PART A – PHYSICS

1. A wire has a mass $0.4 \pm 0.004(g)$ and length $8 \pm 0.08(cm)$. The maximum percentage error in the measurement of its density is 4%. The radius of the wire is $r \pm \Delta r$, find Δr :

(A) 0.02 r (B) 0.01 r
(C) 0.03 r (D) 0.1 r

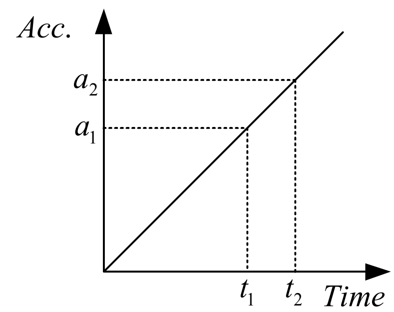
2. If velocity (v), force (F) and time (T) are taken to be fundamental quantity and K is the dimensionless constant of proportionality, find dimensional formula for mass:

(A) $[KVFT]$ (B) $[KV^{-2}FT]$
(C) $[KV^{-1}FT]$ (D) $[KV^{-1}FT^2]$

3. Three bodies A, B and C fall from rest from the same height. B falls T second after A and C falls T second after B. The time starts at $t = 0$. A falls at $t = 0$. There is such a time t from the start, at which distance between B and C is L. Find the time t :

(A) $\frac{L}{2} + T$ (B) $\frac{L}{gT} + \frac{T}{2}$
(C) $\frac{L}{gT} + \frac{3T}{2}$ (D) $\frac{L}{gT} + 2T$

4. Acceleration-time graph of a particle is shown. Work done by all the forces acting on the particle of mass m in time interval t_1 and t_2, while a_1 is the acceleration at time t_1, is given by :



(A) $\frac{ma_1^2}{4t_1}(t_2^3 - t_1^3)$

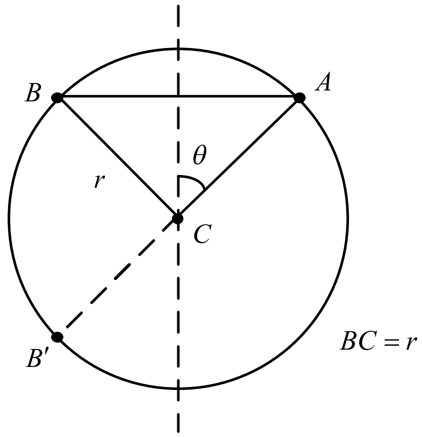
(B) $\frac{ma_1^2}{8t_1^2}(t_2^4 - t_1^4)$

(C) $\frac{ma_1^2}{4t_1^2}(t_2^4 - t_1^4)$

(D) $\frac{ma_1}{2t_1}(t_2^2 - t_1^2)$

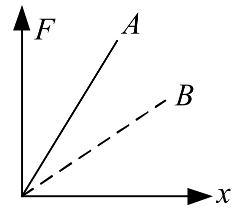
FOR ROUGH WORK

5. Three similar smooth balls attached with massless rods are placed at positions A, B and C in a vertical circular track so that C is at the centre and rod AC makes an angle θ with vertical. If the system rotates from rest such that ball at position B comes to position B' which is diametrically opposite to position A and ball at position A comes to position B, then find the velocity of ball at position B':



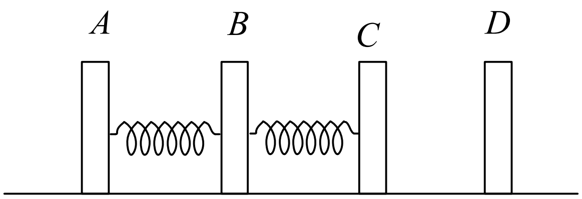
(A) $\sqrt{gr\cos\theta}$ (B) $\sqrt{gr\sin\theta}$
(C) $\sqrt{2gr\cos\theta}$ (D) $\sqrt{2gr\tan\theta}$

6. F is the force required to stretch a spring for a distance x. Line A shows $(F-x)$ curve for a spring of length L. Line B of $(F-x)$ curve will be for a spring of same spring constant, but of a length:



(A) greater than L (B) less than L
(C) equal to L (D) unpredictable

7. Three blocks A, B and C of masses m, m and 2m respectively are connected by the springs of spring constant K and of same length. The masses are moving to the right by uniform velocity V each so that springs are of natural length during motion. C collides with D of mass m inelastically which is at rest. Maximum compression of the spring between A and B is (Just before this moment velocity of A and B remains V):



(A) $V\sqrt{\frac{m}{12K}}$ (B) $V\sqrt{\frac{m}{9K}}$
(C) $V\sqrt{\frac{m}{15K}}$ (D) $V\sqrt{\frac{m}{K}}$

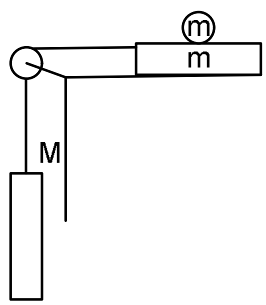
FOR ROUGH WORK

8. A particle moves in $x-y$ plane under the influence of a force such that its linear momentum is

$\vec{P}(t)=A\left[\hat{i}\cos(kt)-\hat{j}\sin(kt)\right]$, where A constants. The angle between the force and the momentum is:

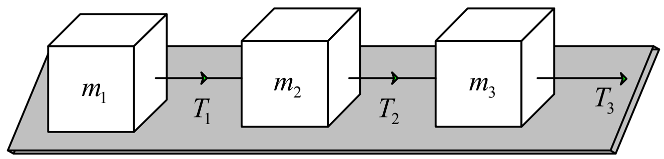
(A) 0° (B) 90°
(C) 30° (D) 45°

9. A plate of mass m is placed on a frictionless surface. The plate is connected to block of mass M through a rope over a massless pulley. A cylinder of mass m is placed on the plate which rolls without slipping. Find the frictional force acting on the cylinder :



(A) $\frac{(M+m)g}{6}$ (B) $\frac{mg}{6}$
(C) $\frac{Mmg}{3(M+2m)}$ (D) $\frac{2(M+m)g}{3}$

10. Three blocks of masses m_1, m_2 and m_3 are connected by massless strings as shown on a frictionless table. They are pulled with a force $T_3 = 40N$. If $m_1 = 10\,kg, m_2 = 6\,kg$ and $m_3 = 4\,kg$, the tension T_2 will be -



(A) 20 N (B) 40 N
(C) 10 N (D) 32 N

11. A wire is loaded by 6 kg at its one end, the increase in length is 12 mm. If the radius of the wire is doubled and all other magnitudes are unchanged, then increase in length will be-

(A) 6 mm (B) 3 mm
(C) 24 mm (D) 48 mm

12. Two capillary tubes of radii 0.2 cm and 0.4 cm are dipped in the same liquid. The ratio of heights through which liquid will rise in the tubes is :

(A) 1 : 2 (B) 2 : 1
(C) 1 : 4 (D) 4 : 1

FOR ROUGH WORK

13. 0.93 watt-hour of energy is supplied to a block of ice weighing 10 g. It is found that

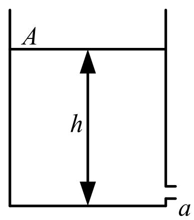
(A) Half of the block melts

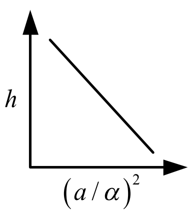
(B) The entire block melts and the water attains a temperature of $4°C$

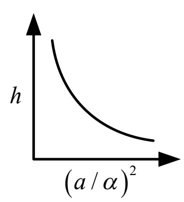
(C) The entire block just melts

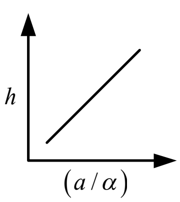
(D) The block remains unchanged

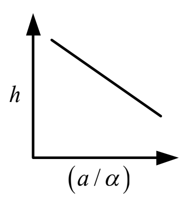
14. The water in the vessel is maintained at A at a height h. The area of the hole at the bottom of the vessel is a. The water in the vessel is poured at the constant rate α. The water in the vessel is poured at the constant rate α. Find the correct graph :



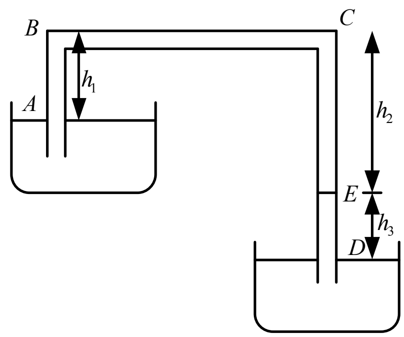
(A)


(B)


(C)


(D)


15. In the siphon as shown, which of the option is not correct, if $h_2 > h_1$ and $h_3 < h_1$?



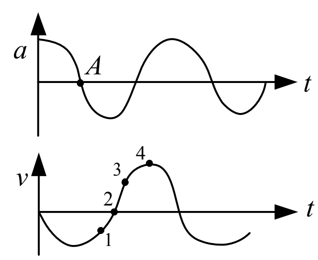
(A) $p_E < p_D$

(B) $p_E > p_C$

(C) $p_B > p_C$

(D) $p_E > p_B$

16. Corresponding to point A on acceleration time $(a-t)$ curve of a particle executing SHM, there is a point 1 or 2 or 3 or 4 on velocity time $(v-t)$ curve of the motion. The starting value of time on $x-$axis is different for two curves. Find the point :



(A) 1

(B) 2

(C) 3

(D) 4

FOR ROUGH WORK

17. Two pendulums of different lengths are in phase at the mean position at a certain instant. The minimum time after which they will be again in same phase is $\frac{5T}{4}$, where T is the time period of shorter pendulum. Find the ratio of lengths of the two pendulums :

(A) 1 : 16 (B) 1 : 4
(C) 1 : 2 (D) 1 : 25

18. A satellite is equatorial plane is rotating in the direction of earth's rotation with time interval between its two consecutive appearances overhead of an observer as time period of rotation of the earth, T_E. What is the time period of the satellite?

