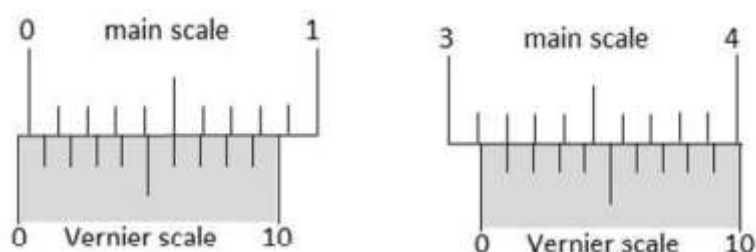


JEE Advanced 2021 (Paper 1)

Physics

Question Paper

Question 1: The smallest division on the main scale of Vernier calipers is 0.1 cm. Ten divisions of the Vernier scale correspond to nine divisions of the main scale. The figure below on the left shows the reading of this caliper with no gap between its two jaws. The figure on the right shows the reading with a solid sphere held between the jaws. The correct diameter of the sphere is;



- a. 3.07 cm
- b. 3.11 cm
- c. 3.15 cm
- d. 3.17 cm

Solution:

Answer: (c)

Least count of Vernier calipers = 0.01 cm

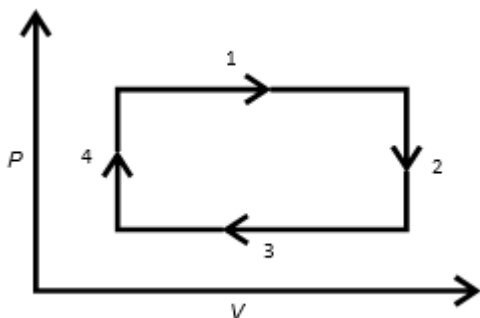
Error in scale = 4 LC = 0.04 cm

Reading = 3.1 cm + 1 L.C = 3.1 cm + 0.01 = 3.11 cm

So correct diameter of the sphere = (3.11 + 0.04) cm = 3.15 cm



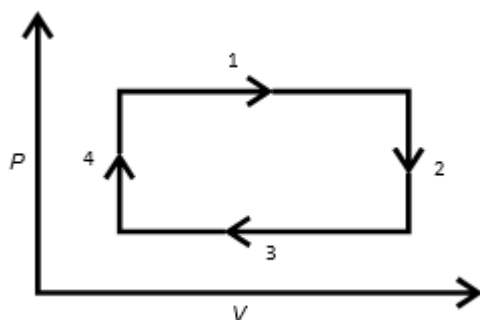
Question 2: An ideal gas undergoes a four step cycle as shown in the P – V diagram below. During this cycle, heat is absorbed by the gas in;



- a. steps 1 and 2
- b. steps 1 and 3
- c. steps 1 and 4
- d. steps 2 and 4

Solution:

Answer: (c)



Given P – V diagram

For process (1)

$$\Delta Q_1 = nC_p\Delta T$$

As $P = \text{constant}$ and V increases so T will increase

So $\Delta Q_1 > 0$

For process (2)

$$\Delta Q_2 = nC_v\Delta T$$

$V = \text{constant}$, $P \downarrow$, So $T \downarrow$



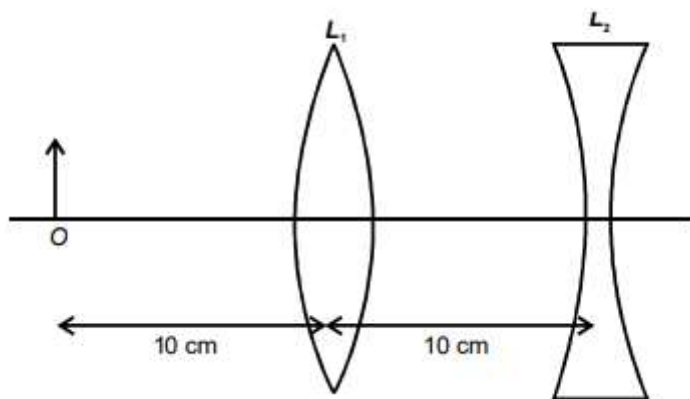
For process (3), $\Delta Q_3 = nC_p\Delta T < 0$

For process (4), $\Delta Q_4 = nC_p\Delta T$

As $\Delta T > 0$

So $\Delta Q_4 > 0$

Question 3: An extended object is placed at point O, 10 cm in front of a convex lens L_1 and a concave lens L_2 is placed 10 cm behind it, as shown in the figure. The radii of curvature of all the curved surfaces in both the lenses are 20 cm. The refractive index of both the lenses is 1.5. The total magnification of this lens system is;



- a. 0.4
- b. 0.8
- c. 1.3
- d. 1.6

Solution:

Answer:(b)

$$\frac{1}{f_1} = \left(\frac{3}{2} - 1\right) \left(\frac{2}{20}\right) = 1/20$$

$$\frac{1}{v_1} - \frac{1}{-10} = \frac{1}{20}$$

$$\frac{1}{v_1} = \frac{1}{20} - \frac{1}{10} = \frac{-1}{20}$$

$$v_1 = -20 \text{ cm}$$

$$m_1 = v/u_1 = -20/-10 = 2$$

Again,



$$\frac{1}{v_2} - \frac{1}{-30} = \frac{1}{-20}$$

$$\frac{1}{v_2} = -\frac{1}{30} - \frac{1}{20} = -\frac{5}{50} = -\frac{1}{12}$$

$$m_2 = -12/-30 = \frac{2}{5}$$

$$m = m_1 = m_2 = 2 \times (\frac{2}{5}) = 0.8$$

Question 4: A heavy nucleus Q of half-life 20 minutes undergoes alpha-decay with a probability of 60% and beta-decay with a probability of 40%. Initially, the number of Q nuclei is 1000. The number of alpha-decays of Q in the first one hour is;

- a. 50
- b. 75
- c. 350
- d. 525

Solution:

Answer: (d)

$$t_{1/2} = 20 \text{ min}$$

In 60 min, no. of half-life = 3

$$\Rightarrow N_A = [1000 - 1000/2^3] \times 0.6$$

$$= 100 \times (\frac{7}{8}) \times 0.6$$

$$= 525$$

Question Statement for Question Nos. 5 and 6

A projectile is thrown from a point O on the ground at an angle 45° from the vertical and with a speed of $5\sqrt{2}$ m/s. The projectile at the highest point of its trajectory splits into two equal parts. One part falls vertically down to the ground, 0.5 s after the splitting. The other part, t seconds after the splitting, falls to the ground at a distance x meters from the point O. The acceleration due to gravity $g = 10 \text{ m/s}^2$.

Question 5: The value of t is _____.

Solution:

Answer: 0.5

$$H = u^2 \sin^2 \theta / 2g$$



$$= \frac{50}{2 \times 10} \times \frac{1}{2} = \frac{5}{4}$$

$$t = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 5}{4 \times 10}} = (1/2)s = 0.5s$$

Question 6: The value of x is _____.

Solution:

Answer: 7.5

$$X = 3R/2 \text{ as } X_{cm} = R$$

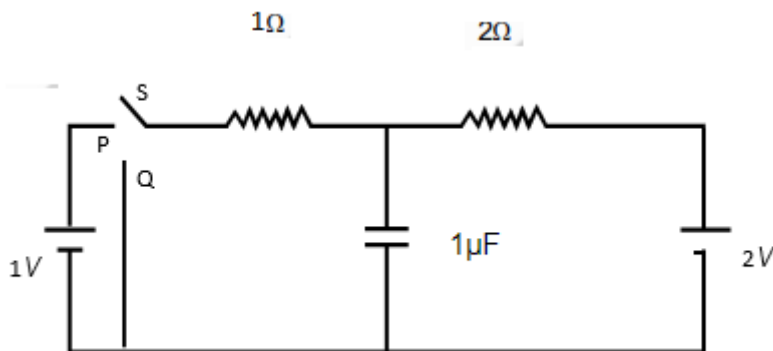
$$R = u^2 \sin^2 \theta / g$$

$$= 50/10 = 5$$

$$\Rightarrow X = 3R/2 = 15/2 = 7.5 \text{ m}$$

Question Statement for Question Nos. 7 and 8

In the circuit shown below, the switch S is connected to position P for a long time so that the charge on the capacitor becomes $q_1 \mu\text{C}$. Then S is switched to position Q. After a long time, the charge on the capacitor is $q_2 \mu\text{C}$.



Question 7: The magnitude of q_1 is

Solution:

Answer: 01.33

With switch S at position P after long time potential difference across capacitor branch

$$\frac{\frac{2}{2} + \frac{1}{1}}{\frac{1}{2} + \frac{1}{1}} = \frac{2 \times 1}{3} = (4/3)v$$

Charge on capacitor $q_1 \mu\text{C} = (4/3) \mu\text{C}$

$$\Rightarrow q_1 = 4/3 = 1.33$$



Question 8: The magnitude of q_2 is ____.

Solution:

Answer: 00.67

With switch S at position Q after long time potential difference across the capacitor
= potential difference across a resistance of 1 ohm.

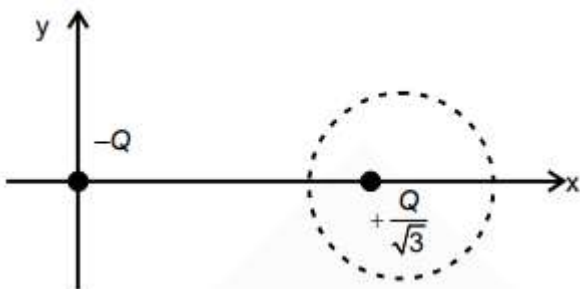
$$= \left(\frac{2}{3}\right) V$$

$$\text{Charge on capacitor } q_2 \mu\text{C} = \left(\frac{2}{3}\right) \mu\text{C}$$

$$\Rightarrow q_2 = 0.67$$

Question Statement for Question Nos. 9 and 10

Two-point charges $-Q$ and $+Q / \sqrt{3}$ are placed in the xy -plane at the origin $(0, 0)$ and a point $(2, 0)$, respectively, as shown in the figure. This results in an equipotential circle of radius R and potential $V = 0$ in the xy -plane with its centre at $(b, 0)$. All lengths are measured in meters.



Question 9: The value of R is ____ meter

Solution:

Answer: ($R = 01.73$)

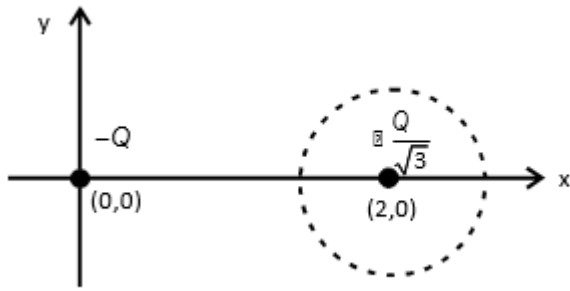
Question 10: The value of b is ____ meter

Solution:

Answer: ($b = 03.00$)

Solution for Q. Nos. 9 & 10



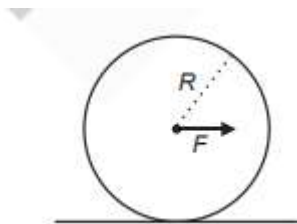


$$V(x, y) = \frac{1}{4\pi\epsilon_0} \left[-\frac{Q}{\sqrt{x^2+y^2}} + \frac{Q}{\sqrt{3}\sqrt{(x-2)^2+y^2}} \right]$$

$$\Rightarrow 3(x-1)^2 + 3y^2 = x^2 + y^2$$

$$\Rightarrow (x-3)^2 + y^2 = (\sqrt{3})^2$$

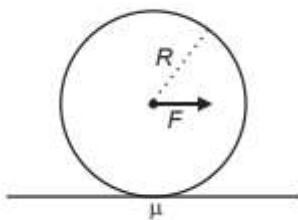
Question 11: A horizontal force F is applied at the centre of mass of a cylindrical object of mass m and radius R , perpendicular to its axis as shown in the figure. The coefficient of friction between the object and the ground is μ . The centre of mass of the object has an acceleration a . The acceleration due to gravity is g . Given that the object rolls without slipping, which of the following statement(s) is(are) correct?



