dot{Cl}(g) \quad \Delta H^\circ = 85 \text{ kcal mol}^{-1}$$

$$H - Cl(g) \longrightarrow \dot{H}(g) + \dot{Cl}(g) \quad \Delta H^\circ = 103 \text{ kcal mol}^{-1}$$

Question 13. The correct match of the C-H bonds (shown in bold) in Column J with their BDE in Column K is;

Column J	**Column K**
Molecule	**BDE (kcal mol^{-1})**
(P) H-CH(CH3)2	**(i) 132**

(Q) H-CH2Ph	**(ii) 110**
(R) H-CH=CH2	**(iii) 95**
(S) H-C°CH	**(iv) 88**

a. P – iii, Q – iv, R – ii, S – i
b. P – i, Q – ii, R – iii, S – iv
c. P – iii, Q – ii, R – i, S – iv
d. P – ii, Q – i, R – iv, S – iii

Solution:

Answer: (a)

$$H-CH(CH_3)_2 \longrightarrow H\dot{C}(CH_3)-CH_3$$

$$H-CH_2Ph \longrightarrow {}^{\bullet}CH_2-Ph \quad \text{(Benzyl radical)}$$

$$H-CH=CH_2 \rightarrow {}^{\bullet}CH=CH_2$$

$$H-C\equiv CH \rightarrow {}^{\bullet}C\equiv CH$$

Order of stability of free radical

Q > P > R > S

Stability of free radical α 1 / Bond energy

∴ Order of bond energy :

S > R > P > Q

Question 14. For the following reaction,

$$CH_4(g) + Cl_2(g) \overset{light}{\rightarrow} CH_3Cl(g) + HCl(g)$$

the correct statement is;

a. Initiation step is exothermic with DH° = -58 kcal mol^{-1}
b. Propagation step involving $^{\cdot}CH_3$ formation is exothermic with DH° = -2 kcal mol^{-1}
c. Propagation step involving CH_3Cl formation is endothermic with DH° = +27 kcal mol^{-1}
d. The reaction is exothermic with DH° = -25 kcal mol^{-1}

Solution:

Answer: (d)

(1) $Cl_2 \rightarrow 2Cl^{\bullet}$ (Initiation step) $\Delta H = 58$ kcal/mol

(2) $CH_4 + Cl^{\bullet} \rightarrow {}^{\bullet}CH_3 + HCl$

(3) ${}^{\bullet}CH_3 + Cl_2 \rightarrow CH_3Cl + Cl^{\bullet}$

$\left.\right]$ Propagation step

Step (1) → Endothermic (bond breaking)

Step (2) → ΔH = 105 – 103 = 2 kcal/mol (Endothermic)

Step (3) → ΔH = 58 – 85 = –27 kcal/mol (Exothermic)

For complete reaction

$$CH_4(g) + \ Cl_2(g) \overset{light}{\rightarrow} \ CH_3Cl(g) + \ HCl(g)$$

ΔH = 58 + 105 – 85 – 103

= –25 kcal/mol

Question Statement for Questions 15 and 16.

The reaction of $K_3[Fe(CN)_6]$ with freshly prepared $FeSO_4$ solution produces a dark blue precipitate called Turnbull's blue. The reaction of $K_4[Fe(CN)_6]$ with the $FeSO_4$ solution in the complete absence of air produces a white precipitate X, which turns blue in the air. Mixing the $FeSO_4$ solution with $NaNO_3$, followed by slow addition of concentrated H_2SO_4 through the side of the test tube produces a brown ring.

Question 15. Precipitate X is

a. $Fe_4[Fe(CN)_6]_3$
b. $Fe_4[Fe(CN)_6]$
c. $K_2Fe[Fe(CN)_6]$
d. $KFe[Fe(CN)_6]$

Solution:

Answer: (c)

Question 16. Among the following, the brown ring is due to the formation of

a. $[Fe(NO)2(SO4)2]^{2-}$
b. $[Fe(NO)2(H2O)4]^{3+}$
c. [Fe(NO)4(SO4)2]
d. $[Fe(NO)(H2O)5]^{2}$

Solution:

Answer: (d)

Solution of Question Nos. 15 and 16

$$Fe^{2+} + K_3[Fe(CN)_6] \rightarrow \underset{\text{Turnbull's blue ppt.}}{Fe_3[Fe(CN)_6]_2} \downarrow$$

$$Fe^{2+} + K_4[Fe(CN)_6] \xrightarrow[\text{of air}]{\text{in absence}} \underset{\text{White ppt. (X)}}{K_2Fe[Fe(CN)_6]} \downarrow$$

In air Fe^{2+} gets oxidised to Fe^{3+}

$$Fe^{3+} + [Fe(CN)_6]^{4-} \longrightarrow \underset{\text{Prussian blue}}{Fe_4[Fe(CN)_6]_3} \downarrow$$