(A) T_E (B) $2T_E$
(C) $\frac{T_E}{2}$ (D) $\frac{2T_E}{3}$

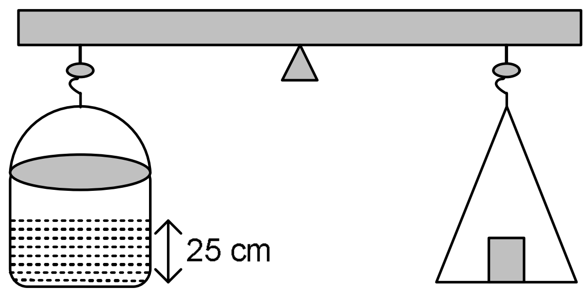
19. Time period of revolution of a nearest satellite around a planet of radius R is T. Period of revolution around another planet, whose radius is 3R but having same density is:

(A) T (B) 3T
(C) 9T (D) $3\sqrt{3}T$

20. A large horizontal surface moves up and down in S.H.M. with an amplitude of 1 cm. If a mass of 10 kg (which is placed on the surface) is to remain continually in contact with it, the maximum frequency will be -

(A) 0.5 Hz
(B) 1.5 Hz
(C) 5 Hz
(D) 10 Hz

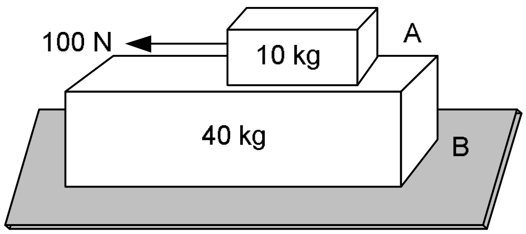
21. A cylinder containing water upto a height of 25 cm has a hole of cross-section $\frac{1}{4}cm^2$ in its bottom. It is counterpoised in a balance. What is the initial change in the balancing weight when water begins to flow out?



(A) Increase of 12.5 gm-wt
(B) Increase of 6.25 gm-wt
(C) Decrease of 12.5 gm-wt
(D) Decrease of 6.25 gm-wt

FOR ROUGH WORK

22. A 40 kg slab rests on a frictionless floor as shown in the figure. A 10 kg block rests on the top of the slab. The static coefficient of friction between the block and slab is 0.60 while the kinetic friction is 0.40. The 10 kg block is acted upon by a horizontal force 100 N. If $g = 9.8\,m/s^2$, the resulting acceleration of the slab will be -



(A) $0.98\,m/s^2$ (B) $1.47\,m/s^2$

(C) $1.52\,m/s^2$ (D) $6.1\,m/s^2$

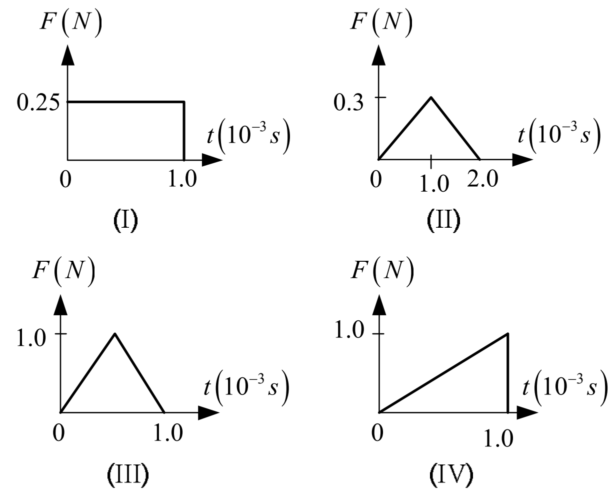
23. A body takes time t to reach the bottom of an inclined plane of angle θ with the horizontal. If the plane is made rough, time taken now is 2t. The coefficient of friction of the rough surface is :

(A) $\frac{3}{4}\tan\theta$ (B) $\frac{2}{3}\tan\theta$

(C) $\frac{1}{4}\tan\theta$

(D) $\frac{1}{2}\tan\theta$

24. Figures I, II, III and IV depict variation of force with time



The impulse is highest in the case of situation depicted. Figure -

(A) I and II (B) III and I

(C) III and IV (D) Only IV

25. A particle moving in a straight line covers half the distance with speed of 3 m/s. The other half of the distance is covered in two equal time intervals with speed of 4.5 m/s and 7.5 m/s respectively. The average speed of the particle during this motion is :

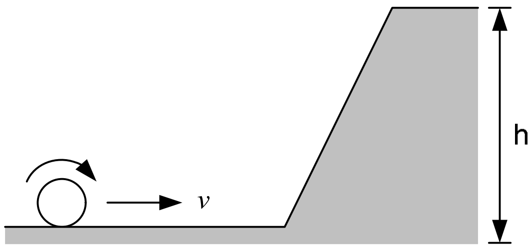
(A) 4.0 m/s (B) 5.0 m/s

(C) 5.5 m/s

(D) 4.8 m/s

FOR ROUGH WORK

26. A solid sphere is rolling on a frictionless surface, shown in figure with a translational velocity v m/s. If sphere climbs up to height h then value of v should be -



(A) $\geq \sqrt{\frac{10}{7} gh}$ (B) $\geq \sqrt{2gh}$

(C) $2gh$ (D) $\frac{10}{7} gh$

27. Two point masses of 0.3 kg and 0.7 kg are fixed at the ends of a rod of length 1.4 m and of negligible mass. The rod is set rotating about an axis perpendicular to its length with a uniform angular speed. The point on the rod through which the axis should pass in order that the work required for rotation of the rod is minimum, is located at a distance of

(A) 0.4 m from mass of 0.3 kg

(B) 0.98 m from mass of 0.3 kg

(C) 0.70 m from mass of 0.7 kg

(D) 0.98 m from mass of 0.7 kg

28. A smooth sphere A is moving on a frictionless horizontal plane with angular speed ω and centre of mass velocity v. It collides elastically and head on with an identical sphere B at rest. Neglect friction everywhere. After the collision their angular speeds are ω_A and ω_B respectively. Then -

(A) $\omega_A < \omega_B$

(B) $\omega_A = \omega_B$

(C) $\omega_A = \omega$

(D) $\omega = \omega_B$

29. Two identical cylindrical vessels with their bases at same level each contains a liquid of density ρ. The height of the liquid in one vessel is h_1 and that in the other vessel is h_2. The area of either base is A. The work done by gravity in equalizing the levels when the two vessels are connected, is

(A) $(h_1 - h_2) g\rho$

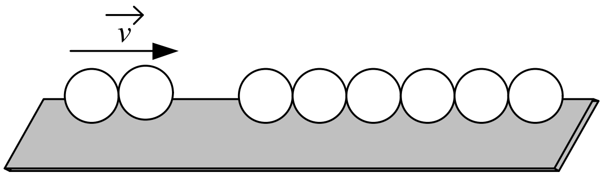
(B) $(h_1 - h_2) gA\rho$

(C) $\frac{1}{2}(h_1 - h_2)^2 gA\rho$

(D) $\frac{1}{4}(h_1 - h_2)^2 gA\rho$

FOR ROUGH WORK

30. Six identical balls are lined in a straight groove made on a horizontal frictionless surface as shown. Two similar balls each moving with a velocity v collide elastically with the row of 6 balls from left. What will happen



(A) One ball from the right rolls out with a speed 2v and the remaining balls will remains at rest

(B) Two balls from the right will roll out with speed v each and the remaining balls will remain stationary

(C) All the six balls in the row will roll out with speed v/6 each and the two colliding balls will come to rest

(D) The colliding balls will come to rest and no ball rolls out from right.

END OF PHYSICS PART

FOR ROUGH WORK

PART A – CHEMISTRY

31. In an isothermal expansion of a gaseous sample the correct relation is (consider w (work) with sign according to new IUPAC convention)
[The reversible and irreversible processes are carried out between same initial and final states.]

(A) $w_{rev} > w_{irrev}$ (B) $w_{irrev} > w_{rev}$

(C) $q_{rev} < q_{irrev}$

(D) can not be predicted

32. What will be the value of a, b, c, d, e and f in the following equation-

$$aMnO_4^- + bC_2O_4^{2-} + cH^+ \longrightarrow dMn^{2+} + eCO_2 + fH_2O$$

(A) 2, 2, 10, 8, 5, 16

(B) 2, 5, 16, 2, 10, 8

(C) 2, 5, 10, 2, 8, 16

(D) 2, 8, 16, 2, 5, 10

33. Which of the following statement is FALSE about resonance?

(A) It increases the stability of a molecule

(B) It leads to similar type of bonds

(C) It increase the reactivity of the molecule

(D) All resonating species don't contribute equally to the hybrid structure

34. The correct decreasing order of preference of functional groups during the IUPAC nomenclature of polyfunctional compounds is -

(A) –COOH, –CHO, –OH, $–NH_2$

(B) $–NH_2$, –OH, –CHO, –COOH

(C) –COOH, –OH, $–NH_2$, –CHO

(D) –COOH, $–NH_2$, –CHO, –OH

35. $A_2\ (g) + 4B_2\ (g) \rightleftharpoons 2AB_4\ (g) + Q$.

What are the favourable conditions for the formation of AB_4.

(A) low temperature and high pressure

(B) high temperature and low pressure

(C) low temperature and low pressure

(D) high temperature and high pressure.

36. Which of the following statement is incorrect?

(A) in a pair of geometrical isomers, the cis- has the higher refractive index than trans isomer

(B) it is possible, by suitable mean, to convert the cis-isomer into the trans-isomer while it is more difficult to convert the trans-isomer into the cis.

(C) the trans isomer CHCl=CHCl will have a zero dipole moment.

(D) the melting point of the cis-isomer is higher than that of the trans-isomers.

FOR ROUGH WORK

37. 1 mole of argon is expanded isothermally and irreversibly (not against vacuum) from 10L to 100L. Which of the following is incorrect for the process?