- a. For the same F , the value of a does not depend on whether the cylinder is solid or hollow
- b. For a solid cylinder, the maximum possible value of a is $2g$
- c. The magnitude of the frictional force on the object due to the ground is always mg
- d. For a thin-walled hollow cylinder, $a = F/2m$

Solution:

Answer: (b, d)



For a solid cylinder,



$$F \times R = \frac{3}{2}mR^2 \times \frac{a}{R}$$

$$\Rightarrow a = 2F/3m$$

For hollow cylinder,

$$F \times R = 2mR^2 \times \frac{a}{R}$$

$$\Rightarrow a = F/3m$$

For solid cylinder

$$f = F - m \times (2F/3m) = (F/3) \leq \mu mg$$

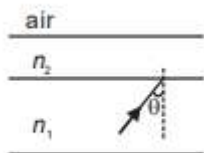
$$\Rightarrow F \leq 3\mu mg$$

Therefore,

$$A \leq (2/3m) \times (3\mu mg)$$

$$\Rightarrow a_{\max} = 2\mu g$$

Question 12: A wide slab consisting of two media of refractive indices n_1 and n_2 is placed in the air as shown in the figure. A ray of light is incident from medium n_1 to n_2 at an angle, where $\sin \theta$ is slightly larger than $1/n_1$. Take the refractive index of air as 1. Which of the following statement(s) is(are) correct?



- The light ray enters air if $n_2 = n_1$
- The light ray is finally reflected back into the medium of refractive index n_1 if $n_2 < n_1$
- The light ray is finally reflected back into the medium of refractive index n_1 if $n_2 > n_1$
- The light ray is reflected back into the medium of refractive index n_1 if $n_2 = 1$

Solution:

Answer: (b, c, d)

$$\sin \theta > 1/n_1 \dots (i)$$

$$\text{And } n_1 \sin \theta = 1 \times \sin r \dots (ii)$$

$$\Rightarrow \sin r > 1$$

\Rightarrow refraction into air is not possible



Question 13: A particle of mass $M = 0.2$ kg is initially at rest in the xy -plane at a point $(x = -l, y = -h)$, where $l = 10$ m and $h = 1$ m. The particle is accelerated at time $t = 0$ with a constant acceleration $a = 10$ m/s² along the positive x -direction. Its angular momentum and torque with respect to the origin, in SI units, are represented by \vec{L} and $\vec{\tau}$, respectively. $\hat{i}, \hat{j}, \hat{k}$,

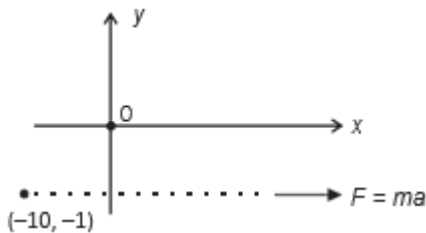
and are unit vectors along the positive x, y and z -directions respectively. If $\hat{k} = \hat{i} \times \hat{j}$

then which of the following statement(s) is(are) correct?

- The particle arrives at the point $(x = l, y = -h)$ at time $t = 2$ s
- $\vec{\tau} = 2\hat{k}$ when the particle passes through the point $(x = l, y = -h)$
- $\vec{L} = 4\hat{k}$ when the particle passes through the point $(x = l, y = -h)$
- $\vec{\tau} = \hat{k}$ when the particle passes through the point $(x = 0, y = -h)$

Solution:

Answer: (a, b, c)



$$t = \sqrt{\frac{2 \times 20}{10}} = 2 \text{ s}$$

$$\vec{\tau} = (0.2 \times 10 \times 1)\hat{k} = 2\hat{k}$$

$$\vec{L} = (0.2 \times (10 \times 2) \times 1)\hat{k} = 4\hat{k}$$

Question 14: Which of the following statement(s) is(are) correct about the spectrum of hydrogen atom?

- The ratio of the longest wavelength to the shortest wavelength in the Balmer series is 9/5
- There is an overlap between the wavelength ranges of Balmer and Paschen series



c. The wavelengths of Lyman series are given by $(1 + 1/m^2)\lambda_0$ where λ_0 is the shortest wavelength of Lyman series and m is an integer

d. The wavelength ranges of the Lyman and Balmer series do not overlap

Solution:

Answer: (a, d)

For Balmer series:

$$\frac{1}{\lambda} = R \left[\frac{1}{2^2} - \frac{1}{n^2} \right]$$

n = 3,4,5 ----

$$\frac{1}{\lambda_{max}} = R \left[\frac{1}{4} - \frac{1}{9} \right]$$

$$\frac{1}{\lambda_{min}} = R \left[\frac{1}{4} \right]$$

$$\frac{\lambda_{max}}{\lambda_{min}} = \frac{9}{5}$$

For Lyman Series

$$\frac{1}{\lambda} = R \left(1 - \frac{1}{n^2} \right)$$

n = 2,3,4 ----

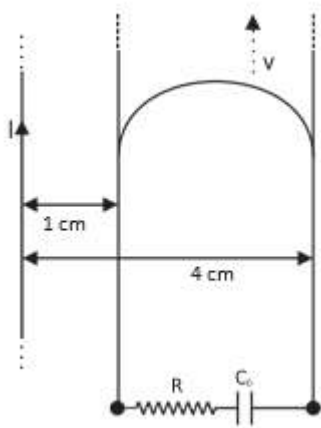
$$1/\lambda_{min} = R$$

$$\Rightarrow \lambda = \lambda_0 n^2 / (n^2 - 1)$$

Question 15: A long straight wire carries a current, $I = 2$ ampere. A semi-circular conducting rod is placed beside it on two conducting parallel rails of negligible resistance. Both the rails are parallel to the wire. The wire, the rod and the rails lie in the same horizontal plane, as shown in the figure. Two ends of the semi-circular rod are at distances 1 cm and 4 cm from the wire. At time $t = 0$, the rod starts moving on the rails with a speed $v = 3.0$ m/s (see the figure).

A resistor $R = 1.4$ and a capacitor $C_0 = 5.0$ F are connected in series between the rails. At time $t = 0$, C_0 is uncharged. Which of the following statement(s) is(are) correct? [$\mu_0 = 4 \times 10^{-7}$ SI units. Take $\ln 2 = 0.7$]



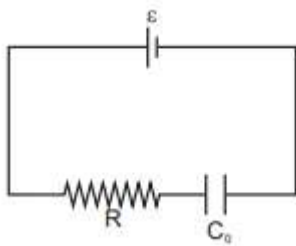


- a. Maximum current through R is 1.2×10^{-6} ampere
- b. Maximum current through R is 3.8×10^{-6} ampere
- c. Maximum charge on capacitor C_0 is 8.4×10^{-12} coulomb
- d. Maximum charge on capacitor C_0 is 2.4×10^{-12} coulomb

Solution:

Answer: (a, c)

The equivalent circuit of the given arrangement is,



$$\epsilon = \frac{\mu_0 I v}{2\pi} I n \frac{b}{a}$$

$$= 1.68 \times 10^{-6} \text{ V}$$

$$\text{At } t = 0, i_{\max} = \epsilon/R = 1.68 \times 10^{-6}/1.4$$

$$= 1.2 \times 10^{-6} \text{ A}$$

$$\text{At } t = \infty, q_{\max} = C_0 \epsilon = 8.4 \times 10^{-12} \text{ C}$$

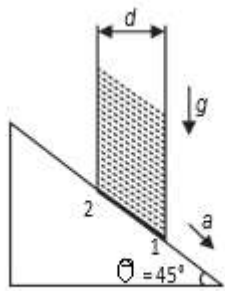
Question 16: A cylindrical tube, with its base as shown in the figure, is filled with water. It is moving down with constant acceleration along a fixed inclined plane with an angle = 45° . P_1

and P_2 are pressures at points 1 and 2, respectively, located at the base of the tube. Let

$$\beta = P_1 - P_2 / \rho g d, \text{ where } \rho \text{ is the density of water, } d \text{ is the inner diameter of the tube}$$

and g is the acceleration due to gravity. Which of the following statement(s) is(are) correct?

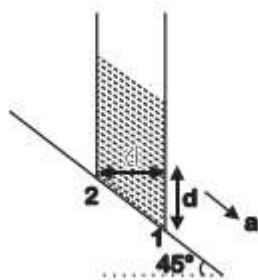




- a. $\beta = 0$ when $a = g/\sqrt{2}$
- b. $\beta > 0$ when $a = g/\sqrt{2}$
- c. $\beta = \frac{\sqrt{2}-1}{\sqrt{2}}$ when $a = g/2$
- d. $\beta = 1/\sqrt{2}$ when $a = g/\sqrt{2}$

Solution:

Answer: (a, c)



$$P_1 = P_2 - a \cos 45^\circ d + \rho(g - a \sin 45^\circ)d$$

$$\Rightarrow P_1 - P_2 / \rho g d = 1 - \sqrt{2}a/g$$

$$\Rightarrow \beta = 0 \text{ for } a = g/\sqrt{2}$$

$$\beta = \frac{\sqrt{2}-1}{\sqrt{2}} \text{ when } a = g/2$$

Question 17: An α -particle (mass 4 amu) and a singly charged sulfur ion (mass 32 amu) are initially at rest. They are accelerated through a potential V and then allowed to pass into a region of a uniform magnetic field which is normal to the velocities of the particles. Within this region, the α -particle and the sulfur ion move in circular orbits of radii r_α and r_s respectively. The ratio r_s/r_α is _____.

Solution:

Answer: (4)

$$r = mv_0/qB$$



$$\left(\frac{1}{2}\right)mv_0^2 = qV$$

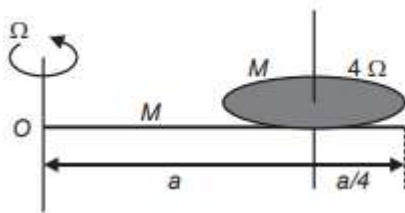
$$r = \sqrt{2mqV}/qB$$

$$r = \frac{1}{B} \sqrt{\frac{2mV}{q}}$$

$$\frac{r_s}{r_\alpha} = \sqrt{\frac{m_s}{q_s}} \times \sqrt{\frac{q_\alpha}{m_\alpha}} = \sqrt{2} \times \sqrt{8}$$

$$\frac{r_s}{r_\alpha} = 4$$

Question 18: A thin rod of mass M and length a is free to rotate in a horizontal plane about a fixed vertical axis passing through point O . A thin circular disc of mass M and of radius $a/4$ is pivoted on this rod with its centre at a distance $a/4$ from the free end so that it can rotate freely about its vertical axis, as shown in the figure. Assume that both the rod and the disc have uniform density and they remain horizontal during the motion. An outside stationary observer finds the rod rotating with an angular velocity and the disc rotating about its vertical axis with angular velocity 4Ω . The total angular momentum of the system about the point O is $\left(\frac{Ma^2\Omega}{48}\right)n$. The value of n is _____.



Solution:

Answer: 49

$$L_s = L_{disc} + L_{rod}$$

$$L_{disc} = \vec{r} \times \vec{p} + I_{cm}4\Omega$$

$$= \frac{Ma^2}{32} \times 4\Omega + \frac{3a}{4} \times \frac{3a}{4} \times M\Omega$$

$$= (11/16) Ma^2 \Omega$$

$$L_{rod} = Ma^2 \Omega/3$$



$$L_{syStatement} = \frac{Ma^2}{3}\Omega + \frac{11}{16}Ma^2\Omega$$

$$= (49/48) Ma^2\Omega$$

$$n = 49$$

Question 19: A small object is placed at the centre of a large evacuated hollow spherical container. Assume that the container is maintained at 0 K. At time $t = 0$, the temperature of the object is 200 K. The temperature of the object becomes 100 K at $t = t_1$ and 50 K at $t = t_2$. Assume the object and the container to be ideal black bodies. The heat capacity of the object does not depend on temperature. The ratio (t_1/t_2) is _____.