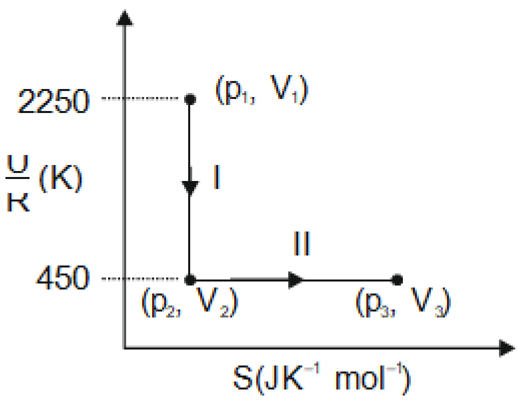
$$2NO_3^- + 4H_2SO_4 + 6Fe^{2+} \rightarrow 6Fe^{3+} + 2NO\uparrow + 4SO_4^{2-} + 4H_2O$$

$$[Fe(H_2O)_6]^{2+} + NO \longrightarrow \underset{\text{Compound responsible for brown ring}}{[Fe(H_2O)_5NO]^{2+}} + H_2O$$

$\therefore$ $X = K_2Fe[Fe(CN)_6]$

Brown ring is due to $[Fe(H_2O)_5NO]^{2+}$

Question 17. One mole of an ideal gas at 900 K, undergoes two reversible processes, I followed by II, as shown below. If the work done by the gas in the two processes are the same, the value of $In\frac{v_3}{v_2}$ is ____.



(U: internal energy, S: entropy, p: pressure, V: volume, R: gas constant)

(Given: molar heat capacity at constant volume, C of the gas is 5 / 2 R)

Solution:

Answer: (10)

Process I is adiabatic reversible

Process II is a reversible isothermal process

Process I - (Adiabatic Reversible)

$\Delta U / R = 450 - 2250$

$\Delta U = -1800\ R$

$W_I = \Delta U = -1800R$

Process II - (Reversible Isothermal Process)

T1 = 900 K

Calculation of T_2 after the reversible adiabatic process

$-1800R = nC_v(T_2 - T_1)$

$-1800R\ 1 \times 5/2\ R(T_2 - 900)$

$T_2 = 180$ K

$W_{II} = -nRT_2 \ln = W$

$-1 \times R \times 180 \ln v_3 / v_2 -1800R$

$\ln v_3 / v_2 10$

Question 18. Consider a helium (He) atom that absorbs a photon of wavelength 330 nm. The change in the velocity (in cm s^{-1}) of the He atom after the photon absorption is_____.

(Assume: Momentum is conserved when the photon is absorbed.

Use: Planck constant = 6.6×10^{-34} J s, Avogadro number = 6 × 1023 mol^{-1}, Molar mass of He = 4 g mol^{-1})

Solution:

Answer: (30)

Momentum of photon = $\frac{h}{\lambda} = \frac{6.6\times10^{-27}}{330\times10^{-7}}$ gm cm s^{-1}

Momentum of 1 mole of He-atoms = $m\Delta v$

$\therefore m\Delta v = N_A \times h / \lambda$

$$4 \times \ \Delta v = \frac{6\times10^{23}\times6.6\times10^{-27}}{330\times10^{-7}}$$

$$\Delta v = \frac{6\times6.6\times10^{2}}{33\times4}$$ = 30 cm s^{-1}

$\therefore$ Change in velocity of He-atoms = 30 cm s^{-1}

Question 19. Ozonolysis of ClO2 produces oxide of chlorine. The average oxidation state of chlorine in this oxide is ____.

Solution:

Answer: (6)

ClO_2 contains an odd electron and is paramagnetic. It reacts with ozone to give O_2 and Cl_2O_6.

$2ClO_2 + 2O_3 \rightarrow Cl_2O_6 + 2O_2$

In Cl_2O_6, the average oxidation state of Cl is +6.

JEE Advanced 2021 Paper 2 Maths Question Paper

Question 1: Let;

$S_1 = \{(i, j, k) : i, j, k \in \{1,2,...,10\}\}$

$S_2 = \{(i, j) : 1 \leq i < j + 2 \leq 10, i, j \in \{1,2,...,10\}\}$

$S_3 = \{(i, j, k, l) : 1 \leq i < j < k < l, i, j, k, l \in \{1,2,...,10\}\}$

$S_4 = \{(i, j, k, l) :$ i, j, k and l are distinct elements in $\{1,2,...,10\}\}$.

If the total number of elements in the set S_r is n_r, r = 1, 2,3,4, then which of the following statements is (are) TRUE?

a. $n_1 = 1000$
b. $n_2 = 44$
c. $n_3 = 220$
d. $n_4/12 = 420$

Solution:

Answer: (a, b, d)

Number of elements in $S_1 = 10 \times 10 \times 10 = 1000$

Number of elements in $S_2 = 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 = 44$

Number of elements in $S_3 = {}^{10}C_4 = 210$

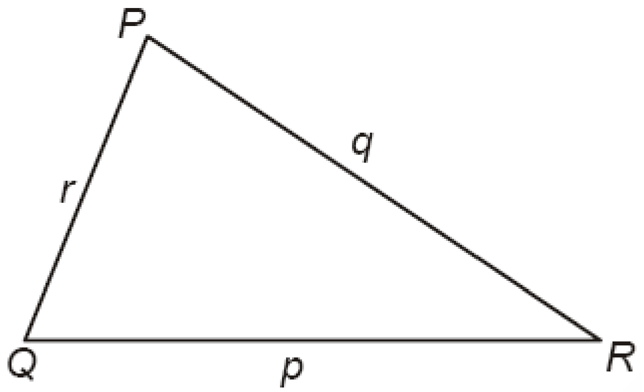
Number of elements in $S_4 = {}^{10}P_4 = 210 \times 4! = 5040$