(A) $\Delta E = 0$ (B) $\Delta H = 0$

(C) heat supplied (q) = 0

(D) $\Delta T = 0$

38. The IUPAC name of the following compounds is : $CH_2 = CH - CH(CH_3)_2$

(A) 3-Methyl-1-butene

(B) 2-Vinyl propane

(C) 1-Isopropylethylene

(D) 1,1-Dimethyl-2-propene

39. Smallest atomic radius among the following is possessed by :

(A) Li (B) Mg (C) Be (D) Sr

40. Hybridisation in CH_4, CH_3^+, CH_3^-, respectively are –

(A) sp^3, sp^2, sp^3 (B) sp^3, sp^3, sp^3

(C) sp^2, sp^2, sp^2 (D) sp^2, sp^3, sp^2

41. All possible alcohol isomers [including stereoisomer(s)] of a compound with molecular formula $C_4H_{10}O$ are

(A) 4 (B) 6

(C) 8 (D) 5

42. Heterolytic fission of an organic covalent bond gives –

(A) Free radicals

(B) Both cation and anion

(C) Only cation (D) Only anion

43. P and Q are two elements which forms P_2Q_3 and PQ_2. If 0.15 mole of P_2Q_3 weighs 15.9 g and 0.15 mole of PQ_2 weighs 9.3 g, then atomic weights of P and Q respectively are:

(A) 13, 9 (B) 24, 20

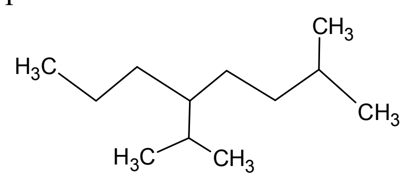
(C) 14, 8 (D) 26, 18

44. Equivalent weight of chlorine molecule in the equation

$$3Cl_2 + 6NaOH \longrightarrow 5NaCl + NaClO_3 + 3H_2O$$

(A) 42.6 (B) 35.5 (C) 59.1 (D) 71

45. The correct IUPAC name of following compound is



(A) 2–Methyl–5–isopropyloctane

(B) 2,6–Dimethyl–5–propylheptane

(C) 5–Isopropyl–2–methyloctane

(D) 4–(1–Methylethyl)–7– methyloctane

FOR ROUGH WORK

46. The De-broglie wavelength of a particle with mass 1 g and velocity 100 m/s is-

(A) $6.63\times10^{-33}\,m$ (B) $6.63\times10^{-34}\,m$

(C) $6.63\times10^{-35}\,m$ (D) $6.65\times10^{-35}\,m$

47. Which one of the following statements is incorrect in relation to ionisation enthalphy ?

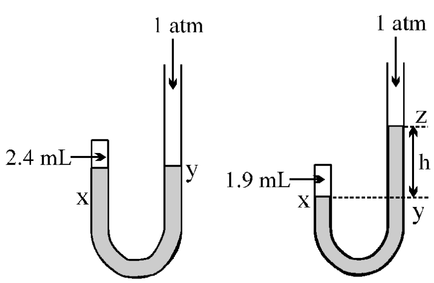
(A) Ionization enthalpy increases for each successive electron

(B) The greatest increase in ionizaiton enthalpy is experienced on removal of electron from core of noble gas configuration

(C) End of valence electrons is marked by a big jump in ionization enthalpy.

(D) Removal of electron from orbitals bearing lower n value is easier than from orbital having higher n value.

48. Given J-tube has 2.4 mL of air at a pressure of 1 atm. On adding mercury, volume of air is reduced to 1.9 mL as shown.

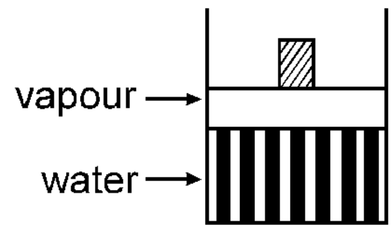


Difference in the level of mercury in two columns is approximately–

(A) 700 mm (B) 200 mm

(C) 900 mm (D) 760 mm

49. Some quantity of water is contained in a container as shown in figure. As neon is added to this system at constant pressure, the amount of liquid water in the vessel



(A) increases (B) decreases

(C) remains same (D) changes unpredictably

50. The shape of sulphate ion is -

(A) hexagonal (B) square planer

(C) trigonal bipyramidal

(D) tetrahedral

51. The vapour density of pure ozone would be

(A) 48 (B) 32 (C) 24 (D) 16

52. An electron will have the highest energy in the set-

(A) 3, 2, 1, 1/2 (B) 4, 2, – 1, 1/2

(C) 4, 1, 0, – 1/2 (D) 5, 0, 0, 1/2

FOR ROUGH WORK

53. Identify the least stable ion amongst the following.

(A) Li^{-} (B) Be^{-} (C) B^{-} (D) C^{-}

54. Which quantum number will determine the shape of the sub-shell–

(A) Principal quantum number

(B) Azimuthal quantum number

(C) Magnetic quantum number

(D) Spin quantum number

55. Compound HN_3 (hydrazoic acid), oxidation state of N atoms are.

(A) 0, 0, 3 (B) 0, 0, – 1

(C) 1, 1, – 3 (D) – 3, – 3, – 3

56. Which is the most ionic

(A) LiF (B) Li_2O

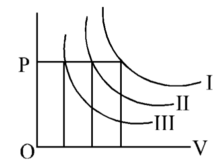
(C) Li_3N (D) All same

57. Which one is the correct expression below for the solution containing 'n' number of weak acids?

(A) $[H^+] = \sqrt{\sum_{i=1}^{n} \frac{K_i}{C_i}}$ (B) $[H^+] = \sqrt{\sum_{i=1}^{n} K_i C_i}$

(C) $[H^+] = \sum_{i=1}^{n} K_i C_i$ (D) none of these

58. I, II, III, are three isotherms respectively at T_1, T_2 and T_3.



Temperature will be in order –

(A) $T_1 = T_2 = T_3$ (B) $T_1 < T_2 < T_3$

(C) $T_1 > T_2 > T_3$ (D) $T_1 > T_2 = T_3$

59. 0.1 M CH_3COOH is diluted at 25°C ($K_a = 1.8 \times 10^{-5}$), then which of the following will be found correct

(A) $[H^+]$ will decrease (B) pH will increase

(C) number of H^+ will increase

(D) all the above are correct

60. Which of the following statement is/are not correct-

(A) CH_3^+ shows sp^2 hybridisation whereas CH_3^- shows sp^3 hybridisation

(B) NH_4^+ has a regular tetrahedral geometry

(C) sp^2 hybridised orbitals have equal s and p character

(D) Hybridised orbitals generally form s-bonds

END OF CHEMISTRY PART

FOR ROUGH WORK

PART C – MATHEMATICS

61. If $z=\dfrac{\tan\alpha-i\left(\sin\dfrac{\alpha}{2}+\cos\dfrac{\alpha}{2}\right)}{1+2i\sin\dfrac{\alpha}{2}}$, then

Re $z=0$, when

(A) $\alpha=2n\pi$ or $n\pi+\dfrac{\pi}{4},n\in I$

(B) $\alpha=n\pi,\ n\in I$

(C) $\alpha=2n\pi+\dfrac{\pi}{2},\ n\in I$

(D) None of these

62. If a, b, c, p, q, r be six complex numbers such that $\dfrac{a}{p}+\dfrac{b}{q}+\dfrac{c}{r}=0$ and $\dfrac{p}{a}+\dfrac{q}{b}+\dfrac{r}{c}=1-i$, then the value of $\dfrac{p^2}{a^2}+\dfrac{q^2}{b^2}+\dfrac{r^2}{c^2}$ is

(A) 0 (B) – 2i (C) 2i (D) – 1

63. If $x \in R$, then $|x-5|<1$ iff $\dfrac{x}{x+10}$ lies in

(A) $\left(\dfrac{1}{4},\dfrac{3}{7}\right)$ (B) $\left(\dfrac{2}{7},\dfrac{3}{8}\right)$

(C) $\left(\dfrac{2}{7},\dfrac{3}{7}\right)$ (D) None of these

64. The equation $(3-x)^4+(2-x)^4=(5-2x)^4$ has

(A) only real roots

(B) only non-real roots

(C) two real and two non-real roots

(D) none of these

65. The number of values of k for which $\{x^2-(k-2)x+k^2\}\{x^2+kx+2k-1\}$ is a perfect square is :

(A) 2 (B) 1 (C) 0 (D) 4

66. α, β, γ, δ are the smallest positive real numbers in ascending order of magnitude which satisfy the equation sin x = k, $|k|\le 1$, then $4\sin\dfrac{\alpha}{2}+3\sin\dfrac{\beta}{2}+2\sin\dfrac{\gamma}{2}+\sin\dfrac{\delta}{2}$ is equal to

(A) $2\sqrt{1-k}$ (B) $\dfrac{1}{2}\sqrt{1+k}$

(C) $2\sqrt{1+k}$

(D) None of these

67. The maximum value of

$5\cos\theta+3\cos\left(\theta+\dfrac{\pi}{3}\right)+3$ is

(A) 5 (B) 10

(C) 11 (D) – 1

FOR ROUGH WORK

68. tan |x| = |tan x| if

(A) $x \in \left(-\left(\frac{2n+1}{2}\right)\pi, -n\pi\right]$

(B) $x \in \left(\left(\frac{2n-1}{2}\right)\pi, n\pi\right]$

(C) $x \in \left[-n\pi, -\left(\frac{2n-1}{2}\right)\pi\right) \text{ or } \left[n\pi, \pi\left(\frac{2n+1}{2}\right)\right)$

(D) none of these

69. If $A = \{(x, y) : x^2 + y^2 = 25\}$ and $B = \{(x, y) : x^2 + 9y^2 = 144\}$, then $A \cap B$ contains

(A) One point
(B) Three points
(C) Two points
(D) Four points

70. Let R be a relation defined on the set of integers given by aRb if $a = 2^k b$ for some integer k, then R is

(A) an equivalence relation
(B) Reflexive but not symmetric
(C) Reflexive and transitive but not symmetric
(D) Reflexive and symmetric but not transitive.