Solution:

Answer: 9

$$\text{Heat radiated} = e\sigma AT^4$$

$$= KT^4$$

$$-mS(dT/dt) = KT^4$$

$$-mS \int_{200}^{100} \frac{dT}{T^4} = Kt_1$$

$$t_1 = \frac{1}{K_1} \left[\frac{1}{100^3} - \frac{1}{200^3} \right] = \frac{1}{K_1} \left[\frac{7}{200^3} \right]$$

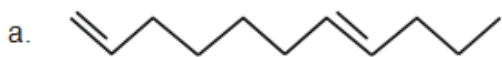
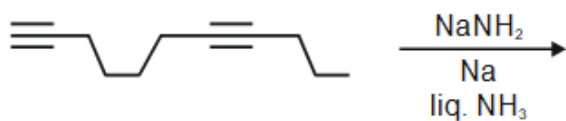
$$t_2 = \frac{1}{K_1} \left[\frac{1}{50^3} - \frac{1}{200^3} \right] = \frac{1}{K_1} \left[\frac{63}{200^3} \right]$$

$$\frac{t_2}{t_1} = 9$$



JEE Advanced 2021 Paper 1 Chemistry Question Paper

Question 1. The major product formed in the following reaction is:

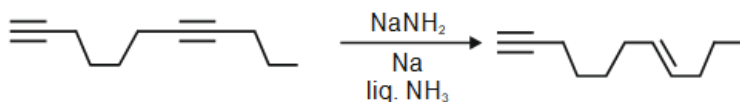


Solution:

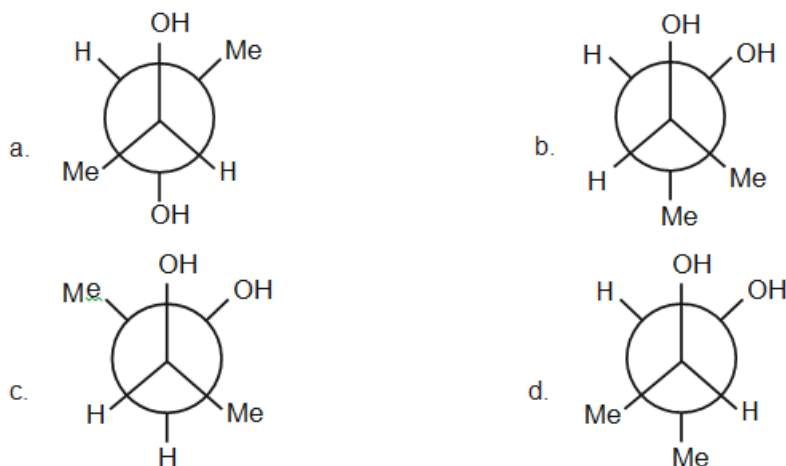
Answer: (b)

It is a case of Birch reduction. Alkynes on reaction with alkali metal in liq. NH_3 gives trans-alkene. But terminal alkynes do not get reduced.





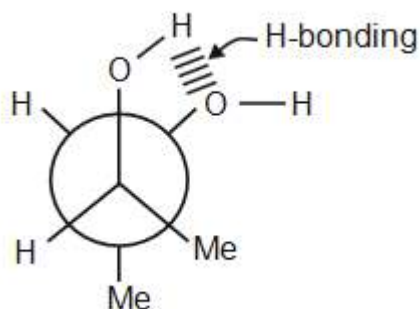
Question 2. Among the following, the conformation that corresponds to the most stable conformation of meso-butane-2,3-diol is;



Solution:

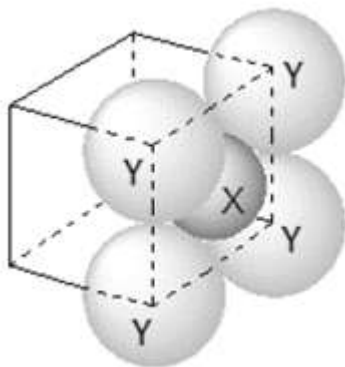
Answer: (b)

Meso compounds have a plane of symmetry. In case of butan-2, 3-diol, gauche form is the most stable due to intramolecular H-bonding.



Question 3. For the given close-packed structure of a salt made of cation X and anion Y shown below (ions of only one face are shown for clarity), the packing fraction is approximately (packing fraction = packing efficiency / 100)





- a. 0.74
- b. 0.63
- c. 0.52
- d. 0.48

Solution:

Answer: (b)

a = edge length of unit cell

$$2r_y = a$$

$$2(r_x - r_y) = \sqrt{2}a$$

$$2r_x + a = \sqrt{2}a$$

$$2r_x = a(\sqrt{2} - 1)$$

$$r_x = 0.207 a$$

Packing fraction = $3 \times \text{vol. of } x + \text{vol. of } y / \text{vol. of unit cell}$

$$\frac{3 \times \frac{4}{3} \times \pi r_x^3 + \frac{4}{3} \times \pi r_y^3}{a^3}$$

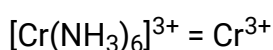
$$\frac{4 \times \pi \times (0.207a)^3 + \frac{4}{3} \times \pi \times (0.5a)^3}{a^3}$$

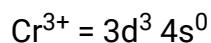
Question 4. The calculated spin only magnetic moments of $[\text{Cr}(\text{NH}_3)_6]^{3+}$ and $[\text{CuF}_6]^{3-}$ in BM, respectively, are (Atomic numbers of Cr and Cu are 24 and 29, respectively).

- a. 3.87 and 2.84
- b. 4.90 and 1.73
- c. 3.87 and 1.73
- d. 4.90 and 2.84

Solution:

Answer: (a)



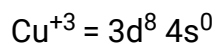
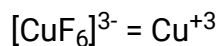


It has 3 unpaired electrons

$$\mu = n \sqrt{(n+2)} \text{ BM}$$

$$\mu = 3 \sqrt{(3+2)} \text{ BM}$$

$$\mu = 3.87 \text{ BM}$$



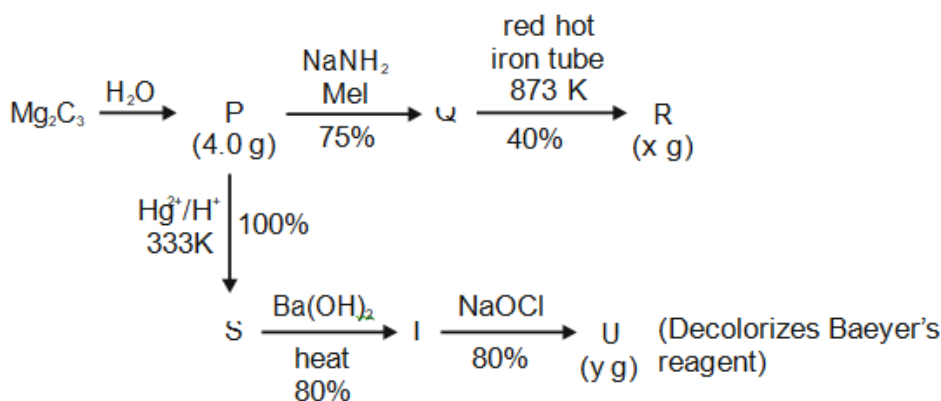
It has 2 unpaired electrons

$$\mu = 2 \sqrt{(2+2)} \text{ BM}$$

$$= 2.84 \text{ BM}$$

Question Stem for Question 5 and 6:

For the following reaction scheme, percentage yields are given along the arrow:



x g and y g are the masses of R and U, respectively. (Use: Molar mass (in g mol⁻¹) of H, C and O as 1, 12 and 16, respectively)

Question 5. The value of x is_____.

Solution:

Answer: (1.62)

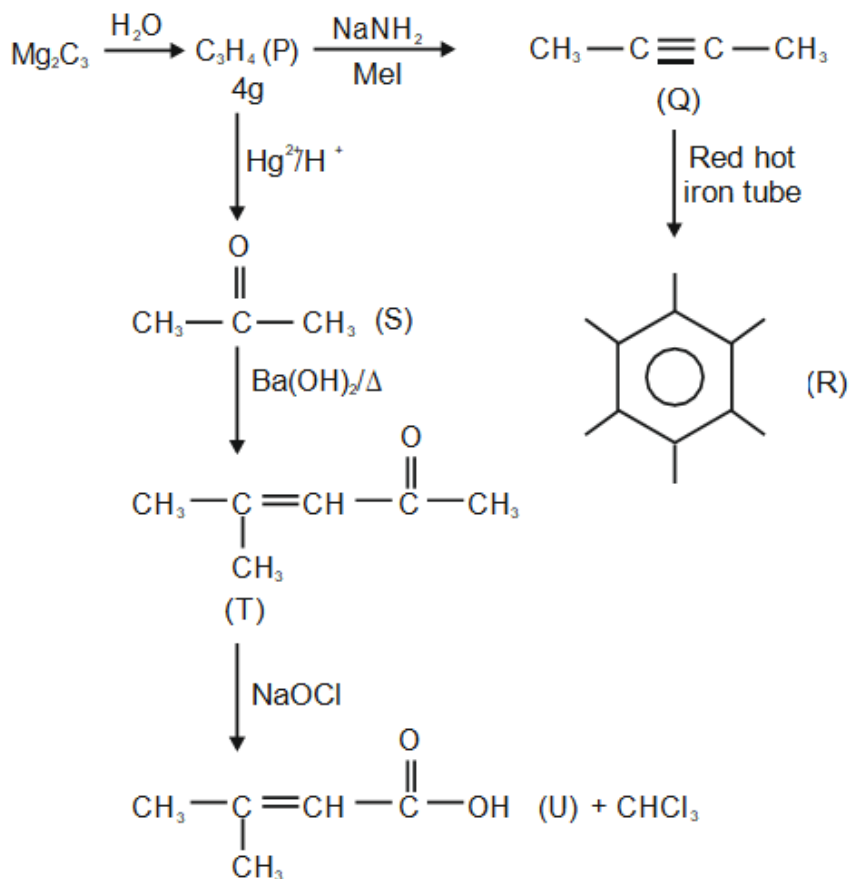
Question 6. The value of y is _____.

Solution:

Answer: (3.20)

Solution for both Questions 5 and 6





4 g of C₃H₄ = 0.1 mol

From 0.1 mol of P, 0.01 mol of R will be produced

⇒ 1.62 g of R is produced

From 0.1 mol of P, 0.032 mol of U is produced

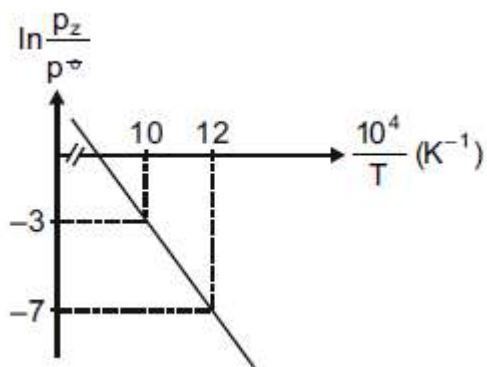
= 3.2 g of U is produced

Question statement for Questions 7 and 8.

For the reaction, $\text{X(s)} \rightleftharpoons \text{Y(s)} + \text{Z(g)}$, the plot of $\ln \frac{p_z}{p^\ominus}$ Versus $10^4 / T$ is given below (in solid

line), where p_z is the pressure (in bar) of the gas Z at temperature T and $p^\ominus = 1$ bar.