Question 2: Consider a triangle PQR having sides of lengths p, q, and r opposite to the angles P, Q, and R, respectively. Then which of the following statements is (are) TRUE?

a. $\cos P \geq 1 - p^2/2qr$
b. $\cos R \geq ((q-r)/(p+q))\cos P + ((p-r)/(p+q))\cos Q$
c. $(q+r)/p < 2\sqrt{(\sin Q \sin R)}/\sin P$
d. if $p<q$ and $p<r$, then $\cos Q > p/r$ and $\cos R > p/q$

Solution:

Answer: (a, b)



(a) $\cos P = (q^2+r^2 - p^2)/2qr$

And $(q^2+r^2)/2 \geq \sqrt{(q^2 . r^2)}$ (AM ≥ GM)

$\Rightarrow (q^2+r^2) \geq 2qr$

So $\cos P \geq (2qr - p^2)/2qr$

$\cos P \geq 1 - p^2/2qr$

(b) $((q-r) \cos P + (p-r) \cos Q)/(p+q) = ((q \cos P + p \cos Q) - r(\cos P + \cos Q))/(p+q)$

$= r(1- \cos P - \cos Q)/(p+q)$

$= (r(q-p \cos R) - (p-q \cos R))/(p+q)$

$= ((r-p-q)+(p+q)\cos R)/(p+q)$

$= \cos R + (r-q-p)/(p+q) \leq \cos R$ (since $r < p+q$)

(c) $(q+r)/p = (\sin Q + \sin R)/\sin P \geq 2\sqrt{(\sin Q \sin R)}/\sin P$

(d) If $p < q$ and $q < r$

So, p is the smallest side, therefore one of Q or R can be obtuse

So, one of cos Q or cos R can be negative

Therefore, cos Q > p/r and cos R > p/q cannot hold always.

Question 3: Let f: [-π/2, π/2] → R be a continuous function such that f(0) = 1 and $\int_0^{\pi/3} f(t)\,dt = 0$. Then which of the following statements is (are) TRUE?

a. The equation f(x) - 3 cos 3x = 0 has at least one solution in (0, π/3)
b. The equation f(x) - 3 sin 3x = -6/π has at least one solution in (0, π/3)

c. $\lim_{x\to 0} \frac{x\int_0^x f(t)dt}{1-e^{x^2}} = -1$

d. $\lim_{x\to 0} \frac{\sin x\int_0^x f(t)dt}{x^2} = -1$

Solution:

Answer: (a, b, c)

f(0) = 1, $\int_0^{\pi/3} f(t)\,dt = 0$

(a) Consider a function g(x) = $\int_0^x f(t)dt$ - sin 3x. g(x) is continuous and differentiable function

And g(0) = 0

g(π/3) = 0

By Rolle's theorem g'(x) = 0 has at least one solution in (0, π/3)

f(x) - 3 cos 3x = 0 for some x ∈ (0, π/3)

(b) Consider a function

h(x) = $\int_0^x f(t)dt$ + cos 3x + 6x/π

h(x) is continuous and differentiable function and h(0) = 1

h(π/3) = 1

By Rolle's theorem h'(x) = 0 for at least one x ∈ (0, π/3)

f(x) - 3 sin 3x + 6/π = 0 for some x ∈ (0, π/3)

(c) $\lim_{x\to 0} \frac{x\int_0^x f(t)dt}{1-e^{x^2}}$

(0/0 form)

By L'Hospital rule

= $\lim_{x\to 0} \frac{xf(x)+\int_0^x f(t)dt}{-2xe^{x^2}}$, (0/0 form)

$= \lim_{x\to0} \frac{xf'(x)+f(x)+f(x)}{-4x^2e^{x^2}-2e^{x^2}}$

= (0+2f(0))/(0-2)

= -1

(d) $\lim_{x\to0} \frac{\sin x\int_0^x f(t)dt}{x^2}$, (0/0 form)

$= \lim_{x\to0} \frac{\sin x\ .f(x)+\cos x\int_0^x f(t)dt}{2x}$

$= \lim_{x\to0} \frac{\cos x\ .f(x)+\sin x.f'(x)+\cos x.f(x)-\sin x.\int_0^x f(t)dt}{2}$

= (1+0+1-0)/2

= 1

Question 4: For any real numbers α and β, let $y_{\alpha,\beta}(x)$, x ∈ R, be the solution of the differential equation dy/dx + αy = $xe^{\beta x}$, y(1) = 1. Let S = {$y_{\alpha,\beta}(x)$, α, β ∈ R } . Then which of the following functions belong(s) to the set S?

a. f(x) = $(x^2/2)e^{-x}$ + (e - ½)e^{-x}
b. f(x) = $(-x^2/2)e^{-x}$ + (e + ½)e^{-x}
c. f(x) = $(e^x/2)$(x-½) + (e - $e^2/4$)e^{-x}
d. f(x) = $(e^x/2)$(½ -x) + (e + $e^2/4$)e^{-x}

Solution:

Answer: (a, c)

dy/dx + αy = $xe^{\beta x}$

Integrating factor (I.F) = $e^{\int \alpha dx} = e^{\alpha x}$

So, the solution is y.$e^{\alpha x}$ = ∫$xe^{\beta x}$ $e^{\alpha x}$ dx

y.$e^{\alpha x}$ = ∫$xe^{(\beta+\alpha)x}$ dx

If α + β ≠ 0

$ye^{\alpha x}$ = x $e^{(\alpha+\beta)x}/(\alpha+\beta)$ - $e^{(\alpha+\beta)x}/(\alpha+\beta)^2$ + C

y = [$e^{\beta x}/(\alpha+\beta)$][x - 1/(α+β)] + $Ce^{-\alpha x}$ ...(i)