71. For an A.P., $S_{2n} = 3S_n$. The value of $\frac{S_{3n}}{S_n}$ is equal to

(A) 4
(B) 6
(C) 8
(D) 10

72. The sum of the series $i - 2 - 3i + 4 + 5i - ....$ upto 100 terms is

(A) $100(1-i)$
(B) $25\,i$
(C) $50(1+i)$
(D) $50(1-i)\,10$

73. The number of ways in which we can choose two positive integers from 1 to 100 such that their product is multiple of 3 is

(A) ${}^{100}C_2 - {}^{33}C_2$
(B) ${}^{100}C_2 - {}^{67}C_2$
(C) ${}^{33}C_2$
(D) ${}^{67}C_2$

74. An examination paper contains 6 questions of which 3 have 4 possible answers each, 2 have 3 possible answers each and the remaining one question ha 5 possible answers. The total number of possible answers to all the questions is

(A) 3240
(B) 94
(C) 78
(D) 2880

FOR ROUGH WORK

75. The number of ways in which one can select three distinct integers between 1 and 30 both inclusive, whose sum is even is

(A) 455 (B) 1575

(C) 1120

(D) 2030

76. If eccentricities of the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$ and the hyperbola $\frac{x^2}{64}-\frac{y^2}{b^2}=1$ are reciprocals or each other, then $b^2=$

(A) 192

(B) 64

(C) 16

(D) 32

77. A conic section is defined by the equation $x=-1+\sec t, y=2+3\tan t$. The foci of the conic are

(A) $\left(-1-\sqrt{10}, 2\right), \left(-1+\sqrt{10}, 2\right)$

(B) $\left(\sqrt{10}, 0\right), \left(-\sqrt{10}, 0\right)$

(C) $\left(-1-\sqrt{8}, 2\right), \left(-1+\sqrt{8}, 2\right)$

(D) $\left(-1, 2-\sqrt{8}\right), \left(-1, 2+\sqrt{8}\right)$

78. A bridge is in the shape of a semi-ellipse. It is 400 meters long and has a maximum height of 10 meters at the middle point. The height of the bridge at a point distance 80 meters from one end is

(A) 4 meters (B) 2 meters

(C) 8 meters (D) $\sqrt{91}$ meters

79. Length of common chord of the circles $x^2+y^2+2x+6y=0$ and $x^2+y^2-4x-2y-6=0$ is

(A) $\sqrt{106}$ (B) $\frac{1}{5}\sqrt{106}$

(C) $2\sqrt{106}$ (D) $\frac{2}{5}\sqrt{106}$

80. A ray of light along $x+\sqrt{3}y=\sqrt{3}$ gets reflected upon reaching the x-axis, the equation of the reflected ray is

(A) $\sqrt{3}y=x-1$ (B) $y=\sqrt{3}x-\sqrt{3}$

(C) $\sqrt{3}y=x-\sqrt{3}$

(D) $y=x+\sqrt{3}$

81. $\sin^6 x+\cos^6 x$ lies between

(A) $\frac{1}{4}$ and 1 (B) $\frac{1}{4}$ and 2

(C) 0 and 1 (D) None of these

FOR ROUGH WORK

82. The general solution of the equation $\sin^{50}x - \cos^{50}x = 1$ is

(A) $2n\pi + \frac{\pi}{2}$ (B) $2n\pi + \frac{\pi}{3}$

(C) $n\pi + \frac{\pi}{2}$ (D) $n\pi + \frac{\pi}{3}$

83. The value of $\left(\frac{1+i}{\sqrt{2}}\right)^8 + \left(\frac{1-i}{\sqrt{2}}\right)^8$ is equal to

(A) 4 (B) 6 (C) 8 (D) 2

84. $2^{\frac{1}{4}} \cdot 4^{\frac{1}{8}} \cdot 8^{\frac{1}{16}} \cdot 16^{\frac{1}{32}}$.... is equal to

(A) 1 (B) 2 (C) $\frac{3}{2}$ (D) $\frac{5}{2}$

85. The sum of positive terms of the series $10 + 9\frac{4}{7} + 9\frac{1}{7} + \ldots$ is :

(A) $\frac{352}{7}$ (B) $\frac{437}{7}$

(C) $\frac{852}{7}$ (D) None of these

86. An *n*-digit number is a positive number with exactly *n* digits. Nine hundred distinct *n*-digit numbers are to be formed using only the three digits 2, 5 and 7. The smallest value of *n* for which this is possible is

(A) 6 (B) 7 (C) 8 (D) 9

87. Two distinct chords drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$, where $pq \neq 0$, are bisected by the x-axis. Then

(A) $|p| = |q|$ (B) $p^2 = 8q^2$

(C) $p^2 < 8q^2$ (D) $p^2 > 8q^2$

88. The imaginary part of $\frac{2z+1}{iz+1}$ is – 2, then the locus of the point representing z in the complex plane is

(A) Circle (B) A straight line

(C) A parabola (D) None of these

89. Given five line segments of length 2, 3, 4, 5, 6 units. Then the number of triangles that can be formed by joining these lines is

(A) ${}^5C_3 - 3$ (B) ${}^5C_3 - 1$

(C) ${}^5C_3$ (D) ${}^5C_3 - 2$

90. The number of ways in which a committee of 3 ladies and 4 gentlemen can be appointed from a meeting consisting of 8 ladies and 7 gentlemen, if Mrs. *X* refuses to serve in a committee if Mr. *Y* is a member is

(A) 1960 (B) 1540

(C) 3240 (D) None of these

END OF TEST PAPER

FOR ROUGH WORK

(Paper – 2 MAIN)

XI ALL BATCHES - SOLUTIONS

PART A - PHYSICS

Sol.1. **(B)** $\rho = \frac{m}{\pi r^2 L}$

$\frac{\Delta\rho}{\rho} \times 100 = \left[\frac{\Delta m}{m} + 2\frac{\Delta r}{r} + \frac{\Delta L}{L}\right] \times 100 = \left(0.01 + 2\frac{\Delta r}{r} + 0.01\right) \times 100 = 4$ (given)

or $100\frac{\Delta r}{r} = 1$ $\Delta r = 0.01r$

Sol.2. **(C)** Let M be the quantity $M = KV^xF^yT^z$

$[M] = [LT^{-1}]^x [MLT^{-2}]^y [T]^z$

Equating the exponents, $y = 1$

$x + y = 0$ and $-x - 2y + z = 0$

On solving $x = -1, y = 1$ and $z = 1$ So, $M = KV^{-1}FT$.

Sol.3. **(C)** According to problem, $L = \frac{1}{2}g(t-T)^2 - \frac{1}{2}g(t-2T)^2$ On solving, $t = \frac{L}{gT} + \frac{3}{2}T$

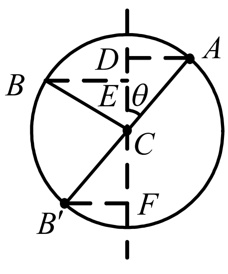
Sol.4. **(B)** $W = K_f - K_i = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}m\left[\left(\frac{1}{2}a_2t_2\right)^2 - \left(\frac{1}{2}a_1t_1\right)^2\right]$

$= \frac{1}{8}m\left[\frac{t_2^4}{t_1^2}a_1^2 - a_1^2t_1^2\right]$ $\left(\text{since, } a_2 = \frac{t_2}{t_1}a_1\right)$

$= \frac{ma_1^2}{8t_1^2}(t_2^4 - t_1^4)$

Sol.5. **(C)** AB' is a straight line

So, $\angle ACB = \angle BCB' = 90^\circ$ Given, $AC = BC = B'C = r$



$CD = r\cos\theta$ $CE = r\cos(90^\circ - \theta) = r\sin\theta$

$ED = r(\cos\theta - \sin\theta)$

Decrease in PE of ball A $= mg \cdot ED = mgr(\cos\theta - \sin\theta)$

$CF = r\cos\theta,$ $EF = r(\cos\theta + \sin\theta)$

Decrease in PE of ball at position B $= mg \cdot EF = mgr(\cos\theta + \sin\theta)$

Increase in KE = Decrease in PE

$\frac{1}{2} \cdot 2mv^2 = mgr(\cos\theta + \sin\theta) + mgr(\cos\theta - \sin\theta)$

$v = \sqrt{2gr\cos\theta}$

Sol.6. **(A)** Slope of the curve is the spring constant, K. (Slope $\propto K$)

If the slope has decreased, length will be greater than L, since, $K \propto \left(\frac{1}{L}\right)$

Sol.7. **(C)** Collision (inelastic) between C and D. $2mV = (2m+m)V_1$ $V_1 = \frac{2V}{3}$

$V_1 < V$, springs will start getting compressed. At the maximum compression of springs, all blocks will have common velocity V_2.

$mV + mV + (3m)V_1 = (5m)V_2$ $V_2 = \frac{4}{5}V$ (Since, $V_1 = \frac{2V}{3}$)

At the instant when C and D collide and acquires velocity $V_1 = \frac{2V}{3}$, A and B still moves with velocity V at that instant (given).