(Given, $\frac{d(\ln K)}{d(\frac{1}{T})} = -\frac{\Delta H^\ominus}{R}$, where the equilibrium constant $K = \frac{p_z}{p^\ominus}$ and the gas constant, $R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$)

Question 7. The value of standard enthalpy, ΔH^\ominus (in kJ mol^{-1}) for the given reaction is

_____.

Solution:

Answer: (166.28)



$$\text{Given } K = \frac{p_z}{p^\ominus}$$

$$\ln K = \ln A - \frac{\Delta H^\ominus}{RT}$$

$$\Rightarrow \ln \frac{p_z}{p^\ominus} = \ln A - \frac{\Delta H}{RT}$$

$$\text{Slope of } \ln \frac{p_z}{p^\ominus} \text{ vs } \frac{1}{T} \text{ is } \frac{d \left[\ln \left(\frac{p_z}{p^\ominus} \right) \right]}{d \left(\frac{1}{T} \right)} = \frac{-\Delta H^\ominus}{R}$$

$$\text{From the graph, we have } \frac{-\Delta H^\ominus}{R} = -2 \times 10^4$$

$$\Rightarrow \Delta H^\ominus = 2 \times 10^4 \times 8.314 \text{ J}$$

$$\Delta H^\ominus = 166.28 \text{ kJ mol}^{-1}$$

Question 8. The value of ΔS^\ominus (in $\text{J K}^{-1} \text{ mol}^{-1}$) for the given reaction, at 1000 K is _____.



Solution:

Answer: (141.34)

$$-RT \ln K = \Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$$

$$\ln K = \Delta H^\circ / RT + \Delta S^\circ / R$$

$$\Delta S^\circ / R = 17$$

$$\Delta S^\circ = 17R$$

$$= 141.338 \text{ J K}^{-1}$$

Question Stem for Questions 9 and 10.

The boiling point of water in a 0.1 molal silver nitrate solution (solution A) is $x^\circ\text{C}$. To this solution A, an equal volume of 0.1 molal aqueous barium chloride solution is added to make a new solution B. The difference in the boiling points of water in the two solutions A and B is $y \times 10^{-2}^\circ\text{C}$.

(Assume: Densities of the solutions A and B are the same as that of water and the soluble salts dissociate completely. Use: Molal elevation constant (Ebullioscopic constant), $K_b = 0.5 \text{ K kg mol}^{-1}$; Boiling point of pure water as 100°C .)

Question 9. The value of x is _____.

Solution:

Answer: (100.1)

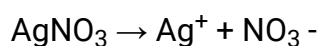
Question 10. The value of $|y|$ is _____.

Solution:

Answer: (2.5)

Given molality of AgNO_3 solution is 0.1 molal (solution-A)

$$\Delta T_b = i k_b m$$



van't Hoff factor (i) for $\text{AgNO}_3 = 2$

$$\Delta T_b = 2 \times 0.5 \times 0.1$$

$$(T_s - T^\circ) = 0.1$$

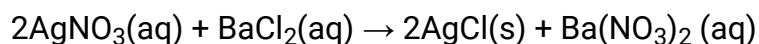
$$(T_s)_A = 100.1^\circ\text{C}, \text{ so } x = 100.1$$

Now solution - A of equal volume is mixed with 0.1 molal BaCl_2 solution to get solution-B. AgNO_3 reacts with BaCl_2 to form $\text{AgCl}(s)$.

0.1 mole of AgNO_3 present in 1000 gram solvent or 1017 gram or 1017 mL solution,



milli moles of AgNO_3 in V ml 0.1 molal solution is nearly $0.1 V$. Similarly in BaCl_2 .



0.1 V	0.1 V	0	0
0	0.05 V	0.1 V	0.05 V

$$\Delta T_b = \left[\frac{0.05V \times 3}{2V} + \frac{0.05V \times 3}{2V} \right] \times 0.5 = 0.075$$

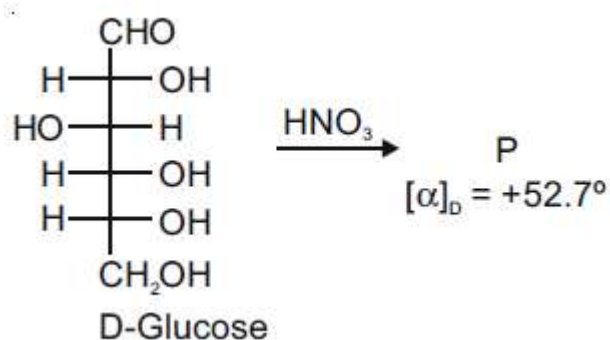
$$(T_s)_B = 100.075^\circ\text{C}$$

$$(T_s)_A - (T_s)_B = 100.1 - 100.075 = 0.025^\circ\text{C}$$

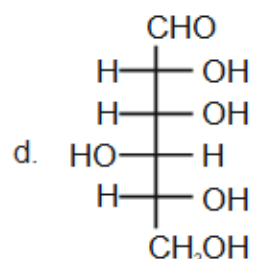
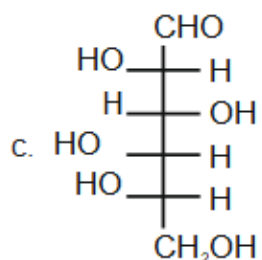
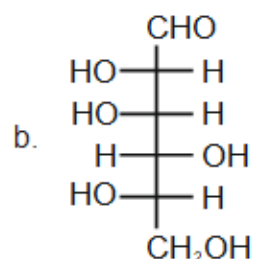
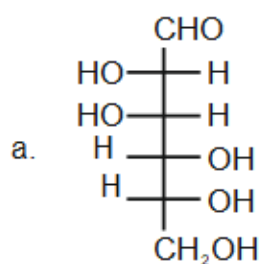
$$= 2.5 \times 10^{-2}^\circ\text{C}$$

So $x = 100.1$ and $|y| = 2.5$

Question 11. Given:



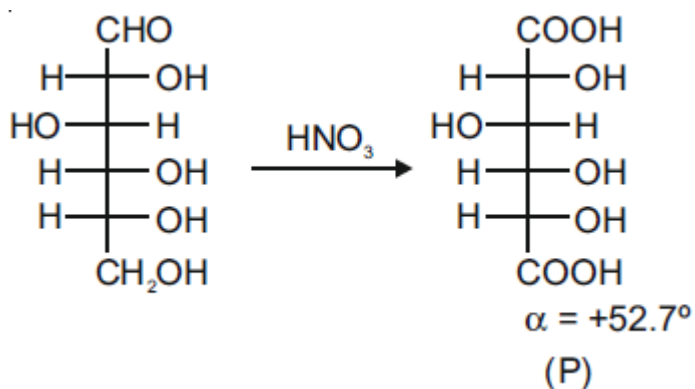
The compound(s), which on reaction with HNO_3 will give the product having a degree of rotation, $[\alpha]_D = -52.7^\circ$ is(are);



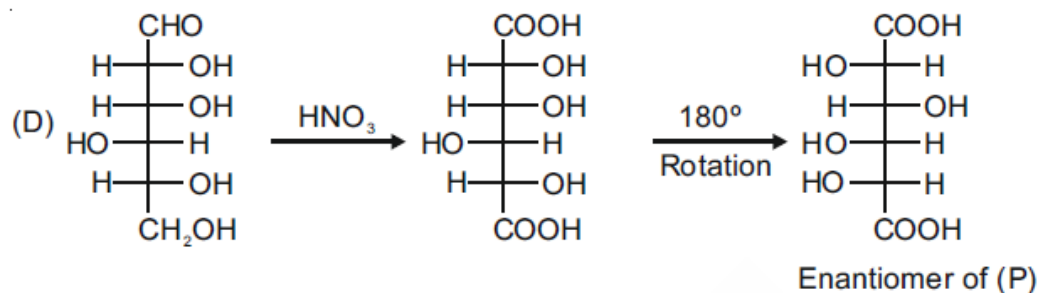
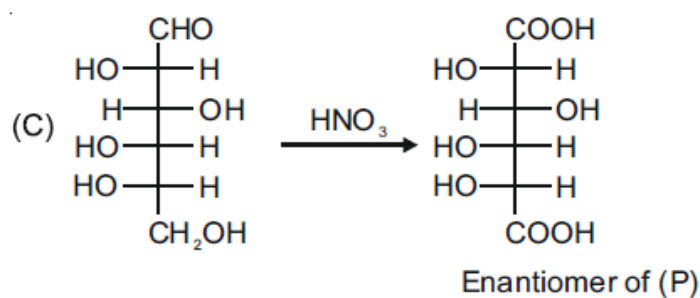
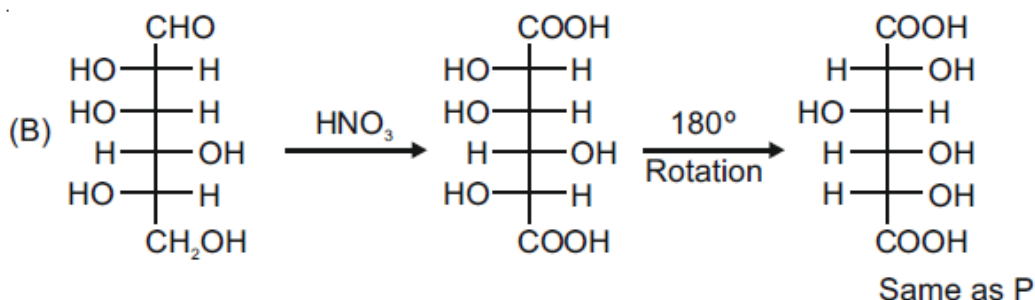
Solution:

Answer: (c, d)





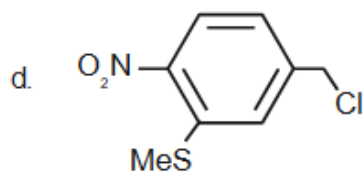
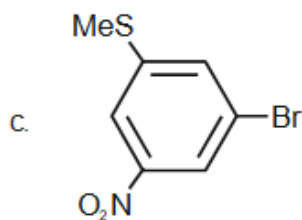
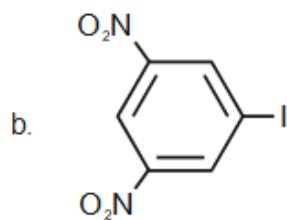
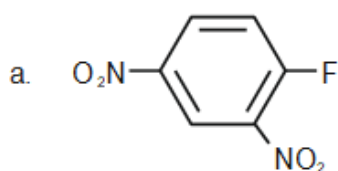
The enantiomer of (P) will have -52.7° rotation. So the reactant must be an isomer of D-glucose which can give the mirror image of (P)



So answer must be C and D

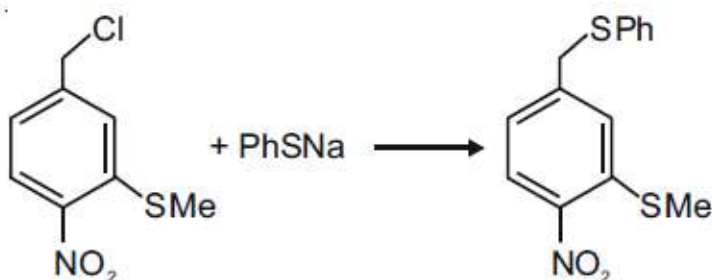
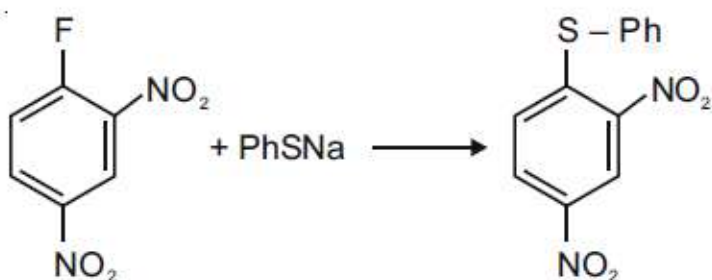
Question 12. The reaction of Q with PhSNa yields an organic compound (major product) that gives a positive Carius test on treatment with Na_2O_2 followed by the addition of BaCl_2 . The correct option(s) for Q is(are).