Put α = β = 1 in (i)

y = $(e^x/2)$(x - ½) + Ce^{-x}

y(1) = 1

$1 = (e/2)(½) + C/e$

$\Rightarrow C = e - e^2/4$

So, $y = (e^x/2)(x-½) + (e - e^2/4)e^{-x}$

If $\alpha + \beta = 0$ and $\alpha = 1$

$dy/dx + y = xe^{-x}$

$I.F = e^x$

$ye^x = \int x\, dx$

$ye^x = x^2/2 + C$

$y = e^{-x}x^2/2 + Ce^{-x}$

$y(1) = 1$

$1 = 1/2e + C/e$

$\Rightarrow C = e - ½$

$y = e^{-x}x^2/2 + (e - ½)e^{-x}$

Question 5: Let O be the origin and $\overrightarrow{OA} = 2\hat{i} + 2\hat{j} + \hat{k}$, $\overrightarrow{OB} = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\overrightarrow{OC} = \frac{1}{2}(\overrightarrow{OB} - \lambda\overrightarrow{OA})$ for some λ> 0. If $\left|\overrightarrow{OB} \times \overrightarrow{OC}\right| = \frac{9}{2}$, then which of the following statements is (are) TRUE ?

a. Projection of $\overrightarrow{OC}$ on $\overrightarrow{OA}$ is -3/2

b. Area of the triangle OAB is 9/2
c. Area of the triangle ABC is 9/2

d. The acute angle between the diagonals of the parallelogram with adjacent sides $\overrightarrow{OA}$ and $\overrightarrow{OC}$ is π/3

Solution:

Answer: (a, b, c)

$$\overrightarrow{OA} = 2\hat{i} + 2\hat{j} + \hat{k}$$

$$\overrightarrow{OB} = \hat{i} - 2\hat{j} + 2\hat{k}$$

$$\overrightarrow{OC} = \frac{1}{2}(\overrightarrow{OB} - \lambda\overrightarrow{OA})$$

$$\overrightarrow{OB} \times \overrightarrow{OC} = \overrightarrow{OB} \times \frac{1}{2}(\overrightarrow{OB} - \lambda\overrightarrow{OA})$$

= $\frac{-\lambda}{2}\overrightarrow{OB} \times \overrightarrow{OA} = \frac{\lambda}{2}(\overrightarrow{OA} \times \overrightarrow{OB})$

Now, $\overrightarrow{OA} \times \overrightarrow{OB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{vmatrix} = 6\hat{i} - 3\hat{j} - 6\hat{k}$

So, $\overrightarrow{OB} \times \overrightarrow{OC} = \frac{3\lambda}{2}(2\hat{i} - \hat{j} - 2\hat{k})$

$$\left|\overrightarrow{OB} \times \overrightarrow{OC}\right| = \left|\frac{9\lambda}{2}\right| = \frac{9}{2}$$

So, λ = 1 (since λ >0)

$$\overrightarrow{OC} = \frac{1}{2}(\overrightarrow{OB} - \overrightarrow{OA})$$

$$\overrightarrow{OC} = \frac{1}{2}(-\hat{i} - 4\hat{j} + \hat{k})$$

(a) Projection of vector OC on vector OA = $\frac{\overrightarrow{OC}.\overrightarrow{OA}}{\left|\overrightarrow{OA}\right|}$

= ½ (-2-8+1)/3

= -3/2

(b) Area of triangle OAB = $\frac{1}{2}\left|\overrightarrow{OA} \times \overrightarrow{OB}\right|$ = 9/2

(c) Area of the triangle ABC is = $\frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \frac{1}{2}\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -4 & 1 \\ -\frac{5}{2} & -4 & -\frac{1}{2} \end{vmatrix}\right|$

= $\frac{1}{2}\left|6\hat{i}-3\hat{j}-6\hat{k}\right|$

= 9/2

(d) Acute angle between the diagonals of the parallelogram with adjacent sides

$\overrightarrow{OA}$ and $\overrightarrow{OC}$ = θ

$$\frac{(\overrightarrow{OA}+\overrightarrow{OC}).(\overrightarrow{OA}-\overrightarrow{OC})}{\left|\overrightarrow{OA}+\overrightarrow{OC}\right|\left|\overrightarrow{OA}-\overrightarrow{OC}\right|}=\cos\theta$$

cos θ = $\frac{(\frac{3}{2}\hat{i}+\frac{3}{2}\hat{k}).(\frac{5}{2}\hat{i}+4\hat{j}+\frac{1}{2}\hat{k})}{\frac{3}{2}\sqrt{2}\times\sqrt{\frac{90}{4}}}$

= 18/3√2 ×√90

θ ≠ π/3

Question 6: Let E denote the parabola y^2 = 8x. Let P = (-2, 4), and let Q and Q' be two distinct points on E such that the lines PQ and PQ' are tangents to E. Let F be the focus of E. Then which of the following statements is (are) TRUE?