Now, $\frac{1}{2}mV^2 + \frac{1}{2}mV^2 + \frac{1}{2}\times 3m \times V_1^2 = \frac{1}{2}\times 5m \times V_2^2 + \frac{1}{2}Kx^2 + \frac{1}{2}Kx^2$

On solving $Kx^2 = \frac{mV^2}{15}$ or $x = V\sqrt{\frac{m}{15K}}$

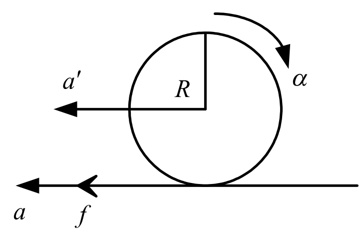
Sol.8. **(B)** $\vec{F} = \frac{d\vec{P}}{dt} = -kA\left[\sin(kt)\hat{t} + \cos(kt)\hat{j}\right]$ $\vec{P} = A\cos(kt)\hat{i} - A\sin(kt)\hat{j}$ Here, $\vec{F}\cdot\vec{P} = 0$

So, angle between $\vec{F}$ and $\vec{P}$ should be $90°$

Sol.9. **(C)** $Mg - T = Ma$ ....(1) and $T = (2m)a$ ....(2)

From eqs. (1) and (2), $a = \frac{Mg}{(M+2m)}$

$a' = \frac{f}{m}$ $\alpha = \frac{\tau}{I} = \frac{fR}{(mR^2/2)}$



For no slipping, $a = a' + \alpha R$

$\frac{Mg}{(M+2m)} = \frac{f}{m} + \frac{2fR}{mR}$ Which gives $f = \frac{Mmg}{3(M+2m)}$

Sol.10. **(D)** $T_2 = (m_1 + m_1)\times\frac{T_3}{m_1+m_2+m_3} = \frac{(10+6)\times 40}{20} = 32N$

Sol.11. **(B)** $t \propto \frac{1}{r^2}$. If radius of the wire is doubled then increment in length will become $\frac{1}{4}$ times i.e., $\frac{12}{4} = 3\,mm.$

Sol.12. **(B)** $h \propto \frac{1}{r}$ $\therefore$ $r_1h_1 = r_2h_2$ $\Rightarrow \frac{h_1}{h_2} = \frac{r_2}{r_1} = \frac{0.4}{0.2} = 2:1$

Sol.13. **(C)** Energy supplied $= 0.93\times 3600J = 3348J$

Heat required to melt 10 g of ice $= 10\times 80\times 4.18 = 3344J$

Hence, block of ice just melt.

Sol.14. **(A)** The level is maintained if rate of in flow of water is equal to rate of out flow of water.

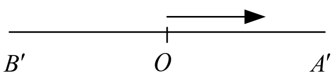
$\alpha = av = a\sqrt{2gh}$

$h = \frac{\alpha^2}{2ga^2} = \text{constant} = \frac{1}{2g}\left(\frac{\alpha}{a}\right)^2$ $h \propto \left(\frac{\alpha}{a}\right)^2$ So, option (a) is correct.

Sol.15. (D) If $p_0 =$ atmospheric pressure $p_A = p_0 = p_B + \rho h_1 g$

$p_D = p_0 = p_E + \rho h_3 g$ Since, $h_3 < h_1$. So, $p_E > p_B$

Sol.16. (D) On $(a-t)$ curve at point A, acceleration is zero, but acceleration is negative after some time.



Acceleration $\propto -$(displacement). At O but towards A′, the displacement is positive and hence, acceleration is negative. Here the velocity is maximum and positive.
At point 4 of $(v-t)$ curve, the velocity is maximum and positive and acceleration is zero.

Sol.17. (D) Let $T_1 = T$ and $T_2 = KT$ $\frac{2\pi}{T}t - \frac{2\pi}{KT}t = 2\pi$ $t\left(\frac{K-1}{KT}\right) = 1$

$\frac{5T}{4}\left(\frac{K-1}{KT}\right) = 1$ which gives $K = 5$ If length of first pendulum $= \ell$

$T_1 = 2\pi\sqrt{\ell / g}$ $T_2 = 5 \times 2\pi\sqrt{\ell / g} = 2\pi\sqrt{25\ell / g}$

So, ratio of lengths = 1 : 25

Sol.18. (C) For observer, $T' = \frac{2\pi}{\omega_S - \omega_E} = \frac{T_S T_E}{T_E - T_S} = T_E$ (given) or $T_E^2 = 2_S T_E$ $T_S = T_E / 2$

Sol.19. (A) Time period of satellite which is very near to planet

$$T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R^3}{G\frac{4}{3}\pi R^3 \rho}} \qquad \therefore \qquad T \propto \sqrt{\frac{1}{\rho}}$$

i.e., time period of nearest satellite does not depends upon the radius of planet, It only depends upon the density of the planet.
In the problem, density is same so time period will be same.

Sol.20. (C) For body to remain in contact $a_{\max} = g$

$$\therefore \quad \omega^2 A = g \Rightarrow 4\pi^2 n^2 A = g \Rightarrow n^2 = \frac{g}{4\pi^2 A} = \frac{10}{4(3.14)^2\, 0.01} = 25 \Rightarrow n = 5\,Hz$$

Sol.21. (C) Let $A =$ The area of cross-section of the hole

$v =$ Initial velocity of efflux $d =$ Density of water,

Initial volume of water flowing out per second $= Av$

Initial mass of water flowing out per second $= Avd$

Rate of change of momentum $= Avd^2$

Initial downward force on the flowing out water $= Adv^2$

So, equal amount of reaction acts upwards on the cylinder.

$\therefore$ Initial upward reaction $= Adv^2$ [As $v = \sqrt{2gh}$]

$\therefore$ Initial decrease in weight $= Ad(2gh) = 2Adgh = 2 \times \left(\frac{1}{4}\right) \times 1 \times 980 \times 25 = 12.5\, gm.wt.$

Sol.22. (A) Limiting friction between block and slab $= \mu_s m_A g = 0.6 \times 10 \times 9.8 = 58.8\,N$

But applied force on block A is 100 N. So the block will slip over a slab.
Now kinetic friction works between block and slab

$F_k = \mu_k m_A g = 0.4 \times 10 \times 9.8 = 39.2N$

This kinetic friction helps to move the slab

$\therefore$ Acceleration of slab $\frac{39.2}{m_B} = \frac{39.2}{40} = 0.98\,m/s^2$

Sol.23. (A) $\mu = \tan\theta\left(1 - \frac{1}{n^2}\right) = \tan\theta\left(1 - \frac{1}{2^2}\right) = \frac{3}{4}\tan\theta$

Sol.24. (C) Impulse = Area between force and time graph and it is maximum for graph (III) and (IV).

Sol.25. (A) If t_1 and $2t_2$ are the time taken by particle to cover first and second half distance respectively.

$t_1 = \frac{x/2}{3} = \frac{x}{6}$ ....(i)

$x_1 = 4.5t_2$ and $x_2 = 7.5t_2$

So, $x_1 + x_2 = \frac{x}{2} \Rightarrow 4.5t_2 + 7.5t_2 = \frac{x}{2}$

$t_2 = \frac{x}{24}$ ......(ii)

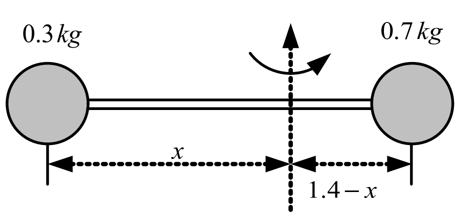
Total time $t = t_1 + 2t_2 = \frac{x}{6} + \frac{x}{12} = \frac{x}{4}$

So, average speed $= 4\,m/\sec$.

Sol.26. (B) As body is moving on a frictionless surface. Its mechanical energy is conserved. When body climbs up the inclined plane it keeps on rotating with same angular speed, as no friction force is present to provide retarding torque so

$\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 \geq \frac{1}{2}I\omega^2 + mgh \Rightarrow v \geq \sqrt{2gh}$

Sol.27. (B) $I = 0.3x^2 + 0.7(1.4 - x)^2$

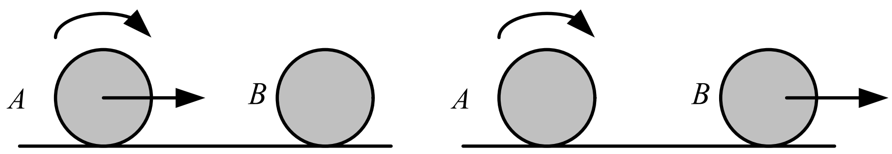


For minimum work moment of inertia of the system should be minimum i.e., $\frac{dI}{dx} = 0$

$\frac{dI}{dx} = 0.3 \times 2x - 0.7 \times 2(1.4 - x) = 0 \Rightarrow x = 0.98\,m$

Sol.28. (C) Since it is head on elastic collision between two identical balls therefore they will exchange their linear velocities i.e., A comes to rest and B starts moving with linear velocity v.

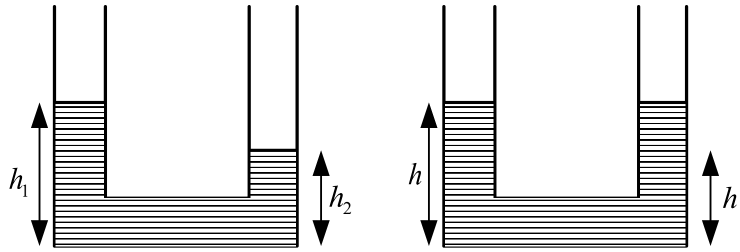
As there is no friction anywhere, torque on both the spheres about their centre of mass is zero and their angular velocities remain unchanged. Therefore $\omega_A = \omega$ and $\omega_B = 0$



Before collision After collision

Hence we can conclude that after collision sphere A will perform pure rotator motion (without translation) and sphere B will perform pure translator motion (without rotation)

Sol.29. (D) If h is the common height when they are connected, by conservation of mass



$$\rho A_1 h_1 + \rho A_2 h_2 = \rho h (A_1 + A_2)$$

$$h = (h_1 + h_2)/2 \quad \text{[as } A_1 = A_2 = A \text{ given]}$$

As $(h_1/2)$ and $(h_2/2)$ are heights of initial centre of gravity of liquid in two vessels, the initial potential energy of the system.

$$U_i = (h_1 A\rho) g \frac{h_1}{2} + (h_2 A\rho) g \frac{h_2}{2} = \rho g A \frac{(h_1^2 + h_2^2)}{2} \quad \text{......(i)}$$

When vessels are connected the height of centre of gravity of liquid in each vessel will be $\frac{h}{2}$

i.e., $\frac{(h_1 + h_2)}{4}$ [as $h = (h_1 + h_2)/2$]

Final potential energy of the system

$$U_F = \left[\frac{(h_1 + h_2)}{2} A\rho\right] g \left(\frac{h_1 + h_2}{4}\right) = A\rho g \left[\frac{(h_1 + h_2)^2}{4}\right] \quad \text{.....(ii)}$$

Work done by gravity

$$W = U_i - U_f = \frac{1}{4}\rho g A\left[2\left(h_1^2 + h_2^2\right) - (h_1 + h_2)^2\right] = \frac{1}{4}\rho g A (h_1 - h_2)^2.$$

Sol.30. (B) Momentum and kinetic energy is conserved only in this case.