Solution:

Answer: (a, d)



The answer should be (a) and (d)

Compounds given in options - b and c do not react with PhSNa.

Question 13. The correct statement(s) related to colloids is(are)

- a. The process of precipitating colloidal sol by an electrolyte is called peptization
- b. Colloidal solution freezes at a higher temperature than the true solution at the same concentration
- c. Surfactants form micelle above critical micelle concentration (CMC). CMC depends on temperature
- d. Micelles are macromolecular colloids

Solution:

Answer: (b, c)

(a) The process of precipitating colloidal sol by an electrolyte is called peptization - False, (It is a process of converting precipitate into colloid)

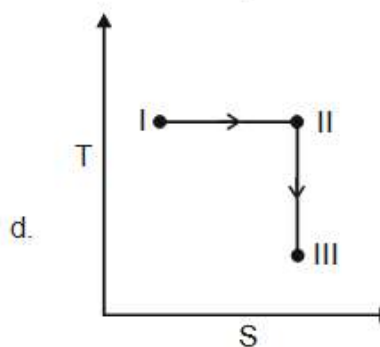
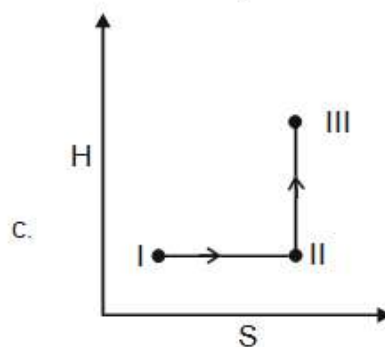
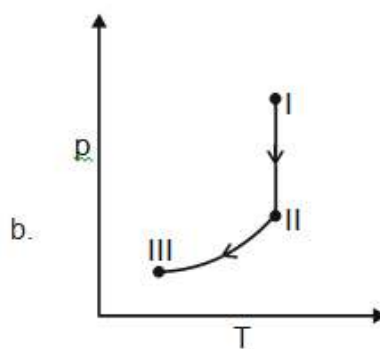
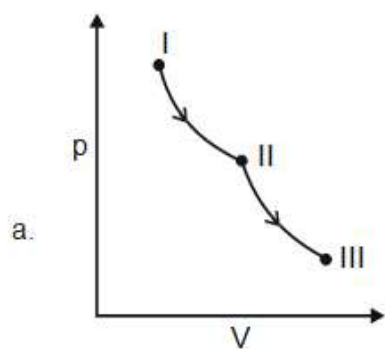


(b) Colloidal solution freezes at a higher temperature than the true solution at the same concentration - True (colligative properties)

(c) Surfactants form micelles above critical micelle concentration (CMC). CMC depends on temperature - True

(d) Micelles are macromolecular colloids - False, As micelles are associated colloids.

Question 14. An ideal gas undergoes a reversible isothermal expansion from the state I to state II followed by a reversible adiabatic expansion from state II to state III. The correct plot(s) representing the changes from the state I to state III is(are) (p: pressure, V: volume, T: temperature, H: enthalpy, S: entropy)



Solution:

Answer: (a, b, d)

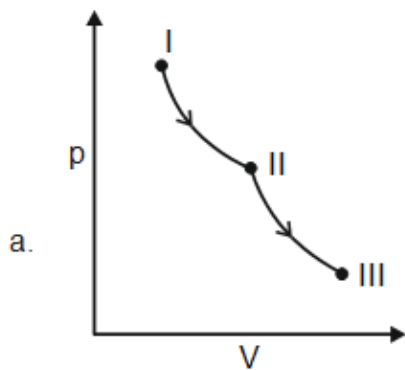
I \rightarrow II \rightarrow reversible, isothermal expansion,

T \rightarrow constant, $\Delta V \rightarrow +ve$, $\Delta S \rightarrow +ve$ $\Delta H \Rightarrow 0$

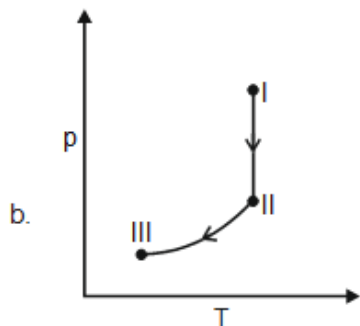
II \rightarrow III \rightarrow Reversible, adiabatic expansion

Q = 0, $\Delta V \rightarrow +ve$, $\Delta S \rightarrow 0$

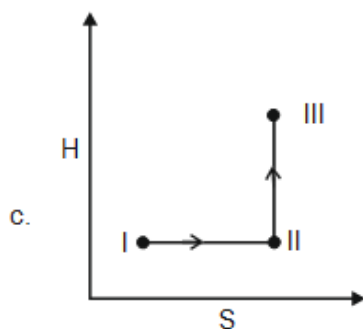




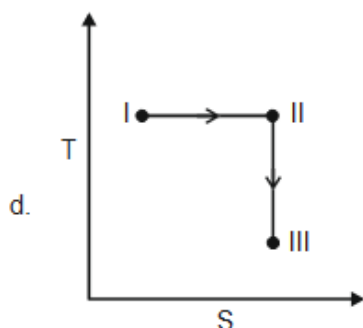
-ve slope - isothermal <
 adiabatic I → II → Isothermal
 II → III → Adiabatic



I → II → T constant
 II → III → adiabatic



I → II → $\Delta S \rightarrow +ve$, $\Delta H \Rightarrow 0$
 II → III → $\Delta S \rightarrow 0$, $\Delta H \rightarrow -ve$



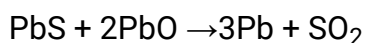
I → II → $\Delta S \rightarrow +ve$, $\Delta T = 0$
 II → III → $\Delta S \rightarrow 0$

Question 15. The correct statement(s) related to the metal extraction processes is(are);

- A mixture of PbS and PbO undergoes self-reduction to produce Pb and SO₂.
- In the extraction process of copper from copper pyrites, silica is added to produce copper silicate.
- Partial oxidation of sulphide ore of copper by roasting, followed by self-reduction produces blister copper.
- In the cyanide process, zinc powder is utilized to precipitate gold from Na[Au(CN)₂].

Solution:

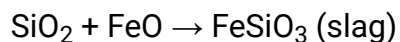
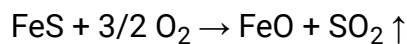
Answer: (a, c, d)



Self-reduction is taking place between PbS and PbO.

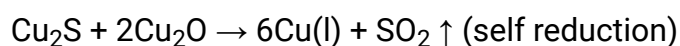
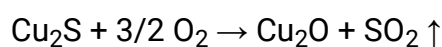


In the Bessemer converter: The raw material for the Bessemer converter is matte, i.e., $\text{Cu}_2\text{S} + \text{FeS}$ (little). Here air blasting is initially done for slag formation and SiO_2 is added from an external source.

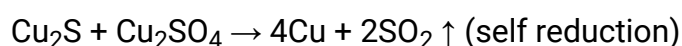
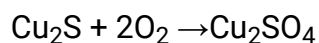


During slag formation, the characteristic green flame is observed at the mouth of the Bessemer converter which indicates the presence of iron in the form of FeO . The disappearance of this green flame indicates that the slag formation is complete. Then air blasting is stopped and slag is removed.

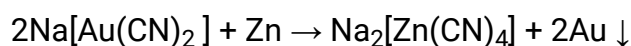
Again air blasting is restarted for partial roasting before self-reduction until two-thirds of Cu_2S is converted into Cu_2O . After this, only heating is continued for the self-reduction process.



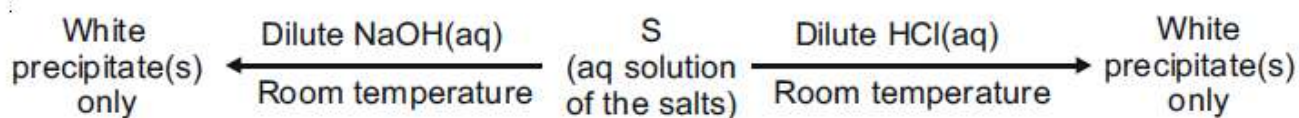
and



Thus the molten Cu obtained is poured into a large container and allowed to cool and during cooling the dissolved SO_2 comes up to the surface and forms blisters. It is known as blister copper.



Question 16. A mixture of two salts is used to prepare a solution S, which gives the following results:



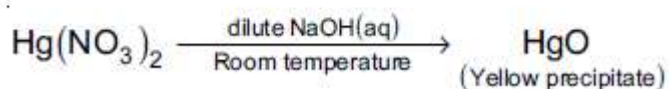
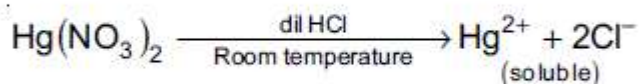
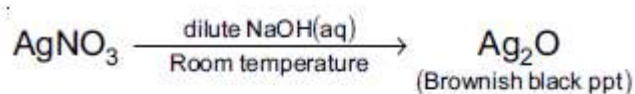
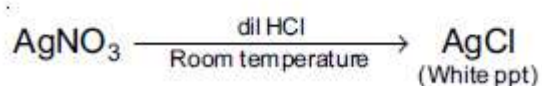
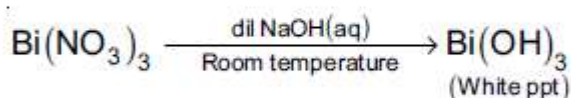
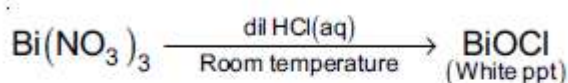
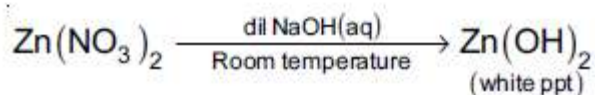
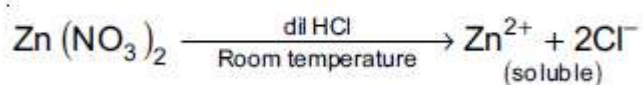
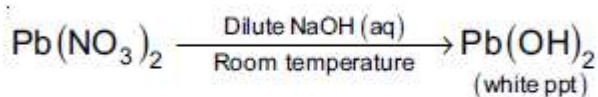
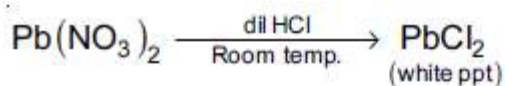
The correct option(s) for the salt mixture is(are)

- $\text{Pb(NO}_3)_2$ and $\text{Zn(NO}_3)_2$
- $\text{Pb(NO}_3)_2$ and $\text{Bi(NO}_3)_2$
- AgNO_3 and $\text{Bi(NO}_3)_3$
- $\text{Pb(NO}_3)_2$ and $\text{Hg(NO}_3)_2$

Solution:

Answer: (a, b)



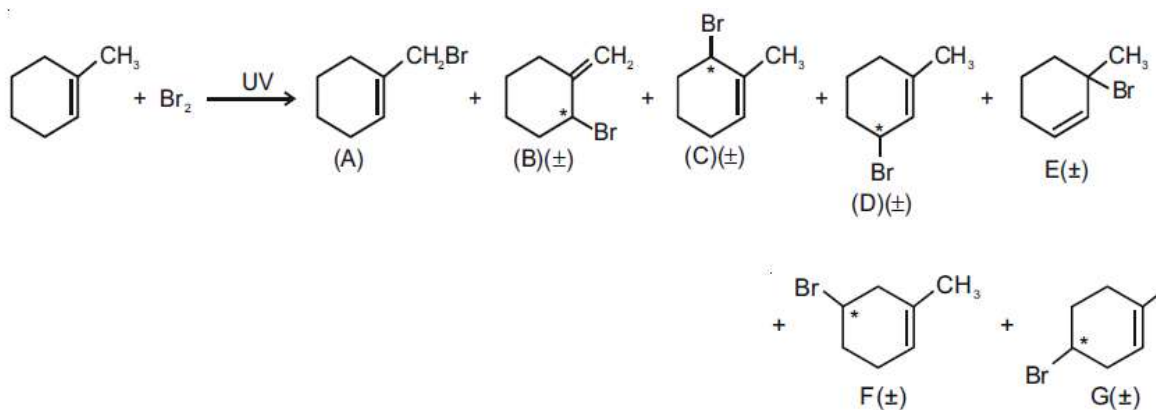


Question 17. The maximum number of possible isomers (including stereoisomers) which may be formed on mono-bromination of 1-methylcyclohex-1-ene using Br_2 and UV light is _____.