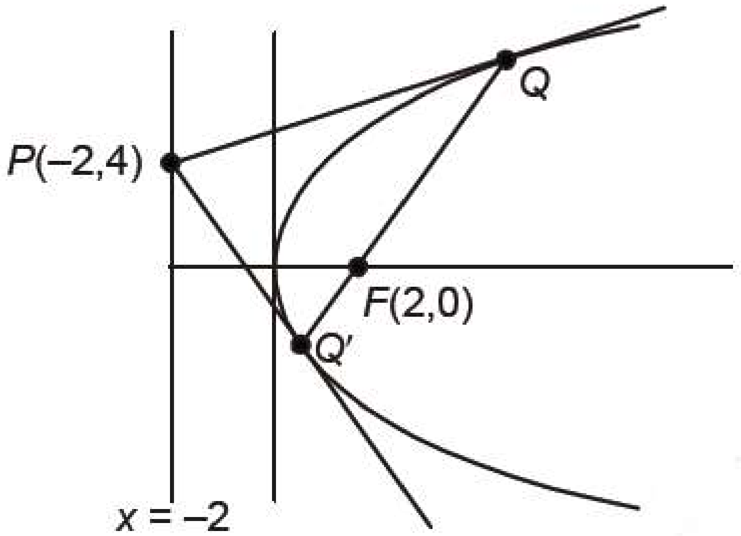
a. The triangle PFQ is a right-angled triangle
b. The triangle QPQ' is a right-angled triangle
c. The distance between P and F is 5√2
d. F lies on the line joining Q and Q'

Solution:

Answer: (a, b, d)

E : y^2 = 8x

P : (-2, 4)



Point P (-2, 4) lies on directrix (x = -2) of parabola $y^2 = 8x$

So, $\angle QPQ' = \pi/2$ and chord QQ' is a focal chord and segment PQ subtends a right angle at the focus.

Slope of QQ' = $2/(t_1+t_2) = 1$

Slope of PF = -1

PF = $4\sqrt{2}$

Question Stem for Question Nos. 7 and 8

Consider the region R = {(x,y)∈ R×R : x ≥ 0 and $y^2 \le 4 - x$. Let F be the family of all circles that are contained in R and have centres on the x-axis. Let C be the circle that has the largest radius among the circles in F. Let (α, β) be a point where circle C meets the curve $y^2 = 4 - x$.

Question 7: The radius of the circle C is

Solution:

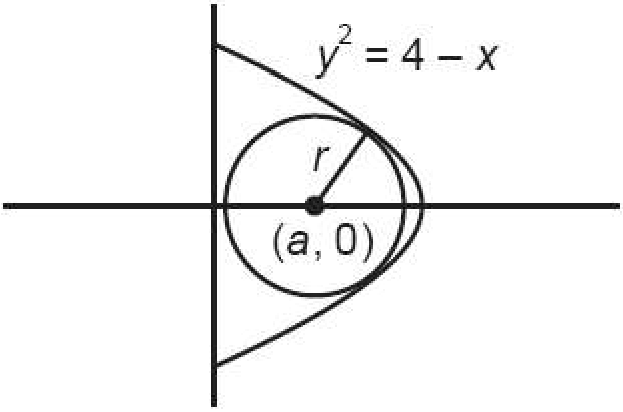
Answer: (1.50)

Question 8: The value of α is

Solution:

Answer: (2.00)

Sol: For comprehension Question 7 and Question 8



Let the circle be,

$(x - a)^2 + y^2 = r^2$

Solving it with parabola

$y^2 = 4 - x$ we get

$(x - a)^2 + 4 - x = r^2$

$x^2 - x(2a + 1) + (a^2 + 4 - r^2) = 0$ ...(1)

D = 0

$\Rightarrow 4r^2 + 4a - 15 = 0$

Clearly $a \geq r$

So $4r^2 + 4r - 15 \leq 0$

$\Rightarrow r_{max} = 3/2 = a$

Radius of circle C is 3/2

From (1) $x^2 - 4x + 4 = 0$

$\Rightarrow x = 2 = \alpha$

Question Stem for Question Nos. 9 and 10

Let $f_1 : (0, \infty) \rightarrow R$ and $f_2 : (0, \infty) \rightarrow R$ be defined by $f_1(x) = \int_0^x \prod_{j=1}^{21} (t - j)^j dt$, x>0 and $f_2(x) = 98(x - 1)^{50} - 600(x - 1)^{49} + 2450$, x > 0, where, for any positive integer n and real number $a_1, a_2, \ldots a_n$. $\prod_{i=1}^{n} a_i$ denotes the product of $a_1, a_2, \ldots a_n$. Let m_i and n_i, respectively, denote the number of points of local minima and the number of points of local maxima of function f_i, i = 1, 2, in the interval $(0, \infty)$.

Question 9: The value of $2m_1 + 3n_1 + m_1n_1$ is

Solution:

Answer: (57.00)

Question 10: The value of $6m_2 + 4n_2 + 8m_2n_2$ is

Solution:

Answer: (06.00)

Solution for Question 9 and 10

$$f_1'(x) = \prod_{j=1}^{21}(x - j)^j$$

$f_1'(x) = (x - 1)(x - 2)^2 (x - 3)^3 ,..., (x - 20)^{20} (x - 21)^{21}$

Checking the sign scheme of $f_1'(x)$ at x = 1, 2, 3, ..., 21, we get

$f_1(x)$ has local minima at x = 1, 5, 9, 13, 17, 21 and local maxima at x = 3, 7, 11, 15, 19

$\Rightarrow m_1 = 6, n_1 = 5$

$f_2(x) = 98(x - 1)^{50} - 600(x - 1)^{49} + 2450$

$f_2'(x) = 98 \times 50(x - 1)^{49} - 600 \times 49 \times (x - 1)^{48}$

$= 98 \times 50 \times (x - 1)^{48} (x - 7)$

$f_2(x)$ has local minimum at x = 7 and no local maximum.