PART B - CHEMISTRY

Sol.31. (B) From the 1st Law of thermodynamics

Sol.32. (B) **(i)** The half reaction for reduction is, $MnO_4^- \longrightarrow Mn^{2+}$

Balancing wrt oxygen by adding $4H_2O$ on R.H.S. and wrt hydrogen by adding $8H^+$ on L.H.S.,

$MnO_4^- + 8H^+ \longrightarrow Mn^{2+} + 4H_2O$

Balancing charge by adding electrons,

$MnO_4^- + 8H^+ + 5e^- \longrightarrow Mn^{+2} + 4H_2O$

(ii) The half reaction for oxidation is,

$C_2O_4^{2-} \longrightarrow 2CO_2$

Balancing charge by adding electrons on R.H.S.

$C_2O_4^{2-} \longrightarrow 2CO_2 + 2e^-$

Now, to equalise the number of electrons, the reduction half reaction is multiplied by 2 and oxidation half reaction by 5, so on adding, we get

$(C_2O_4^{2-} \longrightarrow 2CO_2 + 2e^-) \times 5$

$(MnO_4^- + 8H^+ + 5e^- \longrightarrow Mn^{2+} + 4H_2O) \times 2$

$2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \longrightarrow 2Mn^{++} + 10CO_2 + 8H_2O$

This is the balanced equation.

Sol.33. (C) With increase in stability, reactivity decreases

Sol.34. (A) A/c to the the priority order list

Sol.35. (A) To shift the reaction forward, a/c to le Chatelier's principle,
for an exothermic reaction, low temperature is preferred
when $n_{reactant} > n_{product}$, high pressure is required

Sol.36. (D) Since stability of trans > cis, melting point follows same order.

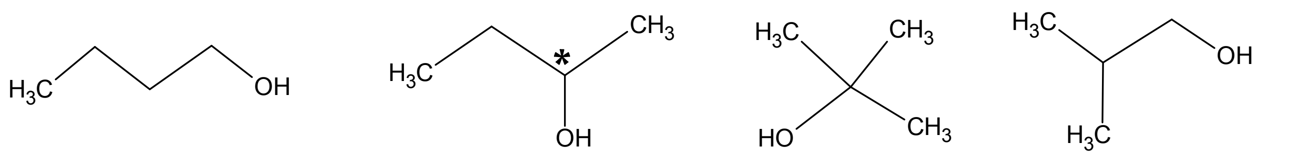
Sol.37. (C) For any isothermal process $\Delta E = 0$, $\Delta H = 0$ and $\Delta T = 0$.

Sol.38. (A) A/c to IUPAC rules for naming Organic Compounds

Sol.39. (C) Be is the first member of II A, others are lower members of II A or I A and hence would be larger than Be.

Sol.40. (A) For CH_4, CH_3^-, steric factor = 4, hence hybridization = sp^3
For, CH_3^+, steric factor = 3, hence hybridization = sp^2

Sol.41. (D) Total = 5



* Optically active, hence 2 isomers

Sol.42. (B) Heterolytic means unequal hence both cation and anion are formed

Sol.43. **(D)** Solve the following equations: $2P + 3Q = \frac{15.9}{0.15}$; $P + 2Q = \frac{9.3}{0.15}$;

Sol.44. **(A)** n=5, M=71

$$Eq.\text{wt}.(\text{for} 1 \text{mole}) = \frac{M}{n}$$

For 3 moles, eq.wt. = 3*(71/5) = 42.6

Sol.45. **(C)** Naming done a/c to the IUPAC rules, remember the ruler of lowest locants, also isopropyl is accepted in the IUPAC nomenclature

Sol.46. **(A)** Use $\lambda = \frac{h}{mv}$, keep the unit conversions in mind

Sol.47. **(D)** Orbitals bearing lower value of n will be more closer to the nucleus and thus electrons will experience greater attraction from nucleus and so its removal will be difficult not easier.

Sol.48. **(C)** A/c to Boyles law, $P_1V_1 = P_2V_2$;

$\therefore$ 1 atm x 2.4 ml = (h x ρ_{Hg} x g) x 1.9 ml;

Please do the appropriate conversions, which gives out the approx. h=900mm

Sol.49. **(B)** A/c to Le Chatelier's principle, addition of inert gases at a constant pressure will shift the equilibrium towards increasing number of gaseous moles hence liquid water decreases

Sol.50. **(D)** sp^3 hybridisation hence tetrahedral geometry

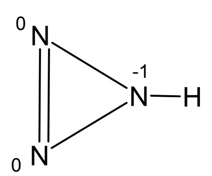
Sol.51. **(C)** $V.D. = \frac{M}{2} = \frac{48}{2} = 24$

Sol.52. **(B)** (B)4d > (D)5s > (C)4p > (A)3d

Sol.53. **(B)** Be has fully filled 2s–subshell ($2s^2$) and therefore, shows least tendency to accept an electron. Thus Be^- is least stable.

Sol.54. **(B)** Azimuthal quantum number defines the name of orbitals (s, p, d, f,…) and hence the shape too.

Sol.55. **(B)**



Sol.56. **(A)** Group I and VII form the most ionic componds

Sol.57. **(B)** Derivational

Sol.58. **(C)** PV = constt, as the temperature is increased, the graph rises higher

Sol.59. **(D)** On the basic of ostwald dilution law, number of H^+ ions will increase but increase in volume will be more. Therefore, $[H^+]$ decreases, pH increases.

Sol.60. **(C)** In sp^2 hybridised orbitals, the % s-character is 33.3 and % p-character is 66.7.

PART C - MATHEMATICS

Sol.61. (A) Multiplying the numerator & denominator by $1 - 2i\sin(\alpha/2)$, we find that

$$\text{Re}(z) = \frac{\tan\alpha - 2\sin\frac{\alpha}{2}\left(\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}\right)}{1 + 4\sin^2\left(\frac{\alpha}{2}\right)} = \frac{\tan\alpha - 2\sin^2\left(\frac{\alpha}{2}\right) - 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{1 + 4\sin^2\left(\frac{\alpha}{2}\right)}$$

$\therefore$ Re(z) = 0

$\Rightarrow \tan\alpha - 2\sin^2\frac{\alpha}{2} - 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} = 0 \Rightarrow \frac{\sin\alpha}{\cos\alpha} - (1 - \cos\alpha) - \sin\alpha = 0$

$\Rightarrow \sin\alpha - \cos\alpha + \cos^2\alpha - \sin\alpha\cos\alpha = 0 \Rightarrow (\sin\alpha - \cos\alpha) . (1 - \cos\alpha) = 0$

Now $\cos\alpha = 1 \Rightarrow \cos\alpha = \cos 0 \Rightarrow \alpha = 2n\pi, n \in I$

and $\sin\alpha = \cos\alpha \Rightarrow \tan\alpha = 1 \Rightarrow \alpha = n\pi + \frac{\pi}{4}, n \in I$

(When $\sin\alpha = \cos\alpha$, then $\cos\alpha \neq 0$)

Sol.62. (B) Let $\frac{p}{a} = A, \frac{q}{b} = B \text{ and } \frac{r}{c} = C$, then $\frac{a}{p} + \frac{b}{q} + \frac{c}{r} = 0 \Rightarrow \frac{1}{A} + \frac{1}{B} + \frac{1}{C} = 0 \Rightarrow BC + CA + AB = 0$

Also, $A + B + C = 1 - i$ $\left(\because \frac{p}{a} + \frac{q}{b} + \frac{r}{c} = 1 - i\right)$

$\Rightarrow (A + B + C)^2 = (1 - i)^2 \Rightarrow A^2 + B^2 + C^2 + 2(AB + BC + CA) = 1 + i^2 - 2i$

$\Rightarrow A^2 + B^2 + C^2 + 2 \times 0 = -2i$

Sol.63. (B) $|x - 5| < 1 \Leftrightarrow -1 < x - 5 < 1$

$\Leftrightarrow 4 < x < 6 \Leftrightarrow 14 < x + 10 < 16$

$\Leftrightarrow \frac{1}{14} > \frac{1}{x+10} > \frac{1}{16} \Leftrightarrow \frac{-10}{14} < \frac{-10}{x+10} < -\frac{10}{16}$ $\left(\textit{Note that } \frac{x}{x+10} = 1 - \frac{10}{x+10}\right)$

$\Leftrightarrow 1 - \frac{5}{7} < 1 - \frac{10}{x+10} < 1 - \frac{5}{8} \Leftrightarrow \frac{2}{7} < \frac{x}{x+10} < \frac{3}{8} \Leftrightarrow \frac{x}{x+10} \in \left(\frac{2}{7}, \frac{3}{8}\right).$

Sol.64. (C) The given equation is $a^4 + b^4 = (a + b)^4$

where $a = 3 - x$, $b = 2 - x$

$\Rightarrow a^4 + b^4 = (a^2 = 2ab + b^2)(a^2 + 2ab + b^2) \Rightarrow a^4 + b^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$\Rightarrow 4a^2b + 6a^2b^2 + 4ab^3 = 0 \Rightarrow 2ab(2a^2 + 3ab + 2b^2) = 0$

$\Rightarrow a = 0$ or $b = 0$ or $2a^2 + 3ab + 2b^2 = 0 \Rightarrow 3 - x = 0$ or $2 - x = 0$

or $2(3 - x)^2 + 3(3 - x)(2 - x) + 2(2 - x)^2 = 0$

$\Rightarrow x = 3$ or $x = 2$ or $7x^2 - 35x + 44 = 0$

Since $7x^2 - 35x + 44 = 0$ has non-real roots, therefore, the given equation has only two real roots.