Solution:

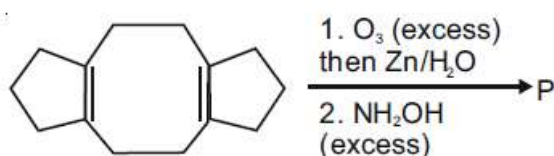
Answer: (13)





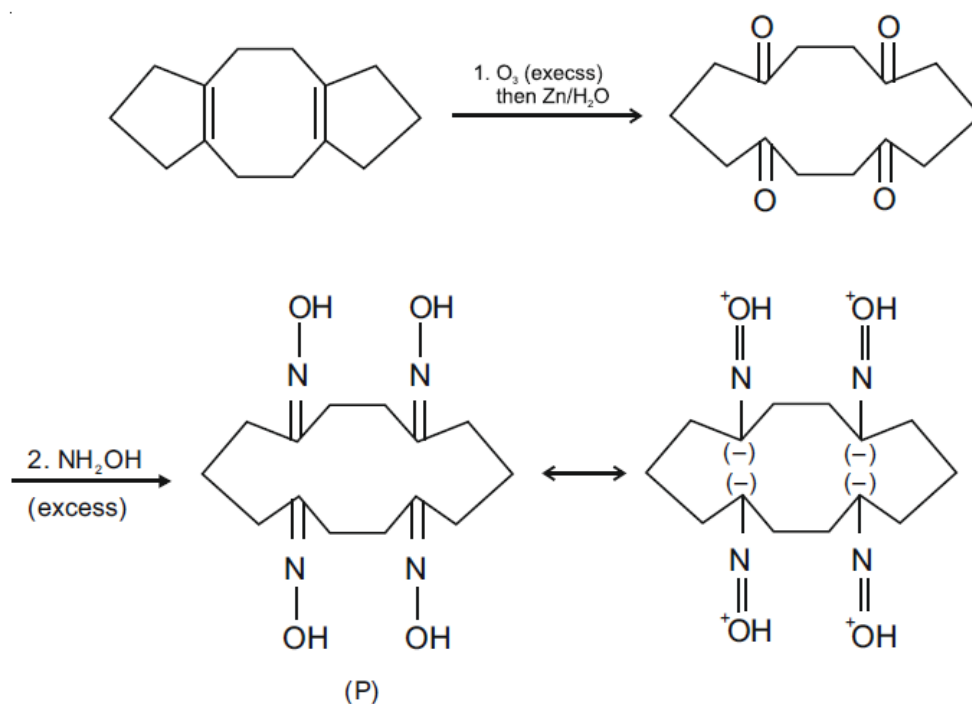
Monobromination of 1-methylcyclohexene in the presence of UV light proceeds by a free radical mechanism. The allyl radicals are formed which are stabilised by resonance. The secondary alkyl radicals are also formed which are stabilised by hyperconjugation. Of the seven products formed, six of them are optically active. So, 13 possible isomers are formed.

Question 18. In the reaction given below, the total number of atoms having sp^2 hybridization in the major product P is _____.



Solution:

Answer: (12)



The total number of atoms having sp^2 hybridisation in the major product (P) = 12

This includes 4 C-atoms, 4 N-atoms and 4 O-atoms.

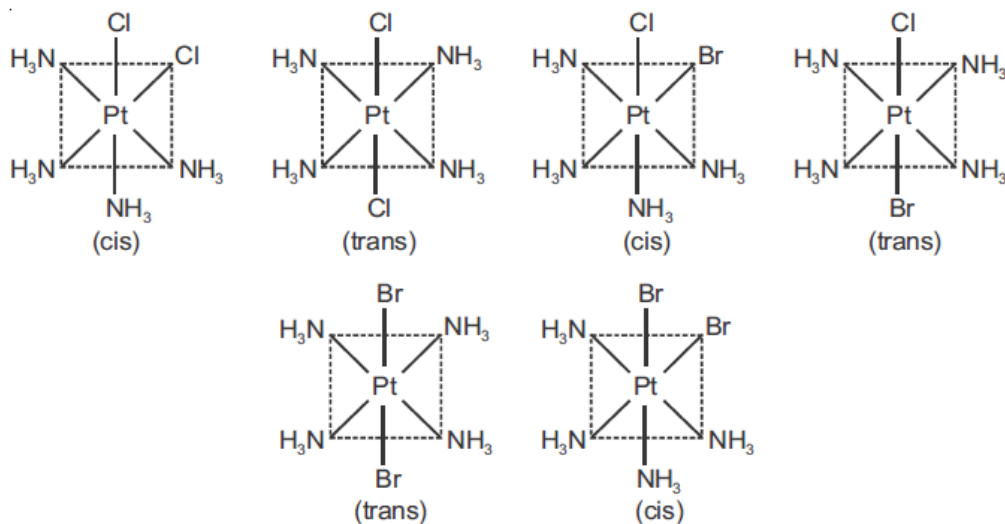


Question 19. The total number of possible isomers for $[\text{Pt}(\text{NH}_3)_4\text{Cl}_2]\text{Br}_2$ is

Solution:

Answer: (6)

The given complex $[\text{Pt}(\text{NH}_3)_4\text{Cl}_2]\text{Br}_2$ has three ionisation isomers and each of them has two geometrical isomers.



JEE Advanced 2021 Paper 1 Maths Question Paper

Question 1: Consider a triangle Δ whose two sides lie on the x-axis and the line $x + y + 1 = 0$. If the orthocenter of Δ is $(1, 1)$, then the equation of the circle passing through the vertices of the triangle Δ is;

- a. $x^2 + y^2 - 3x + y = 0$
- b. $x^2 + y^2 + x + 3y = 0$
- c. $x^2 + y^2 + 2y - 1 = 0$
- d. $x^2 + y^2 + x + y = 0$

Solution:

Answer: b

As we know mirror image of the orthocenter lies on the circumcircle.

Image of $(1, 1)$ in x-axis is $(1, -1)$

Image of $(1, 1)$ in $x + y + 1 = 0$ is $(-2, -2)$.

\therefore The required circle will be passing through both $(1, -1)$ and $(-2, -2)$.

\therefore Only $x^2 + y^2 + x + 3y = 0$ satisfy both.



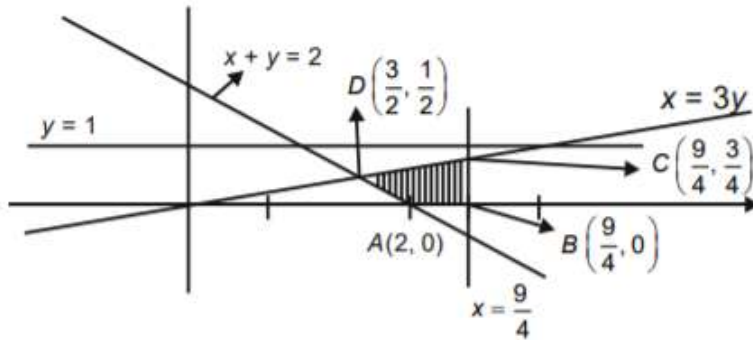
Question 2: The area of the region $\{(x, y): 0 \leq x \leq 9/4, 0 \leq y \leq 1, x \geq 3y, x + y \geq 2\}$

- a. 11/32
- b. 35/96
- c. 37/96
- d. 13/32

Solution:

Answer: a

A rough sketch of the required region is;



\therefore Required area is area of $\triangle ACD$ + Area of $\triangle ABC$

i.e $(\frac{1}{4}) + (3/32) = 11/32$ sq.units.

Question 3: Consider three sets $E_1 = \{1, 2, 3\}$, $F_1 = \{1, 3, 4\}$ and $G_1 = \{2, 3, 4, 5\}$. Two elements are chosen at random, without replacement, from the set E_1 , and let S_1 denote the set of these chosen elements. Let $E_2 = E_1 - S_1$ and $F_2 = F_1 \cup S_1$. Now two elements are chosen at random, without replacement, from the set F_2 and let S_2 denote the set of these chosen elements.

Let $G_2 = G_1 \cup S_2$. Finally, two elements are chosen at random, without replacement from the set G_2 and let S_3 denote the set of these chosen elements. Let $E_3 = E_2 \cup S_3$. Given that $E_1 = E_3$, let p be the conditional probability of the event $S_1 = \{1, 2\}$. Then the value of p is;

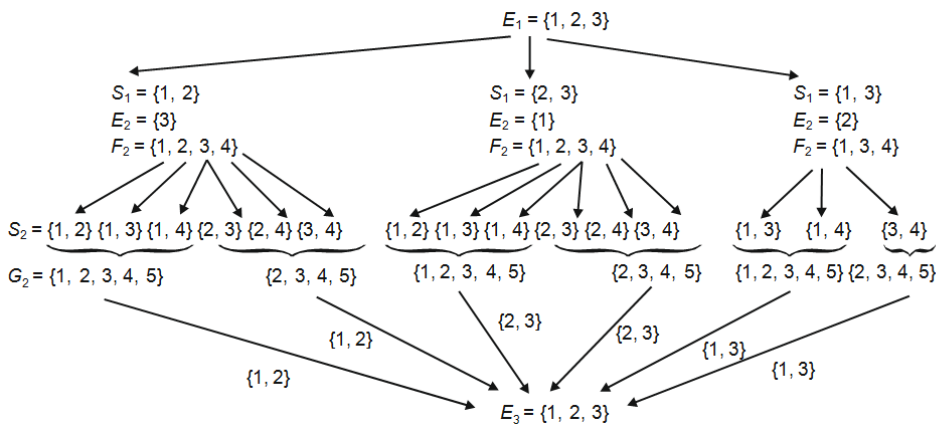
- a. 1/5
- b. 3/5
- c. 1/2
- d. 2/5

Solution:

Answer: a

We will follow the tree diagram,





$$P(E_1 = E_3) = \frac{1}{3} \left[\left(\frac{1}{2} \times \frac{1}{10} \right) + \left(\frac{1}{2} \times 0 \right) + \left(\frac{1}{2} \times \frac{1}{10} \right) + \left(\frac{1}{2} \times \frac{1}{6} \right) + \left(\frac{2}{3} \times \frac{1}{10} \right) + \left(\frac{1}{3} \times 0 \right) \right]$$

$$= \frac{1}{3} \left(\frac{1}{4} \right)$$

$$\text{Required probability} = \frac{1}{3} \left(\frac{1}{2} \times \frac{1}{10} \right) / \left(\frac{1}{3} \times \frac{1}{4} \right)$$

$$= \frac{1}{5}$$

Question 4: Let $\theta_1, \theta_2, \dots, \theta_{10}$ be positive valued angles (in radian) such that $\theta_1 + \theta_2 + \dots + \theta_{10}$

$= 2\pi$. Define the complex numbers $z_1 = e^{i\theta_1}$, $z_k = z_{k-1} e^{i\theta_k}$ for $k = 2, 3, \dots, 10$, where $i =$

$\sqrt{-1}$. Consider the statement P and Q given below:

P: $|z_2 - z_1| + |z_3 - z_2| + \dots + |z_{10} - z_9| + |z_1 - z_{10}| \leq 2\pi$

Q: $|z_2^2 - z_1^2| + |z_3^2 - z_2^2| + \dots + |z_{10}^2 - z_9^2| + |z_1^2 - z_{10}^2| \leq 4\pi$