$\Rightarrow m_2 = 1, n_2 = 0$

$2m_1 + 3n_1 + m_1n_1$

$= 2 \times 6 + 3 \times 5 + 6 \times 5$

$= 57$

$6m_2 + 4n_2 + 8m_2n_2$

$= 6 \times 1 + 4 \times 0 + 8 \times 1 \times 0$

$= 6$

Question Stem for Question Nos. 11 and 12

Let $g_i = [\pi/8, 3\pi/8] \rightarrow R$, i = 1, 2 and f: $[\pi/8, 3\pi/8] \rightarrow R$ be the functions such that $g_1(x) = 1$, $g_2(x) = |4x - \pi|$ and $f(x) = \sin^2 x$, for all $x \in [\pi/8, 3\pi/8]$. Define $S_i = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} f(x).g_i(x)dx$, i = 1,2.

Question 11: The value of $16S_1/\pi$ is

Solution:

Answer: (2.00)

$S_1 = \int_{\pi/8}^{3\pi/8} \sin^2 x \,.\, 1\, dx$

$= \frac{1}{2} \int_{\pi/8}^{3\pi/8}(1 - \cos 2x)dx$

$= \frac{1}{2} (x - \sin 2x/x)_{\pi/8}^{3\pi/8}$

$= \frac{1}{2} (\pi/4 - 0)$

$= \pi/8$

$\Rightarrow 16S_1/\pi = 2$

Question 12:The value of $48S_2/\pi^2$ is

Solution:

Answer: (1.50)

$S_2 = \int_{\pi/8}^{3\pi/8} \sin^2 x\, |4x - \pi|\, dx$

$= \int_{\pi/8}^{3\pi/8} 4 \sin^2 x\, |x - \pi/4|\, dx$

Let $x - \pi/4 = t$

$\Rightarrow dx = dt$

$S_2 = \int_{-\pi/8}^{\pi/8} 4 \sin^2 (\pi/4 + t)|t|\, dt$

$= \int_{-\pi/8}^{\pi/8} 2(1 - \cos 2(\pi/4 + t)\, |t|\, dt$

$= \int_{-\pi/8}^{\pi/8} (2 + 2 \sin 2t)\, |t|\, dt$

$= 2\int_{-\pi/8}^{\pi/8}|t|dt + 2\int_{-\pi/8}^{\pi/8}|t| \sin 2t\, dt$

$= 4\int_0^{\pi/8}t\, dt + 0$

$S_2 = [2t^2]_0^{\pi/8}$

$= \pi^2/32$

$48S_2/\pi^2 = 3/2$

Question Paragraph: Let $M = \{(x, y) \in R \times R : x^2 + y^2 \leq r^2\}$, where $r > 0$. Consider the geometric progression $a_n = 1/2^{n-1}$, $n = 1,2, 3\ldots$ Let $S_0 = 0$ and, for $n \geq 1$, let S_n denote the sum of the first n terms of this progression. For $n \geq 1$, let C_n denote the circle with center $(S_{n-1}, 0)$ and radius a_n, and D_n denote the circle with center (S_{n-1}, S_{n-1}) and radius a_n.

Question 13: Consider M with $r = 1025/513$. Let k be the number of all those circles C_n that are inside M. Let l be the maximum possible number of circles among these k circles such that no two circles intersect. Then,

a. $k + 2l = 22$
b. $2k + l = 26$

c. $2k + 3l = 34$
d. $3k + 2l = 40$

Solution:

Answer: (d)

$a_n = 1/2^{n-1}$

And $S_n = 2(1 - 1/2^n)$

For circles C_n to be inside M.

$S_{n-1} + a_n < 1025/513$

$\Rightarrow S_n < 1025/513$

$\Rightarrow 1 - 1/2^n < 1025/1026$

$\Rightarrow 1 - 1/1026$

$\Rightarrow 2^n < 1026$

$\Rightarrow n \leq 10$

$\therefore$ Number of circles inside be 10 = K

Clearly, alternate circles do not intersect each other i.e., C_1, C_3, C_5, C_7, C_9 do not intersect each other as well as C_2, C_4, C_6, C_8 and C_{10} do not intersect each other hence maximum of 5 set of circles do not intersect each other.

$\therefore l = 5$

$\therefore 3K + 2l = 40$

$\therefore$ Option (D) is correct

Question 14: Consider M with r = $(2^{199}-1)\sqrt{2}/2^{198}$. The number of all those circles D_n that are inside M is;

a. 198
b. 199
c. 200
d. 201

Solution:

Answer: (B)

Since $r = (2^{199}-1)\sqrt{2}/2^{198}$

Now, $\sqrt{2}S_{n-1} + a_n < (2^{199}-1)\sqrt{2}/2^{198}$

$2\sqrt{2}(1 - 1/2^{n-1}) + 1/2^{n-1} < (2^{199}-1)/2^{198}$

$\therefore 2\sqrt{2} - \sqrt{2}/2^{n-2} + 1/2^{n-1} < 2\sqrt{2} - \sqrt{2}/2^{198}$

$(1/2^{n-2})(½ - \sqrt{2}) < -\sqrt{2}/2^{198}$

$(2\sqrt{2}-1)/2.\ 2^{n-2} > \sqrt{2}/2^{198}$

$2^{n-2} < (2 - 1/\sqrt{2})\ 2^{197}$

$n \leq 199$

∴ Number of circles = 199

Option (B) is correct.