Sol.65. (B) The given expression (a biquadratic) will be a perfect square if the equations

$x^2 - (k - 2)x + k^2 = 0$ …..(1)

and $x^2 + kx + 2k - 1 = 0$ …..(2)

either both have equal roots or they have both roots common.

If (1) and (2) have equal roots, then

$(k - 2)^2 - 4k^2 = 0$ and also $k^2 - 4(2k - 1) = 0$, i.e., $3k^2 + 4k - 4 = 0$ and $k^2 - 8k + 4 = 0$, i.e., (k + 2)(3k – 2) = 0 and $(k-4)^2 = 12$ which is not possible for any k.

If (1) and (2) have both roots common then

$$\frac{1}{1} = \frac{-(k+2)}{k} = \frac{k^2}{2k-1} \Rightarrow \frac{k^2}{2k-1} = 1 \text{ and also } \frac{k-2}{k} = -1$$

$\Rightarrow$ $(k-1)^2 = 0$ and $2k = 2 \Rightarrow k = 1$

Thus, k = 1 will make the given expression, a perfect square.

Sol.66. (C) Here, $\beta = \pi - \alpha$, $\gamma = 2\pi + \alpha$, $\delta = 3\pi - \alpha$ (assuming k > 0)

$$\therefore \quad 4\sin\frac{\alpha}{2} + 3\sin\frac{\beta}{2} + 2\sin\frac{\gamma}{2} + \sin\frac{\delta}{2}$$

$$= 4\sin\frac{\alpha}{2} + 3\sin\frac{\pi-\alpha}{2} + 2\sin\frac{2\pi+\alpha}{2} + \sin\frac{3\pi-\alpha}{2}$$

$$= 4\sin\frac{\alpha}{2} + 3\cos\frac{\alpha}{2} + 2\left(-\sin\frac{\alpha}{2}\right) - \cos\frac{\alpha}{2}$$

$$= 2\left(\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}\right) = 2\sqrt{\left(\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}\right)} = 2\sqrt{1+\sin\alpha} = 2\sqrt{1+k}$$

When k < 0, then $\alpha = \pi + \sin^{-1}|k| = \pi + \theta$ where $\theta = \sin^{-1}|k|$, then $\beta = 2\pi - \theta$, $\gamma = 3\pi + \theta$ and $\delta = 4\pi - \theta$. Here, again, we get the same value of given expression.

Sol.67. (B) $5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3 = 5\cos\theta + 3\left(\frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta\right) + 3$

$$= \left(5 + \frac{3}{2}\right)\cos\theta + 3\left(-\frac{\sqrt{3}}{2}\right)\sin\theta + 3 = \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$$

Since the maximum value of $a\cos\theta + b\sin\theta$ is $\sqrt{a^2+b^2}$,

therefore, the maximum value of

$$\frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta = \sqrt{\left(\frac{13}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2} = \sqrt{\frac{169+27}{4}} = \sqrt{\frac{196}{4}} = 7$$

Hence, the maximum value of given expression = 7 + 3 = 10

Sol.68. (C) Since $|\tan x| \geq 0$, therefore, $\tan|x| = |\tan x|$ can hold only if |x| lies either in the first quadrant or in the third quadrant. So (C) is the correct option.

Sol.69. (D) Given curves are $x^2 + y^2 = 25$ …..(1)

and $x^2 + 9y^2 = 144$ …..(2)

Subtracting (1) from (2), we get $8y^2 = 119 \Rightarrow y^2 = \frac{119}{8}$ then from (1),

$$x^2 = 25 - \frac{119}{8} = \frac{81}{8} \Rightarrow x = \pm\frac{9}{\sqrt{8}},\ y = \pm\frac{\sqrt{119}}{\sqrt{8}}$$

$\therefore$ (1) and (2) intersect in four points

$$\left(\frac{9}{\sqrt{8}}, \frac{\sqrt{119}}{\sqrt{8}}\right), \left(-\frac{9}{\sqrt{8}}, -\frac{\sqrt{119}}{\sqrt{8}}\right), \left(\frac{9}{\sqrt{8}}, -\frac{\sqrt{119}}{\sqrt{8}}\right), \left(-\frac{9}{\sqrt{8}}, \frac{\sqrt{119}}{\sqrt{8}}\right).$$

Sol.70. (A) **(i)** As $a = 1a = 2^0 a$, Therefore, a R a for all $a \in 1$. $\therefore$ R is reflexive.

(ii) Let a R b, then

$a = 2^k b$ for some integer k

$\Rightarrow b = 2^{-k} a, -k$ being an integer $(\because k \text{ is an integer})$

$\Rightarrow b\ R\ a$ $\therefore$ R is symmetric.

(iii) Let a R b and b R c

$\Rightarrow a = 2^k b$ and $2^l c$, where k, l are integers $\Rightarrow a = 2^k\left(2^l c\right) = 2^{k+1} c$

$\Rightarrow a$ R c a $k + l$ is an integer $\therefore$ R is reflexive

Hence, R is an equivalence relation.

Sol.71. (B) Let the A.P. be $a + (a+d) + (a+2d) + ...$

Given $\frac{S_{2n}}{S_n} = 3 \Rightarrow \frac{\frac{2n}{2}\{2a + (2n-1)d\}}{\frac{n}{2}\{2a + (n-1)d\}} = 3 \Rightarrow 4a + 4nd - 2d = 6a + 3nd - 3d$

$\Rightarrow 2a = nd + d = (n+1)d$ .....(1)

$\therefore \frac{S_{3n}}{S_n} = \frac{\frac{3n}{2}\{2a + (3n-1)d\}}{\frac{n}{2}\{2a + (n-1)d\}} = \frac{3\{(n+1)d + (3n-1)d\}}{(n+1)d + (n-1)d}$ (using (1)) $= \frac{3(4nd)}{2nd} = 6.$

Sol.72. (D) The sum of the given series = $i + 2i^2 + 3i^3 + 4i^4 + \ldots$ upto 100 terms

Let S $= i + 2i^2 + 3i^3 + 4i^4 + \ldots$ upto $100i^{100}$ ..... (1)

Multiplying both sides by i, we get

iS $= i^2 + 2i^3 + 3i^4 + \ldots\ldots + 100i^{101}$ ..... (2)

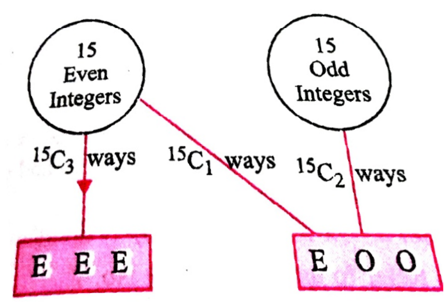
Subtracting (2) from (1), we get S $(1 - i) = i + i^2 + i^3 + \ldots$. Upto 100 terms $-100i^{101}$

$= \frac{i(1-i^{100})}{1-i} - 100i \quad \left(\because i^{100} = \left(i^4\right)^{25} = 1\right) \quad \therefore S = 0 - \frac{100i}{1-i} = \frac{-100i(1+i)}{1-(-1)}.$

Sol.73. (B) From 1 to 100 there are 33 integers which are multiples of 3. The product is a multiple of 3 if atleast one of the two selected integers is a multiple of 3. $\therefore$ Required number = ${}^{100}C_2 - {}^{67}C_2$

Sol.74. (D) $4^3 \times 3^2 \times 5 = 64 \times 9 \times 5.$

Sol.75. (D) The sum is even when either all the three integers are even or one integer is even and two or odd. From 1 to 30, there are 15 even and 15 odd integers. In first case, the number of ways is ${}^{15}C_3$ and in the second case, the number of ways is ${}^{15}C_1 \times {}^{15}C_2$.



$\therefore$ Required number

$= {}^{15}C_3 + {}^{15}C_1\ {}^{15}C_2 = \frac{15 \times 14 \times 13}{1 \times 2 \times 3} + \left(\frac{15 \times 14}{1 \times 2}\right)$

Sol.76. (A) Let 'e' be the eccentricity of the given ellipse then that of the hyperbola in reference will be $\frac{1}{e}$.

$$\therefore \quad 3 = 4\left(1-e^2\right) \quad \text{..... (1)} \quad \text{and} \quad b^2 = 64\left(\left(\frac{1}{e}\right)^2 - 1\right) \quad \text{..... (2)}$$

From (1), $4e^2 = 4-3 \Rightarrow e^2 = \frac{1}{4} \Rightarrow \frac{1}{e^2} = 4$ ..... (3)

From (2) and (3), we get $b^2 = 64\left(4-1\right) = 192.$

Sol.77. (A) Given parametric equations of the conic are

$x = -1 + \sec t$

$y = 2 + 3\tan t$

$\Rightarrow x+1 = \sec t, \frac{y-2}{3} = \tan t$

$\Rightarrow (x+1)^2 - \left(\frac{y-2}{3}\right)^2 = 1 \qquad (\because \sec^2 t - \tan^2 t = 1)$

Which is of the form $\frac{X^2}{1^2} - \frac{Y^2}{3^2} = 1$

Where $X = x+1, Y = y-2, a = 1, b-3.$

As $b^2 = a^2\left(e^2-1\right)$, therefore, $3^2 = 1^2\left(e^2-1\right) \Rightarrow \qquad e^2 = 10 \Rightarrow e = \sqrt{10}$

Foci of the conic are given by

$$X = \pm ae, Y = 0 \Leftrightarrow x+1 = \pm 1 \times \sqrt{10}, y = 2 = 0 \Rightarrow x = -1 \pm \sqrt{10}, y = 2.$$

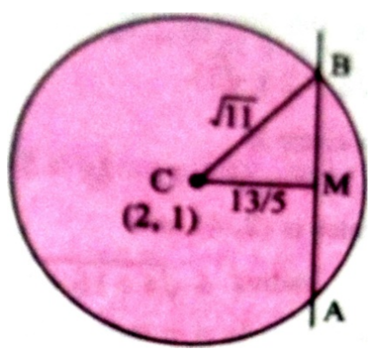
Sol.78. (C) For the ellipse in reference, $2a = 400$ and $b = 10$ and hence, the equation of ellipse can be written as

$\frac{x^2}{(200)^2} + \frac{y^2}{10^2} = 1.$ The point which is 80 m from one end is (200 – 80) m = 120 m from the centre.