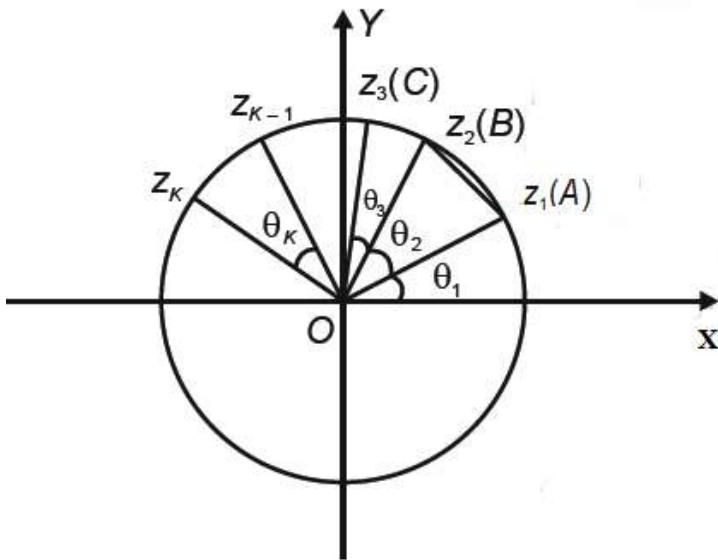
Then,

- a. P is TRUE and Q is FALSE
- b. Q is TRUE and P is FALSE
- c. Both P and Q are TRUE
- d. Both P and Q are FALSE

Solution:

Answer: c





$|z_2 - z_1|$ = length of line AB \leq length of arc AB

$|z_3 - z_2|$ = length of line BC \leq length of arc BC

\therefore Sum of length of these 10 lines \leq Sum of length of arcs (i.e. 2π)

(As $\theta_1 + \theta_2 + \dots + \theta_{10} = 2\pi$)

$\therefore |z_2 - z_1| + |z_3 - z_2| + \dots + |z_1 - z_{10}| \leq 2\pi$

And $|z_k^2 - z_{k-1}^2| = |z_k - z_{k-1}| |z_k + z_{k-1}|$

As we know $|z_k + z_{k-1}| \leq |z_k| + |z_{k-1}| \leq 2$

$|z_2^2 - z_1^2| + |z_3^2 - z_2^2| + \dots + |z_1^2 - z_{10}^2| \leq 2(|z_2 - z_1| + |z_3 - z_2| + \dots + |z_1 - z_{10}|) \leq 2(2\pi)$

\therefore Both P and Q are true.

Question Stem for Question Nos. 5 and 6

Three numbers are chosen at random, one after another with replacement, from the set $S = \{1, 2, 3, \dots, 100\}$. Let p_1 be the probability that the maximum of chosen numbers is at least 81 and p_2 be the probability that the minimum of chosen numbers is at most 40.

Question 5: The value of $(625/4)p_1$ is

Solution:

Answer: 76.25

For p_1 , we need to remove the cases when all three numbers are less than or equal to 80.

So $p_1 = 1 - (80/100)^3$

= 61/125

So $(625/4)p_1 = (625/4) \times 61/125$

= 305/4



$$= 76.25$$

Question 6: The value of $(125/4)p_2$ is

Solution:

Answer: 24.50

For p_2 , we need to remove the cases when all three numbers are greater than 40.

$$\text{So } p_2 = 1 - (60/100)^3$$

$$= 98/125$$

$$\text{So } (125/4)p_2 = (125/4) \times 98/125$$

$$= 24.50$$

Question Stem for Question Nos. 7 and 8

Let α , β and γ be real numbers such that the system of linear equations

$$x + 2y + 3z = \alpha$$

$$4x + 5y + 6z = \beta$$

$$7x + 8y + 9z = \gamma - 1 \text{ is consistent.}$$

Let $|M|$ represent the determinant of the matrix.

$$M = \begin{bmatrix} \alpha & 2 & \gamma \\ \beta & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Let P be the plane containing all those (α, β, γ) for which the above system of linear equations is consistent, and D be the square of the distance of the point $(0, 1, 0)$ from the plane P .

Question 7: The value of $|M|$ is

Solution:

Answer: 1

Question 8: The value of D is

Solution:

Answer: 1.50

Solution for Q.7 and Q.8



$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 0$$

Given system of equation will be consistent even if $\alpha = \beta = \gamma - 1 = 0$, i.e. equations will form homogeneous system.

$$\text{So } \alpha = 0, \beta = 0, \gamma = 1$$

$$M = \begin{vmatrix} 0 & 2 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$$= -1 \times (-1)$$

$$= 1$$

As given equations are consistent

$$x + 2y + 3z - \alpha = 0 \dots P_1$$

$$4x + 5y + 6z - \beta = 0 \dots P_2$$

$$7x + 8y + 9z - (\gamma - 1) = 0 \dots P_3$$

For some scalar λ and μ

$$\mu P_1 + \lambda P_2 = P_3$$

$$\mu(x + 2y + 3z - \alpha) + \lambda(4x + 5y + 6z - \beta) = 7x + 8y + 9z - (\gamma - 1)$$

comparing coefficients

$$\mu + 4\lambda = 7, 2\mu + 5\lambda = 8, 3\mu + 6\lambda = 9$$

$\lambda = 2$ and $\mu = -1$ satisfy all these conditions

comparing constant terms,

$$\alpha\mu - \beta\lambda = -(\gamma - 1)$$

$$\alpha - 2\beta + \gamma = 1$$

So equation of plane is

$$x - 2y + z = 1$$

$$\text{distance from } (0, 1, 0) = \frac{|(-2-1)|}{\sqrt{6}} = \frac{3}{\sqrt{6}}$$

$$D = \left(\frac{3}{\sqrt{6}}\right)^2 = \frac{9}{6} = \frac{3}{2} = 1.50$$

Question Stem for Question Nos. 9 and 10

Consider the lines L_1 and L_2 defined by

$$L_1 : x\sqrt{2} + y - 1 = 0 \text{ and } L_2 : x\sqrt{2} - y + 1 = 0$$



For a fixed constant λ , let C be the locus of a point P such that the product of the distance of P from L_1 and the distance of P from L_2 is λ^2 . The line $y = 2x + 1$ meets C at two points R and S , where the distance between R and S is $\sqrt{270}$.

Let the perpendicular bisector of RS meet C at two distinct points R' and S' . Let D be the square of the distance between R' and S' .

Question 9: The value of λ^2 is

Solution:

Answer: 9

Question 10: The value of D is

Solution:

Answer: 77.14

Solution for Q.9 and Q.10

$$C: |(x\sqrt{2} + y - 1)/\sqrt{3}| |(x\sqrt{2} - y + 1)/\sqrt{3}| = \lambda^2$$

$$\Rightarrow C: |2x^2 - (y-1)^2| = 3\lambda^2$$

C cuts $y-1 = 2x$ at $R(x_1, y_1)$ and $S(x_2, y_2)$

$$\text{So } |2x^2 - 4x^2| = 3\lambda^2$$

$$\Rightarrow x = \pm\sqrt{(3/2)} |\lambda|$$

$$\text{So } |x_1 - x_2| = \sqrt{6}|\lambda| \text{ and } |y_1 - y_2| = 2|x_1 - x_2| = 2\sqrt{6}|\lambda|$$

$$\text{Since } RS^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$$

$$\Rightarrow 270 = 30\lambda^2$$

$$\Rightarrow \lambda^2 = 270/30 = 9$$

Since slope of $RS = 2$ and midpoint of RS is $((x_1+x_2)/2, (y_1+y_2)/2) \equiv (0, 1)$

$$\text{So } R'S' \equiv y-1 = -\frac{1}{2}x$$

$$\text{Solving } y-1 = -\frac{1}{2}x \text{ with 'C' we get } x^2 = (12/7)\lambda^2$$

$$\Rightarrow |x_1 - x_2| = 2\sqrt{(12/7)} |\lambda| \text{ and } |y_1 - y_2| = \frac{1}{2}|x_1 - x_2| = \sqrt{(12/7)} |\lambda|$$

$$\text{Hence } D = (R'S')^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 = (12/7)9 \times 5 \approx 77.14$$

Question 11: For any 3×3 matrix M , let $|M|$ denote the determinant of M . Let $E =$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 8 & 13 & 18 \end{bmatrix},$$



$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix},$$

$$\text{And } F = \begin{bmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{bmatrix}$$

If Q is a nonsingular matrix of order 3×3 , then which of the following statements is(are) TRUE?

a. $F = PEP$ and $P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

b. $|EQ + PFQ^{-1}| = |EQ| + |PFQ^{-1}|$

c. $|(EF)^3| > |EF|^2$

d. Sum of the diagonal entries of $P^{-1}EP + F$ is equal to the sum of diagonal entries of $E + P^{-1}FP$

Solution:

Answer: (a, b, d)

P is formed from I by exchanging second and third rows or by exchanging second and third columns.

So, PA is a matrix formed from A by changing the second and third rows.

Similarly, AP is a matrix formed from A by changing the second and third columns.

Hence, $\text{Tr}(PAP) = \text{Tr}(A) \dots(1)$

(a) Clearly $P.P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

And $PE = \begin{bmatrix} 1 & 2 & 3 \\ 8 & 13 & 18 \\ 2 & 3 & 4 \end{bmatrix}$

$\Rightarrow PEP = \begin{bmatrix} 1 & 3 & 2 \\ 8 & 18 & 13 \\ 2 & 4 & 3 \end{bmatrix} = F$

$\Rightarrow PEP = F \Rightarrow PFP = E \dots(2)$

(b) Since $|E| = |F| = 0$

So, $|EQ + PFQ^{-1}| = |PFPQ + PFQ^{-1}| = |P| |F| |PQ + Q^{-1}| = 0$

Also, $|EQ| + |PFQ^{-1}| = 0$



(c) From (2); $PF = E$ and $|P| = -1$

So, $|F| = |E|$

Also, $|E| = 0 = |F|$

So, $|EF|^3 = 0 = |EF|^2$

(d) since $P^2 = I \Rightarrow P^{-1} = P$

So, $\text{Tr}(P^{-1}EP + F) = \text{Tr}(PEP + F) = \text{Tr}(2F)$

Also $\text{Tr}(E + P^{-1}FP) = \text{Tr}(E + PFP) = \text{Tr}(2E)$

Given that $\text{Tr}(E) = \text{Tr}(F)$

$\Rightarrow \text{Tr}(2E) = \text{Tr}(2F)$

Question 12: Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \frac{x^2-3x-6}{x^2+2x+4}$. Then which of the following statements is (are) TRUE?

- a. f is decreasing in the interval $(-2, -1)$
- b. f is increasing in the interval $(1, 2)$
- c. f is onto
- d. Range of f is $[-3/2, 2]$

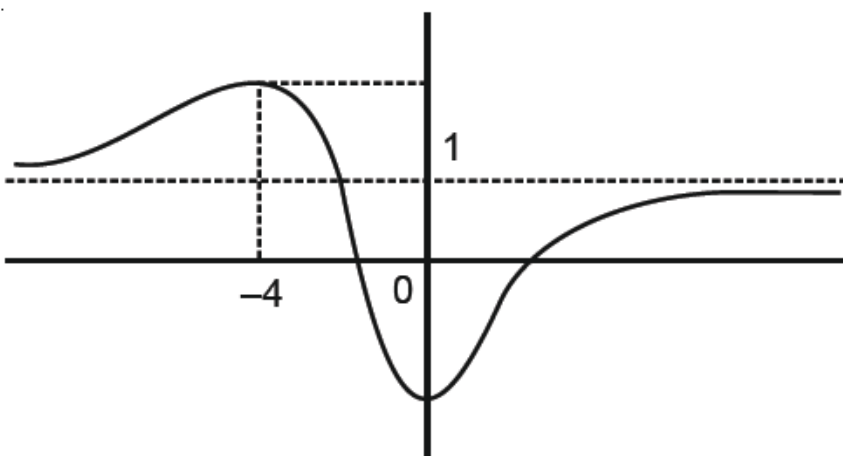
Solution:

Answer: (a, b)

$$f(x) = \frac{x^2-3x-6}{x^2+2x+4}$$

$$\Rightarrow f'(x) = \frac{5x(x+4)}{(x^2+2x+4)^2}$$

$\Rightarrow f(x)$ has local maxima at $x = -4$ and minima at $x = 0$



Range of $f(x)$ is $[-3/2, 11/6]$

Question 13: Let E, F and G be three events having probabilities $P(E) = 1/8$, $P(F) = 1/6$ and $P(G) = 1/4$, and $P(E \cap F \cap G) = 1/10$. For any event H , if H^c denotes its complement, then which of the following statements is (are) TRUE?