Question Paragraph: Let $\psi_1 = [0, \infty) \to R$, $\psi_2 = [0, \infty) \to R$, $f:[0, \infty) \to R$ and $g:[0, \infty) \to R$ be functions such that $f(0) = g(0) = 0$,

$\psi_1(x) = e^{-x} + x, x \geq 0,$

$\psi_2(x) = x^2 - 2x - 2e^{-x} + 2, x \geq 0$

$$f(x) = \int_{-x}^{x} (|t| - t^2)e^{-t^2} dt, x > 0$$

And $g(x) = \int_0^{x^2} \sqrt{t}e^{-t}dt, x > 0$

Question 15: Which of the following statements is TRUE?

a. $f(\sqrt{\ln 3}) + g(\sqrt{\ln 3}) = \frac{1}{3}$
b. For every $x > 1$, there exists an $\alpha \in (1, x)$ such that $\psi_1(x) = 1 + \alpha x$
c. For every $x > 0$, there exists a $\beta \in (0, x)$ such that $\psi_2(x) = 2x (\psi_1(\beta) - 1)$
d. f is an increasing function on the interval $[0, 3/2]$

Solution:

Answer: (c)

Since, $g(x) = \int_0^{x^2} \sqrt{t}e^{-t}dt, x > 0$

Let $t = u^2$

$\Rightarrow dt = 2u\ du$

So $g(x) = \int_0^{x} ue^{-u^2} .2u\ du$

$= 2\int_0^{x} t^2 e^{-t^2} dt$

...(i)

And $f(x) = \int_{-x}^{x}(|t| - t^2)e^{-t^2}dt, x > 0$

Therefore $f(x) = 2\int_0^x(t - t^2)e^{-t^2}dt$ ...(ii)

From equation (i) + (ii) : f(x) + g(x) = $\int_0^x 2te^{-t^2}dt$

Let $t^2 = P$

⇒ 2t dt = dP

f(x) + g(x) = $\int_0^{x^2} e^{-P}dP = \left[-e^{-P}\right]_0^{x^2}$

f(x) + g(x) = $1 - e^{-x^2}$ ...(iii)

∴ f(√ln 3) + g(√ln 3) = $1-e^{-\ln 3}$

= 1-⅓

= ⅔

∴ Option (a) is incorrect.

From equation (ii) : f'(x) = $f'(x) = 2(x - x^2)e^{-x^2} = 2x(1 - x)e^{-x^2}$

Since f(x) is increasing in (0, 1)

∴ Option (d) is incorrect

$\psi_1(x) = e^{-x} + x$

⇒ $\psi_1'(x) = 1- e^{-x} <1$ for x>1

Then for α ∈ (1, x), $\psi_1(x) = 1+ \alpha x$ does not true for α > 1.

∴ Option (b) is incorrect

Now $\psi_2(x) = x^2 - 2x - 2e^{-x} + 2$

$\psi_2'(x) = 2x - 2 + 2e^{-x}$

∴ $\psi_2'(x) = 2\psi_1(x)-2$

From LMVT

$[\psi_2(x) - \psi_2(0)]/(x-0) = \psi_2'(\beta)$ for β ∈(∞, x)

=> $\psi_2(x) = 2x(\psi_1(\beta) - 1)$

Option (c) is correct.

Question 16: Which of the following statements is TRUE?

a. $\psi_1(x) \le 1$, for all x>0
b. $\psi_2(x) \le 0$, for all x>0

c. $f(x) \ge 1 - e^{-x^2} - \frac{2}{3}x^3 + \frac{2}{5}x^5$, for all x∈(0, ½)

d. $g(x) \le (2/3)x^3 - (2/5)x^5 + (1/7)x^7$, for all x∈(0, ½)

Solution:

Answer: (d)

Since $\psi_1(x) = e^{-x} + x$

And for all x>0, $\psi_1(x) > 1$

∴ (a) is not correct

$\psi_2(x) = x^2 + 2 - 2(e^{-x} + x) > 0$ for $x > 0$

∴ (b) is not correct.

Now, $\sqrt{t}e^{-t} = \sqrt{t}(1 - t/1! + t^2/2! - t^3/3! + \ldots\infty)$

And $\sqrt{t}e^{-t} \le t^{1/2} - t^{3/2} + \frac{1}{2}t^{5/2}$

∴ $\int_0^{x^2} \sqrt{t}e^{-t}dt \le \int_0^{x^2} (t^{\frac{1}{2}} - t^{\frac{3}{2}} + \frac{1}{2}t^{\frac{5}{2}})dt$

$= (2/3)x^3 - (2/3)x^5 + (1/7) + (1/7)x^7$

∴ Option (d) is correct

And $f(x) = \int_{-x}^{x} (|t| - t^2)e^{-t^2}dt$

$= 2\int_0^x (t - t^2)e^{-t^2}dt$

$= 2\int_0^x 2te^{-t^2}dt - 2\int_0^x t^2e^{-t^2}dt$

$= 1 - e^{-x^2} - 2\int_0^x t^2e^{-t^2}dt$

Therefore $f(x) \le 1 - e^{-x^2} - 2\int_0^x t^2(1 - t^2)dt$

= $1 - e^{-x^2} - 2\frac{x^3}{3} + \frac{2}{5}x^5$ for all x∈(0, ½)

∴ Option (c) is incorrect.