Substituting $x = 120$ in the equation, we get

$$\left(\frac{120}{200}\right)^2 + \frac{y^2}{10^2} = 1 \qquad \Rightarrow \qquad y = 8.$$

Sol.79. (D) Equation of the common chord of the two circles is



$x^2 + y^2 + 2x + 6y - \left(x^2 + y^2 - 4x - 2y - 6\right) = 0$

or $6x + 8y + 6 = 0$ or $3x + 4y + 3 = 0$

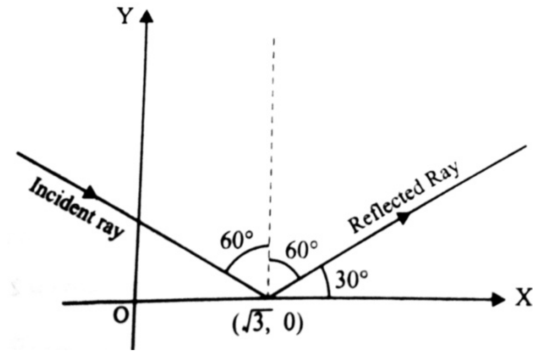
The centre of second circle is (2, 1). Its distance from common chord $= \frac{|6+4+3|}{\sqrt{3^2+4^2}} = \frac{13}{5}.$

Hence, length of common chord is $2\sqrt{11 - \left(\frac{13}{5}\right)^2}.$

Sol.80. (C) Equation of incident ray is

$$x+\sqrt{3}y=\sqrt{3} \quad \text{....(1)}$$

Slop of (1) = $-\frac{1}{\sqrt{3}}=-\tan 30° = \tan(180°-30°)$



$\Rightarrow$ the line (1) makes an angle of 150° with +ve x-axis.

Also the line (1) meets x-axis where y = 0, *i.e.*,

Where $x+0=\sqrt{3} \Rightarrow x=\sqrt{3}$.

So, point of incidence is $(\sqrt{3}, 0)$.

Now, angle of incidence = 150° – 90° = 60°.

$\therefore$ Angle of reflection = 60°.

$\Rightarrow$ Slope of the reflected ray = $\tan 30° = \frac{1}{\sqrt{3}}$.

Hence, equation of the reflected ray is $y-0=\frac{1}{\sqrt{3}}(x-\sqrt{3})$ or $x-\sqrt{3}y-\sqrt{3}=0$.

Sol.81. (A) We have, $\sin^6 x+\cos^6 x=(\sin^2 x)^3+(\cos^2 x)^3$

$=(\sin^2 x+\cos^2 x)^3-3\sin^2 x\cos^2 x(\sin^2 x+\cos^2 x) = 1-3\sin^2 x\cos^2 x$

$= 1-\frac{3}{4}\cdot 4\sin^2 x\cos^2 x = 1-\frac{3}{4}(\sin 2x)^2$

$\Rightarrow$ maximum value of $\sin^6 x + \cos^6 x$ is $1-\frac{3}{4}\times 0=1$

and minimum value is $1-\frac{3}{4}\times 1=\frac{1}{4}$.

Sol.82. (C) We have, $\sin^{50}x - \cos^{50}x = 1$

$\Rightarrow \sin^{50}x = 1 + \cos^{50}x$

Since $\sin^{50} x \le 1$ and $1+\cos^{50} x \ge 1$, therefore, the two sides are equal only if

$\sin^{50} x=1=1+\cos^{50} x$ i.e., $\sin^{50}x = 1$ and $\cos^{50}x = 0$

$\therefore \quad x=n\pi+\frac{\pi}{2}, n\in Z.$

Sol.83. (D) We have, $\left(\frac{1+i}{\sqrt{2}}\right)^8+\left(\frac{1-i}{\sqrt{2}}\right)^8$

$=\left[\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right]^8+\left[\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right]^8$

$=\cos 2\pi+i\sin 2\pi+\cos 2\pi-i\sin 2\pi$

$=2\cos 2\pi=2(1)=2$ [By De-Moivre's theorem]

Sol.84. (B) We have, $2^{\frac{1}{4}} \cdot 4^{\frac{1}{8}} \cdot 8^{\frac{1}{16}} \cdot 16^{\frac{1}{32}} .... = 2^{\frac{1}{4}} \cdot 2^{\frac{2}{8}} \cdot 2^{\frac{3}{16}} \cdot 2^{\frac{4}{32}} ... = 2^{\frac{1}{4}+\frac{2}{8}+\frac{3}{16}+\frac{4}{32}+...}$

Let $S = \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + ...$ $\therefore$ $\frac{1}{2}S = \frac{1}{8} + \frac{2}{16} + \frac{3}{32} + ...$

Subtracting (2) from (1), we get $\frac{1}{2}S = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + ...$

$$\frac{\frac{1}{4}}{1-\frac{1}{2}} = \frac{1}{4} \times \frac{2}{1} = \frac{1}{2}$$

$\therefore$ $S = 1.$ Hence, the given expression $= 2^1 = 2.$

Sol.85. (C) Here $a = 10$, $d = \frac{3}{7}$. Then, $t_n = 10 + (n-1)\left(-\frac{3}{7}\right)$.

t_n is positive if $10 + (n-1)\left(-\frac{3}{7}\right) \geq 0;$

or $70 - 3(n-1) \geq 0$ or $73 \geq 3n;$ or $24\frac{1}{3} \geq n$

$\therefore$ First 24 terms are positive.

$\therefore$ Sum of the positive terms $= S_{24} = \frac{24}{2}\left[2 \times 10 + 23 \times \frac{-3}{7}\right] = 12\left[20 - \frac{69}{7}\right] = \frac{852}{7}.$

Sol.86. (B) Distinct n numbers which can be formed using digits 2, 5 and 7 are 3^n. We have to find n so that

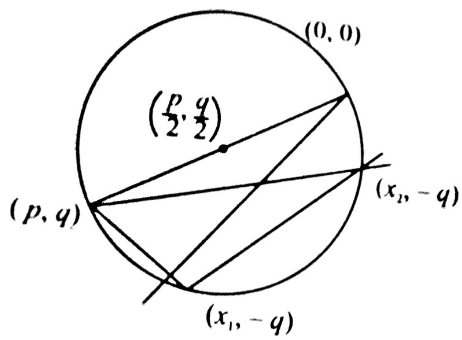
$3^n \geq 900 \Rightarrow 3^{n-2} \geq 100 \Rightarrow n-2 \geq 5 \Rightarrow n \geq 7.$ So, the least value of n is 7.

Sol.87. (D) Given circle is $x^2 + y^2 = px + qy$.

Since the centre of the circle is $\left(\frac{p}{2}, \frac{q}{2}\right)$, so (p, q) and $(0, 0)$ are the end points of a diameter. As the two chords are bisected by x-axis, the chords will cut the circle at the points (x_1, q) and $(x_2, -q)$, where x_1, x_2 are real.

The equation of the line joining these points is $y = -q$.

Solving $y = -q$ and $x^2 + y^2 = px + qy$, we get $x^2 - px + 2q^2 = 0.$



The roots of this equation are x_1 and x_2. Since the roots are real and distinct,

$\therefore$ discriminent > 0 i.e., $p^2 - 8q^2 > 0$ or $p^2 > 8q^2$.

Sol.88. (B) Let $z = x + iy$

$$\therefore \quad \frac{2z+1}{iz+1} = \frac{2(x+iy)+1}{i(x+iy)+1} = \frac{(2x+1)+2iy}{(1-y)+ix} \cdot \left(\frac{1-y-ix}{1-y-ix}\right)$$

$$= \frac{(2x+1)(1-y)+2xy+i\left[-x(2x+1)+2y(1-y)\right]}{(1-y)^2+x^2}$$

$\because$ Imaginary part of $\left(\frac{2z+1}{iz+1}\right) = -2$. $\therefore$ We have, $\frac{-x(2x+1)+2y(1-y)}{(1-y)^2+x^2} = -2$

or $-2x^2 - 2y^2 - x + 2y = -2\left(1+y^2-2y\right) - 2x^2$

i.e., $x + 2y - 2 = 0,$ which is a straight line.

Sol.89. (A) We know that in any triangle the sum of two sides is always greater than the third side.

$\therefore$ The triangle will not be formed if we select segments of lengths (2, 3, 5), (2, 3, 6) or (2, 4, 6).

Hence no. of triangles formed ${}^5C_3 - 3$.

Sol.90. (B) 3 ladies out of 8 can be selected in ${}^8C_3$ ways and s4 gentlemen out of 7 in ${}^7C_4$ ways.

Now each way of selecting 3 ladies is associated with each way of selecting 4 gentlemen.

Hence the required number of ways $= {}^8C_3 \times {}^7C_4 = 56 \times 35 = 1960.$

We now find the no. of committees of 3 ladies and 4 gentlemen in which both Mrs. X and Mr. Y are members. In this case, we can select 2 other ladies from the remaining 7 in ${}^7C_2$ ways and 3 other gentlemen from the remaining 6 in ${}^6C_3$ ways.

$\therefore$ The no. of ways in which both Mrs. *X* and Mr. *Y* are always included $= {}^7C_2 \times {}^8C_3 = 21 \times 20 = 420.$

Hence the required no. of committees in which Mrs. *X* and Mr. *Y* do not serve together

$= 1960 - 420 = 1540$

END OF SOLUTIONS