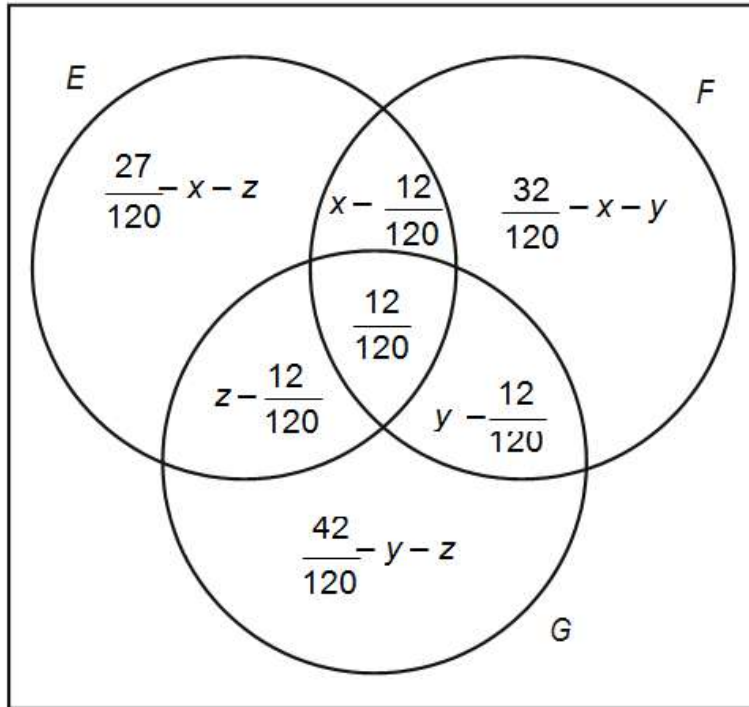
- a. $P(E \cap F \cap G^c) \leq 1/40$
- b. $P(E^c \cap F \cap G) \leq 1/15$
- c. $P(E \cup F \cup G) \leq 13/24$
- d. $P(E^c \cap F^c \cap G^c) \leq 5/12$

Solution:

Answer: (a, b, c)

Let $P(E \cap F) = x$, $P(F \cap G) = y$ and $P(E \cap G) = z$

Clearly $x, y, z \geq 1/10$



Since $x + z \leq 27/120$

$\Rightarrow x, z \leq 15/120$

$x + y \leq 32/120$

$\Rightarrow x, y \leq 20/120$

And $y + z \leq 42/120 \Rightarrow y, z \leq 30/120$

Now $P(E \cap F \cap G^c) = x - 12/120 \leq 3/120 = 1/40$

$P(E^c \cap F \cap G) = y - 12/120 \leq 80/120 = 1/15$

$P(E \cup F \cup G) \leq P(E) + P(F) + P(G) = 13/24$

And $P(E^c \cap F^c \cap G^c) = 1 - P(E \cup F \cup G) \geq 11/24 \geq 5/12$

Question 14: For any 3×3 matrix M , let $|M|$ denote the determinant of M . Let I be the 3×3 identity matrix. Let E and F be two 3×3 matrices such that $(I - EF)$ is invertible. If $G = (I - EF)^{-1}$, then which of the following statements is(are) TRUE?

- a. $|FE| = |I - FE| |FGE|$



- b. $(I - FE)(I + FGE) = I$
- c. $EFG = GEF$
- d. $(I - FE)(I - FGE) = I$

Solution:

Answer: (a, b, c)

$$I - EF = G^{-1}$$

$$G - GEF = I \dots(1)$$

$$\text{And } G - EFG = I \dots(2)$$

Clearly $GEF = EFG$ (option C is correct)

$$\begin{aligned} \text{Also } (I - FE)(I + FGE) &= I - FE + FGE - FE + FGE \\ &= I - FE + FGE - F(G - I)E \\ &= I - FE + FGE - FGE + FE \\ &= I \text{ (option B is correct and D is incorrect)} \end{aligned}$$

$$\begin{aligned} \text{Now, } (I - FE)(I - FGE) &= I - FE - FGE + F(G - I)E \\ &= I - 2FE \end{aligned}$$

$$(I - FE)(-FGE) = -FE$$

$$|(I - FE)(FGE)| = |FE|$$

Question 15: For any positive integer n, let $S_n: (0, \infty) \rightarrow \mathbb{R}$ be defined by $S_n(x) = \sum_{k=1}^n \cot^{-1}((1 + k(k+1)x^2)/x)$ where for any $x \in \mathbb{R}$, $\cot^{-1}(x) \in (0, \pi)$ and $\tan^{-1}(x) \in (-\pi/2, \pi/2)$. Then which of the following statements is (are) TRUE?

- a. $S_{10}(x) = \pi/2 - \tan^{-1}(1 + 11x^2)/10x$, for all $x > 0$
- b. $\lim_{n \rightarrow \infty} \cot(S_n(x)) = x$, for all $x > 0$
- c. The equation $S_3(x) = \pi/4$ has a root in $(0, \infty)$
- d. $\tan(S_n(x)) \leq 1/2$, for all $n \geq 1$ and $x > 0$

Solution:

Answer: (a, b)

$$S_n(x) = \sum_{k=1}^n \tan^{-1} \left(\frac{(k+1)x - kx}{1 + kx \cdot (k+1)x} \right)$$

$$= \sum_{k=1}^n (\tan^{-1}((k+1)x) - \tan^{-1}(kx))$$

$$= \tan^{-1}((n+1)x) - \tan^{-1}(x)$$

$$= \tan^{-1}(nx/(1+(n+1)x^2))$$



Now,

(a) $S_{10}(x) = \tan^{-1}(10x/(1+11x^2)) = \pi/2 - \tan^{-1}((1+11x^2)/10x)$

(b) $\lim_{n \rightarrow \infty} \cot(S_n(x)) = \cot(\tan^{-1}x/x^2) = x$

(c) $S_3(x) = \pi/4 \Rightarrow 3x/(1+4x^2) = 1$

$\Rightarrow 4x^2 - 3x + 1 = 0$ has no real root.

(d) for $x = 1$, $\tan(S_n(x)) = n/(n+2)$ which is greater than $1/2$ for $n \geq 3$ so this option is incorrect.

Question 16: For any complex number $w = c + id$, let $\arg(w) \in (-\pi, \pi]$, where $i = \sqrt{-1}$. Let α and β be real numbers such that for all complex numbers $z = x + iy$ satisfying $\arg(z+\alpha)/(z+\beta) = \pi/4$, the ordered pair (x, y) lies on the circle $x^2 + y^2 + 5x - 3y + 4 = 0$. Then which of the following statements is (are) TRUE?

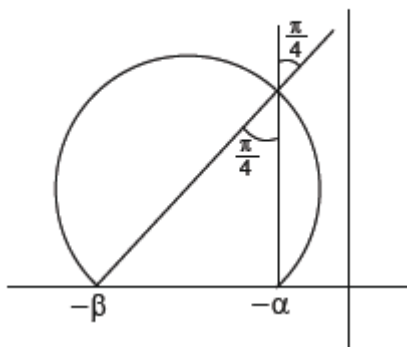
- a. $\alpha = -1$
- b. $\alpha\beta = 4$
- c. $\alpha\beta = -4$
- d. $\beta = 4$

Solution:

Answer: (b, d)

Circle $x^2 + y^2 + 5x - 3y + 4 = 0$ cuts the real axis (x-axis) at $(-4, 0)$, $(-1, 0)$

Clearly $\alpha = 1$ and $\beta = 4$



Question 17: For $x \in \mathbb{R}$, the number of real roots of the equation $3x^2 - 4|x^2 - 1| + x - 1 = 0$ is

Solution:

Answer: 4

$3x^2 - 4|x^2 - 1| + x - 1 = 0$

Let $x \in [-1, 1]$

$\Rightarrow 3x^2 - 4(-x^2+1) + x - 1 = 0$

$\Rightarrow 3x^2 + 4x^2 - 4 + x - 1 = 0$



$$\Rightarrow 7x^2 + x - 5 = 0$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1+140}}{2}$$

Both values are acceptable.

Let $x \in (-\infty, -1) \cup (1, \infty)$

$$x^2 - 4(x^2 - 1) + x - 1 = 0$$

$$\Rightarrow x^2 - x - 3 = 0$$

$$\Rightarrow x = \frac{1 \pm \sqrt{1+12}}{2}$$

Again both are acceptable.

Hence, total number of solution = 4.

Question 18: In a triangle ABC, let $AB = \sqrt{23}$, and $BC = 3$ and $CA = 4$. Then the value of $(\cot A + \cot C)/\cot B$ is;

Solution:

Answer: 2

With standard notations

Given $c = \sqrt{23}$, $a = 3$ and $b = 4$

Now $(\cot A + \cot C)/\cot B = (\cos A/\sin A + \cos C/\sin C)/(\cos B/\sin B)$

$$= ((b^2+c^2-a^2)/2bc \sin A + (a^2+b^2-c^2)/2ab \sin C)/((c^2+a^2-b^2)/2ac \sin B)$$

$$= ((b^2+c^2-a^2)/4\Delta + (a^2+b^2-c^2)/4\Delta)/((c^2+a^2-b^2)/4\Delta)$$

$$= 2b^2/(a^2+c^2-b^2)$$

$$= 2$$

Question 19: Let \vec{u}, \vec{v} and \vec{w} be vectors in three-dimensional space, where \vec{u} and \vec{v} are

unit vectors which are not perpendicular to each other and $\vec{u} \cdot \vec{w} = 1$, $\vec{v} \cdot \vec{w} = 1$ and

$\vec{w} \cdot \vec{w} = 4$. If the volume of the parallelepiped, whose adjacent sides are represented by

the vectors \vec{u}, \vec{v} and \vec{w} is $\sqrt{2}$, then the value of $|3\vec{u} + 5\vec{v}|$ is;

Solution:

Answer: 7



$$\text{Given } [\vec{u} \ \vec{v} \ \vec{w}] = \sqrt{2}$$

$$\text{a.so, } [\vec{u} \ \vec{v} \ \vec{w}]^2 = \begin{vmatrix} \vec{u} \cdot \vec{u} & \vec{u} \cdot \vec{v} & \vec{u} \cdot \vec{w} \\ \vec{v} \cdot \vec{u} & \vec{v} \cdot \vec{v} & \vec{v} \cdot \vec{w} \\ \vec{w} \cdot \vec{u} & \vec{w} \cdot \vec{v} & \vec{w} \cdot \vec{w} \end{vmatrix} = 2$$

Let $\vec{u} \cdot \vec{v} = k$ and substitute rest values, we get

$$\begin{vmatrix} 1 & K & 1 \\ K & 1 & 1 \\ 1 & 1 & 4 \end{vmatrix} = 2$$

$$4K^2 - 2K = 0$$

$$\Rightarrow \vec{u} \cdot \vec{v} = 0 \text{ (rejected)}$$

Or

$$\vec{u} \cdot \vec{v} = \frac{1}{2}$$

$$\text{Therefore } \vec{u} \cdot \vec{v} = \frac{1}{2}$$

$$|3\vec{u} + 5\vec{v}|^2 = 9 + 25 + 30(\frac{1}{2})$$

$$= 49$$

$$\Rightarrow |3\vec{u} + 5\vec{v}| = 7$$