Question 17: A number is chosen at random from the set {1, 2, 3......, 2000}. Let p be the probability that the number is a multiple of 3 or a multiple of 7. Then the value of 500p is;

Solution:

Answer: (214)

E = a number which is multiple of 3 or multiple of 7

n(E) = (3, 6, 9, ..., 1998) + (7, 14, 21, ..., 1995) - (21, 42, 63, ... 1995)

n(E) = 666+ 285 - 95

n(E) = 856

n(E) = 2000

P(E) = 856/2000

P(E) × 500 = 856/4 = 214

Question 18: Let E be the ellipse $x^2/16 + y^2/9 = 1$. For any three distinct points P, Q and Q' on E, let M (P, Q) be the mid-point of the line segment joining P and Q, and M(P, Q') be the mid-point of the line segment joining P and Q'. Then the maximum possible value of the distance between M (P, Q) and M(P, Q'), as P, Q and Q' vary on E, is

Solution:

Answer: (4)

Let P(α), Q(θ), Q'(θ')

M = ½ (4 cos α + 4 cos θ), ½ (3 sin α + 3 sin θ)

M' = ½ (4 cos α + 4 cos θ'), ½ (3 sin α + 3 sin θ')

MM' = ½ $\sqrt{((4 \cos \theta - 4 \cos \theta')^2 + (3 \sin \theta - 3 \sin \theta')^2)}$

MM' = ½ distance between Q and Q'

MM' is not depending on P

Maximum of QQ' is possible when QQ' = major axis

QQ' = 2(4) = 8

MM' = ½ (QQ')

MM' = 4

Question 19: For any real number x, let [x] denote the largest integer less than or equal to x.

If $l = \int_0^{10} \left[\sqrt{\frac{10x}{x+1}}\right] dx$, then the value of 9l is;

Solution:

Answer: (182.00)

$$l = \int_0^{10} \left[\sqrt{\frac{10x}{x+1}}\right] dx$$

y = 10x/(x+1), 0 ≤ x≤10

xy+y = 10x

x = y/(10-y)

0≤ y/(10-y) ≤ 10

y/(10-y) ≥ 0 and (y/(10-y)) - 10 ≤ 0

y/(y-10) ≤ 0 and (11y-100)/(y-10) ≥ 0

y ∈ [0, 10) and y ∈ (-∞, 100/11] ∪ (10, ∞)

y ∈ [0, 100/11]

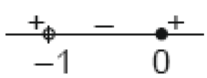
√y ∈ [0, 10/√11]

⇒ [√y] = {0, 1, 2, 3}

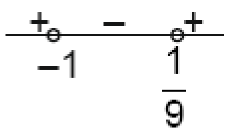
Case 1:

0≤10x/(x+1) < 1

10x/(x+1) ≥ 0 and 10x/(x+1) - 1 < 0



and $\frac{9x-1}{x+1} < 0$



x∈ (-∞, -1) ∪ [0, ∞) and x∈ (-1, 1/9)

x∈ [0, 1/9) then [√(10x/(x+1))] = 0

Case 2:

1 ≤ 10x/(x+1) < 4

$10x/(x+1) - 1 \geq 0$ and $10x/(x+1) - 4 < 0$

$(9x-1)/(x+1) \geq 0$ and $(6x-4)/(x+1) < 0$

$x \in (-\infty, -1) \cup [1/9, \infty)$ and $x \in (-1, 2/3)$

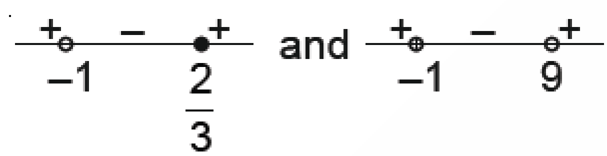
$x \in [1/9, 2/3)$, $[\sqrt{(10x/(x+1))}] = 1$

Case 3:

$4 \leq (10x/x+1) < 9$

$10x/(x+1) - 4 \geq 0$ and $10x/(x+1) < 9$

$(6x-4)/(x+1) \geq 0$ and $(x-9)/(x+1) < 0$



$x \in (-\infty, -1) \cup [2/3, \infty)$ and $x \in (-1, 9)$

$x \in [2/3, 9)$; $[\sqrt{(10x/(x+1))}] = 2$

Case 4:

$x \in [9, 10]$

$\Rightarrow [\sqrt{(10x/(x+1))}] = 3$

$$I = \int_0^{\frac{1}{9}} 0.dx + \int_{\frac{1}{9}}^{\frac{2}{3}} 1.dx + \int_{\frac{2}{3}}^{9} 2.dx + \int_9^{10} 3.dx$$

$I = (2/3 - 1/9) + 2(9 - 2/3) + 3(10-9)$

$I = 5/9 + 50/3 + 3$

$9I = 182$